

Predictive Analysis with Transfer Operators and Analog Methods

Florian Sévellec

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LOPS

http://pageperso.univ-brest.fr/~fsevellec/Teaching/axe_donnees_2019.html



Outline:

- ***Transfer Operator and Markov Chain***

Application to Argo Float Displacements and the Evolution of Intermediate Water Masses

:::Physical Space:::

- ***Transfer Operators and Analog Methods***

Application to the Probabilistic Prediction of Interannual Climate Variations:  **PROCAST**

:::Phase Space:::

Transfer Operator and Markov Chain

Transfer Operator:

The goal is to describe a density (or a probability) at a later time (τ) by its knowledge at the current time (t).

$$\rho(t + \tau) = \mathbf{M}_\tau \rho(t).$$

Hence the solution is given by the knowledge of $\mathbf{M}$, the ***Transfer Operator***.

Markov Chain:

This relation can be reproduced sequentially, assuming the stationarity of $\mathbf{M}$, as a ***Markov Chain***.

$$\rho(t + 2\tau) = \mathbf{M}_\tau \mathbf{M}_\tau \rho(t),$$

.

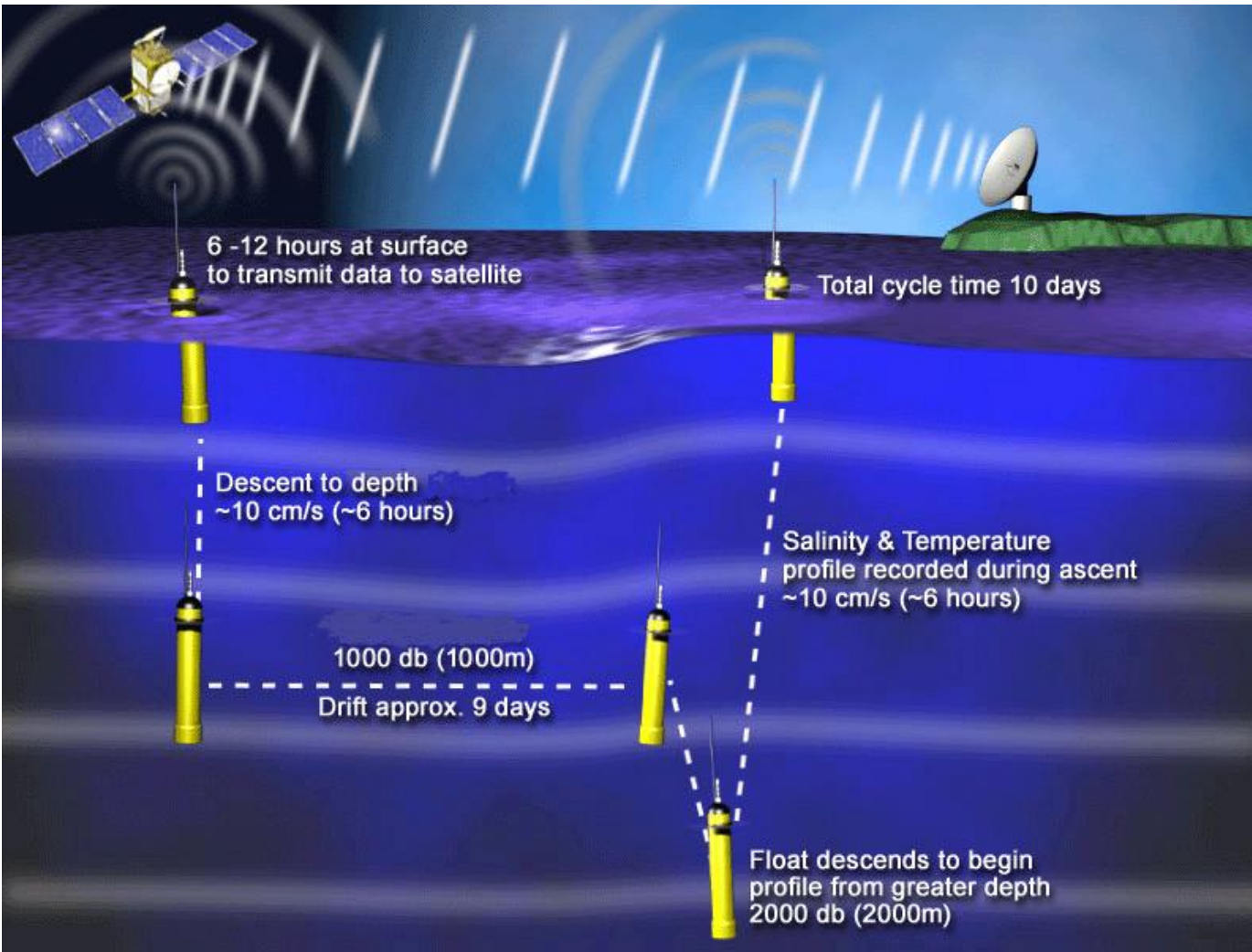
.

$$\rho(t + k\tau) = \mathbf{M}_\tau^k \rho(t).$$



Transfer Operator and Markov Chain

Application to Argo Float Displacements:



Scientific Question:

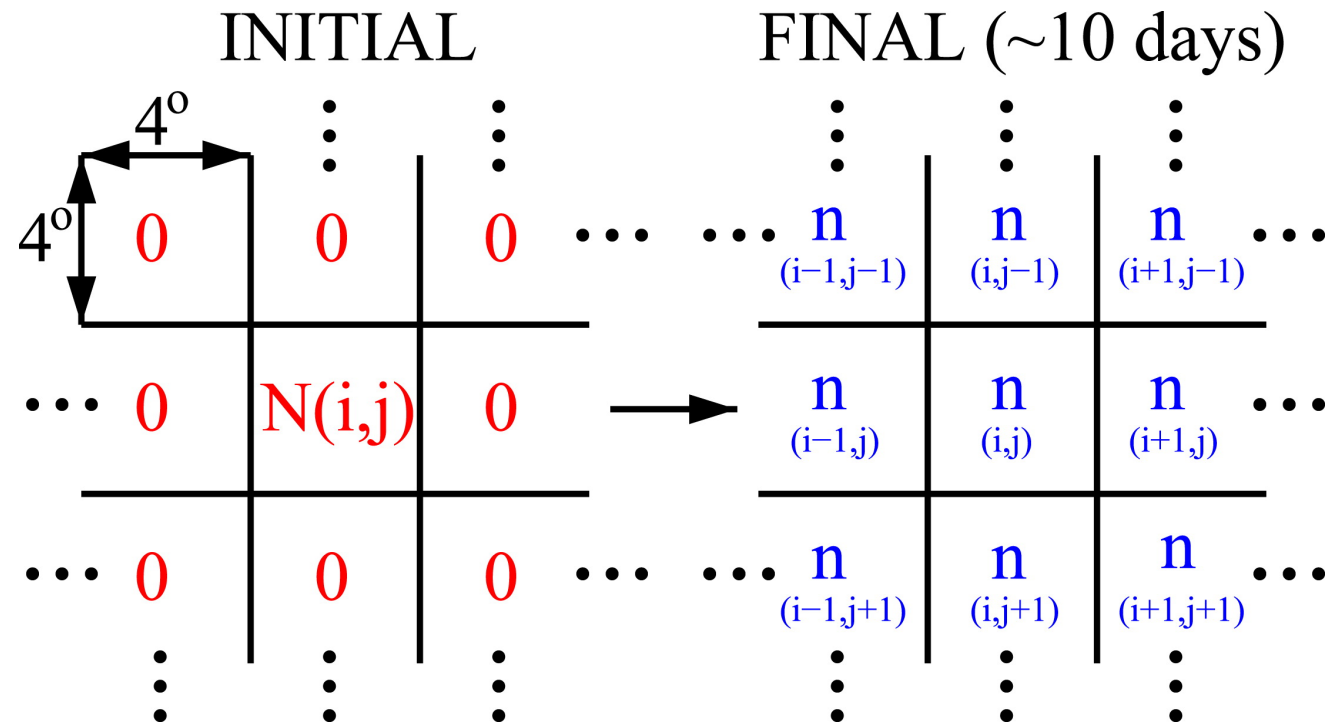
What is the fate of Intermediate Water Masses?

Specific Challenge:

Can we identify the behavior of Intermediate Water Masses using the Argo float displacement during their 10-day journey at 1,000-m depth?

Transfer Operator and Markov Chain

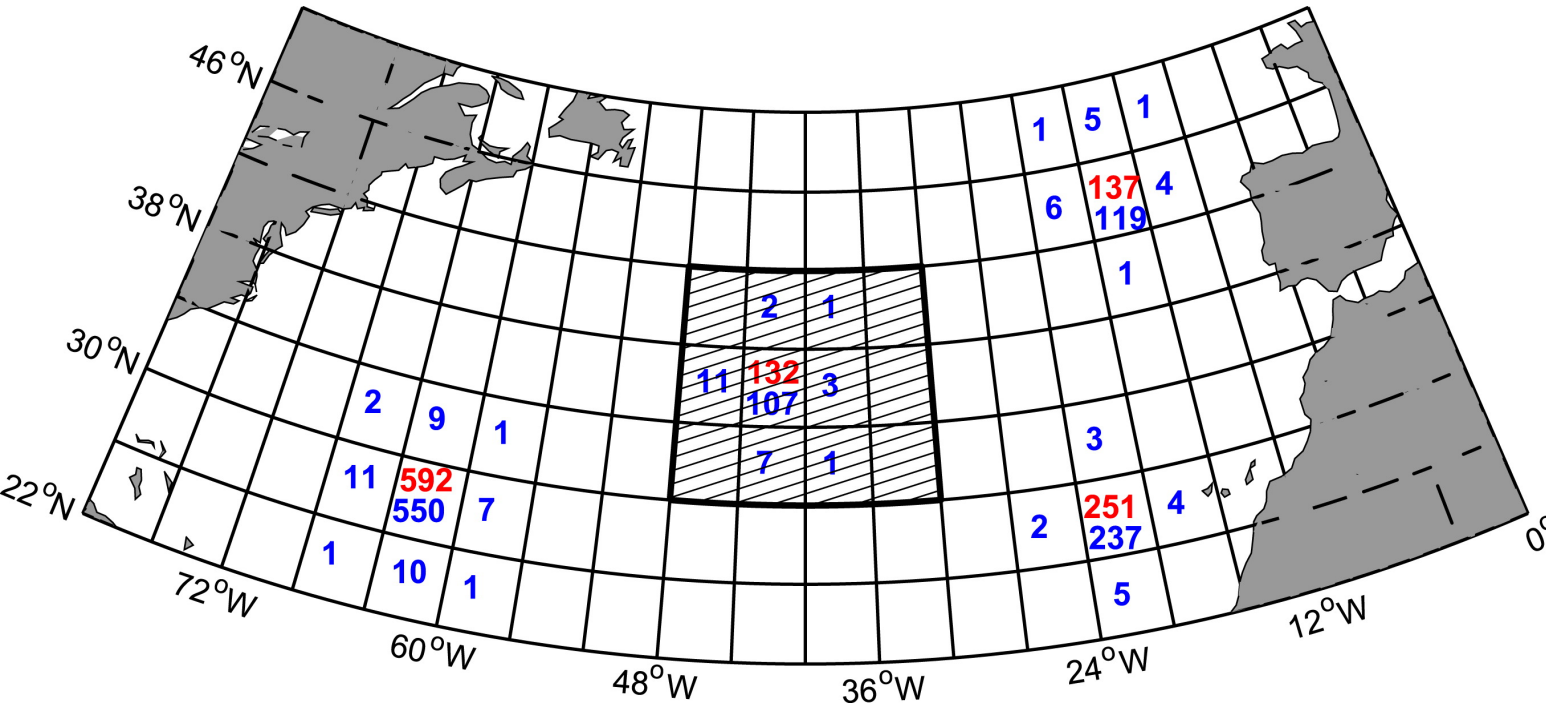
Building the Transfer Operator:



Hence, one could define a probability of transition as n/N

Transfer Operator and Markov Chain

- *Example of probability computation:*



$$\mathbf{M}_{i_0, j_0} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2/132 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/132 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 11/132 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 107/132 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3/132 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 7/132 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/132 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Reproducing this transfer probability for all possible locations $\{i_0, j_0\}$, the transfer operator can be defined as:

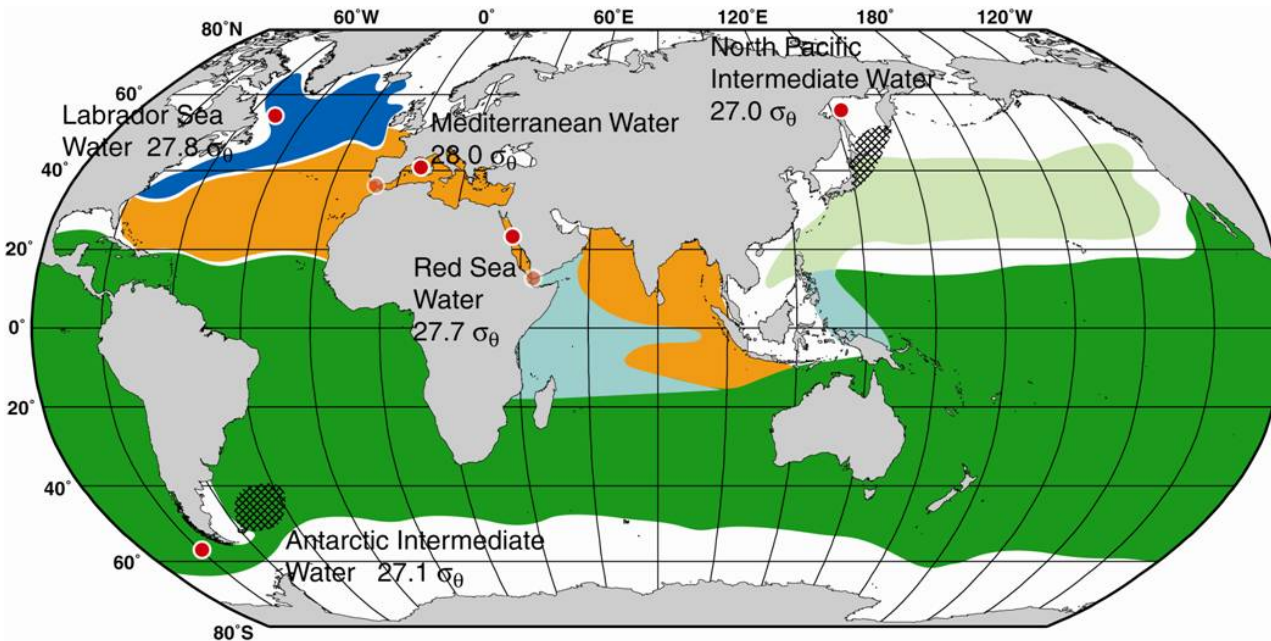
$$\mathbf{M} = \sum_{\{i_0, j_0\}} \mathbf{M}_{i_0, j_0}.$$

- The Transfer Operator contains all the possible journey from one location to an other.
- The non-stationary deterministic journeys of individual Argo float has been transformed into a stationary probabilistic transfer of Argo float density.

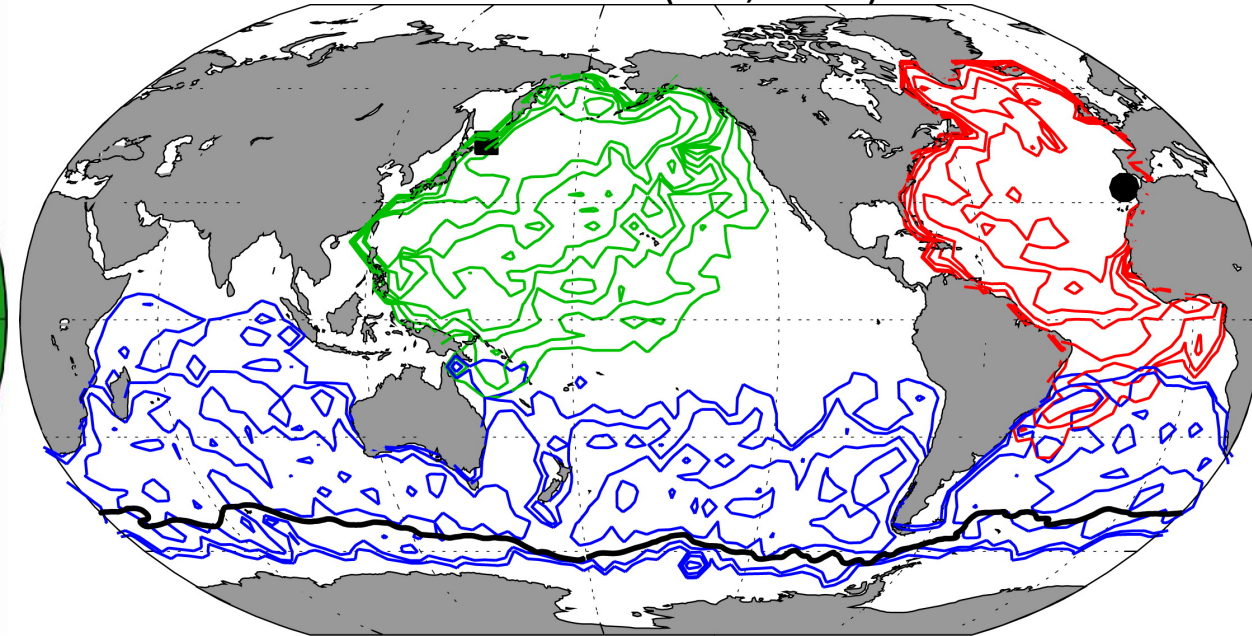
Transfer Operator and Markov Chain

The fate of Intermediate Water Masses:

Talley et al, descriptive Physical Oceanography



Sévellec et al (JPO, 2017)



Not bad...

Transfer Operator and Markov Chain

Intermediate Conclusions

- We have been able to track Intermediate Water Masses based on observations of Argo float journey at depth.
- The uncertainty on the exact location (and time) of the journey might be problematic to define the Markov Chain...
 - The 4x4 grid seems robust enough to define statistically the Argo float fate,
 - but this issue will be more problematic in the next example...

Transfer Operators and Analog Methods

Scientific Context:

- Attributing “forced” vs “natural” variations.
- On interannual to decadal timescales the “natural” variations controlled the GMT changes

Scientific Questions:

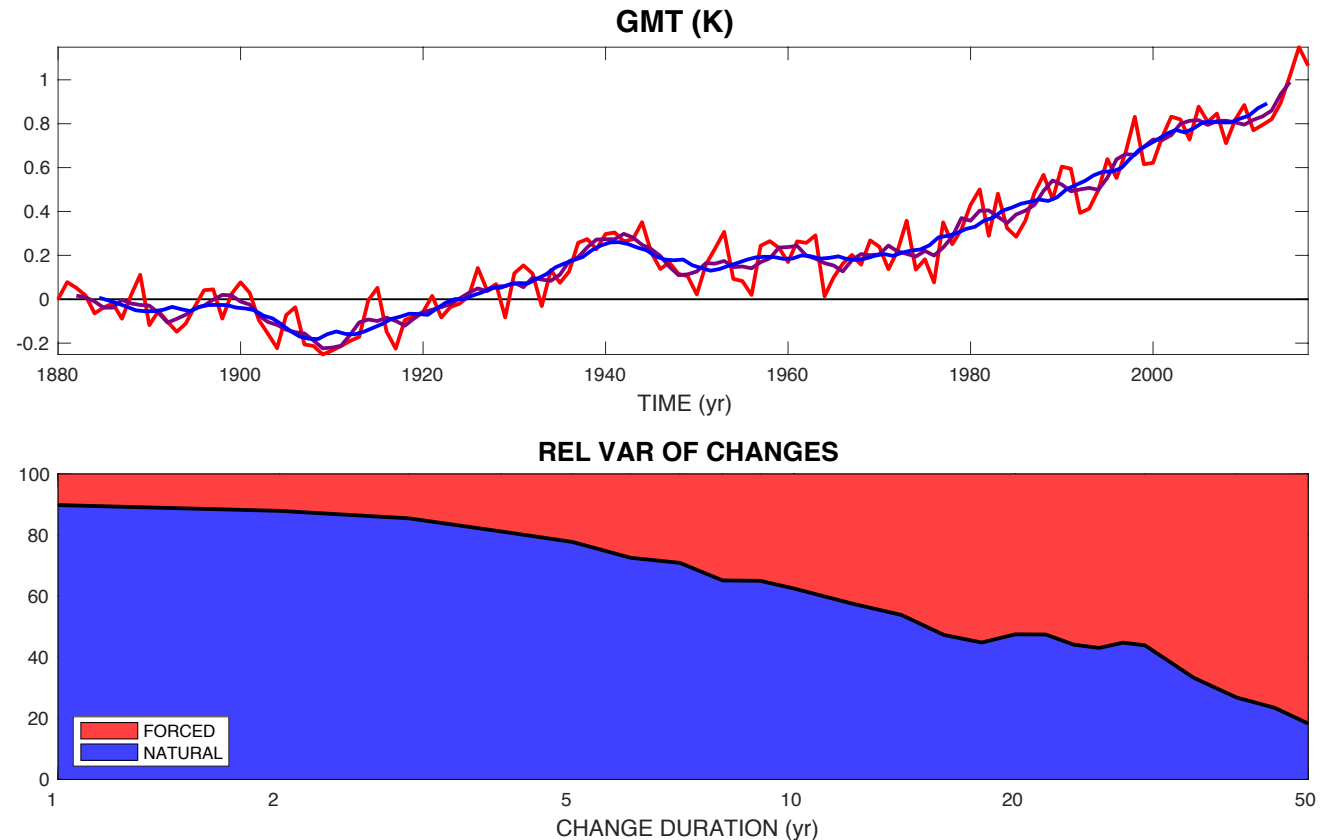
Methodologic

Can we predict global warming variations years ahead?

Pragmatic

What will be the temperature in 2018-2022?

The Global Mean Temperature over the last ~century

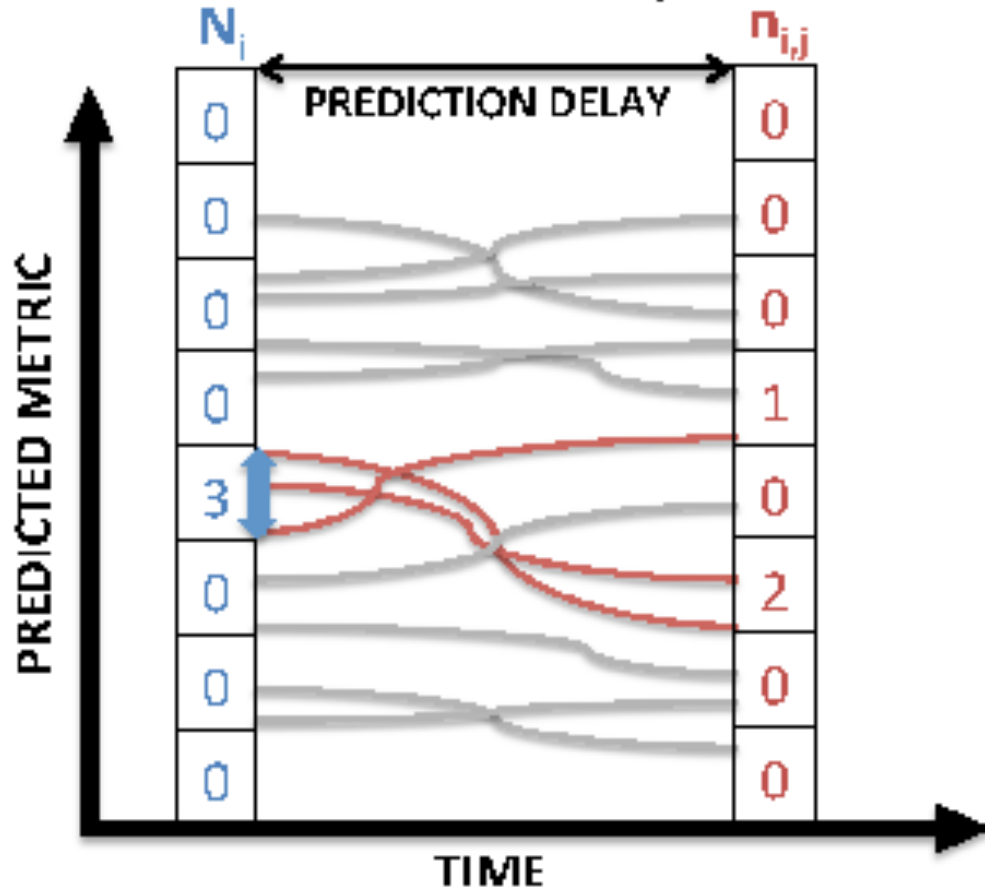


Transfer Operators and Analog Methods

Method: PRObabilistic forecast System

PROCAST

Schematic of the Transfer Operator Method



- The principle is the same as before.
- We use the CMIP5 database as the sudo-observations to train the Transfer Operator.
- Because we focus on a dramatically restricted sub-phase space (1D!), we cannot apply a Markov Chain...
 - Hence we built a different Transfer Operator for different Prediction Delay (τ).
 - The principle of PROCAST becomes fundamentally equivalent than an Analog Method.

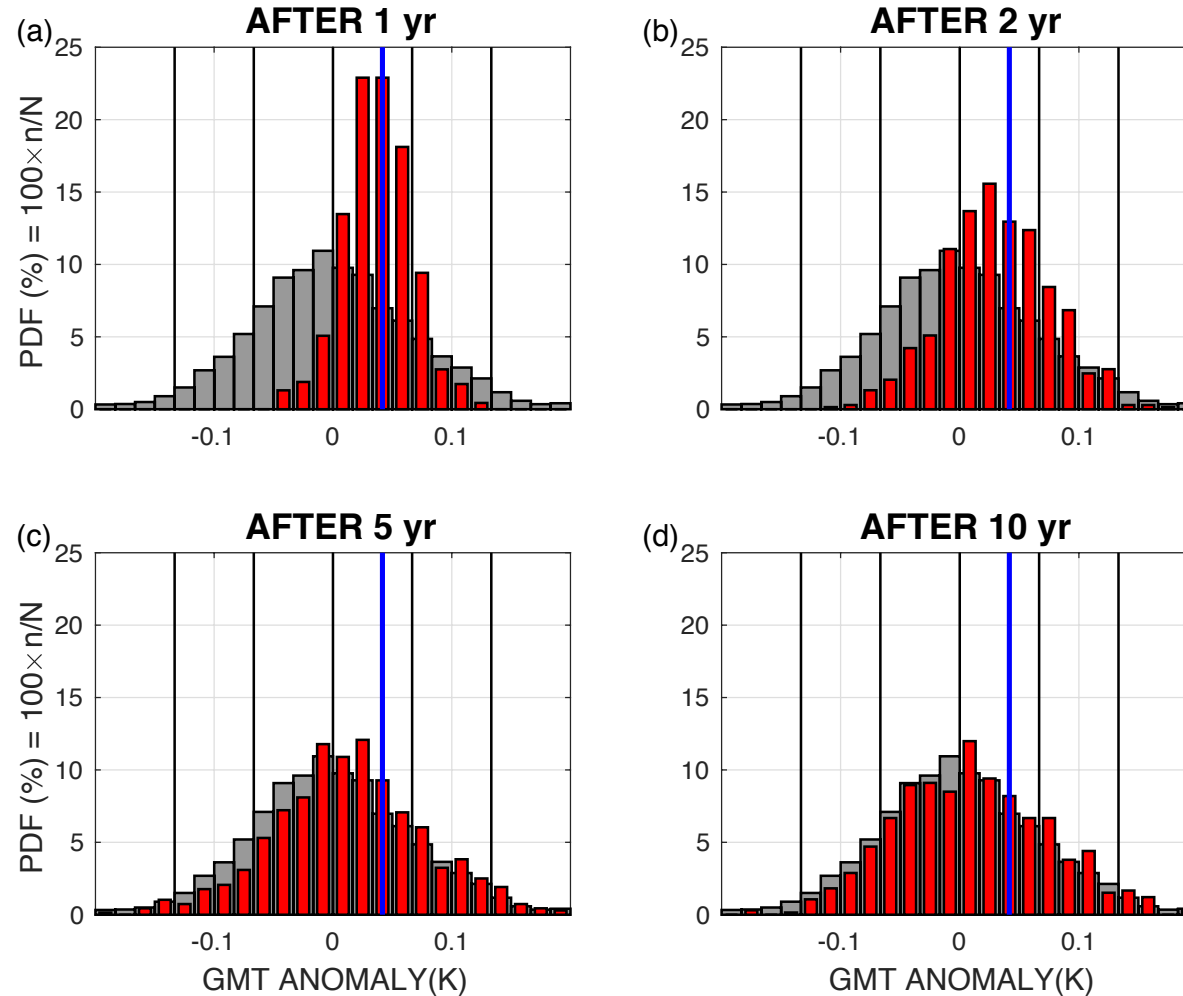
$$\rho(t + k\tau) = \mathbf{M}_{k\tau}\rho(t) \neq \mathbf{M}_{\tau}^k\rho(t).$$

Markov chain is ill-posed if the uncertainty in the “location” propagates faster than the information... A 1D subspace is unequivocally ill-posed...

Transfer Operators and Analog Methods

Example of the probability density function evolution

EXAMPLE OF A 5-yr AVERAGE PROBABILISTIC TRANSITION



Transfer Operators and Analog Methods

PROCAST Hindcast performance

Prediction Skill:

$$\text{Coeff. Det.} = R^2 = 1 - \frac{\overline{\left(\overline{x_i p_i(t)}^i - o(t) \right)^2}^t}{\overline{o(t)^2}^t}.$$

~30%

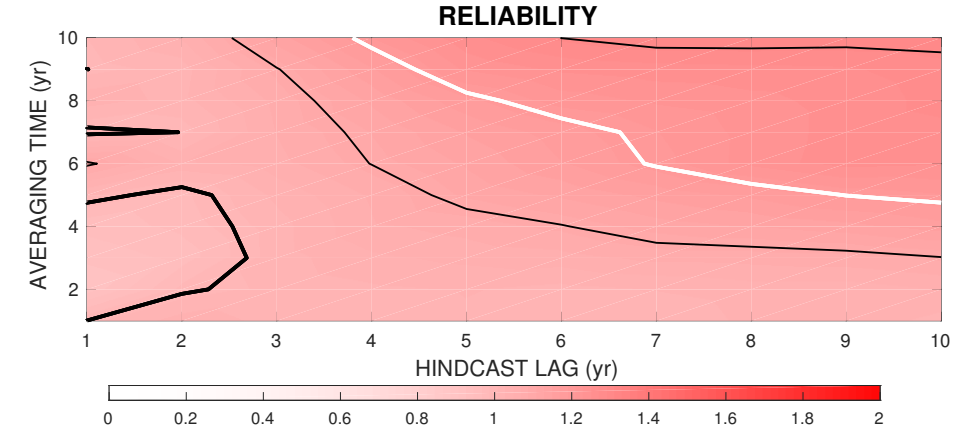
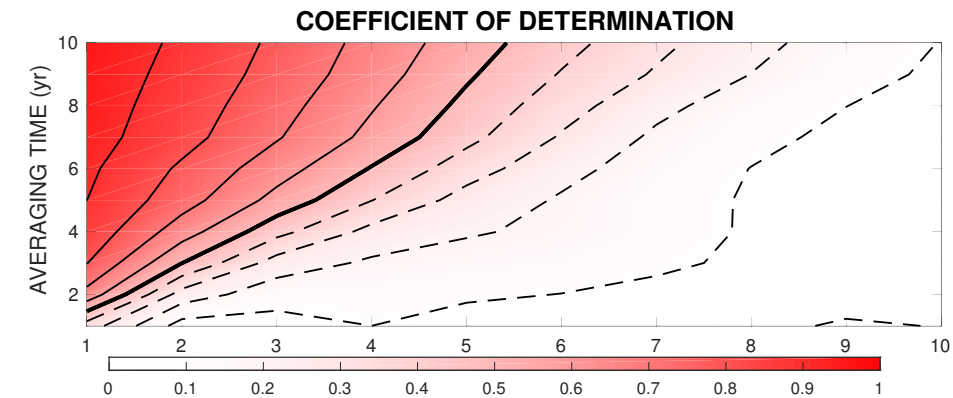
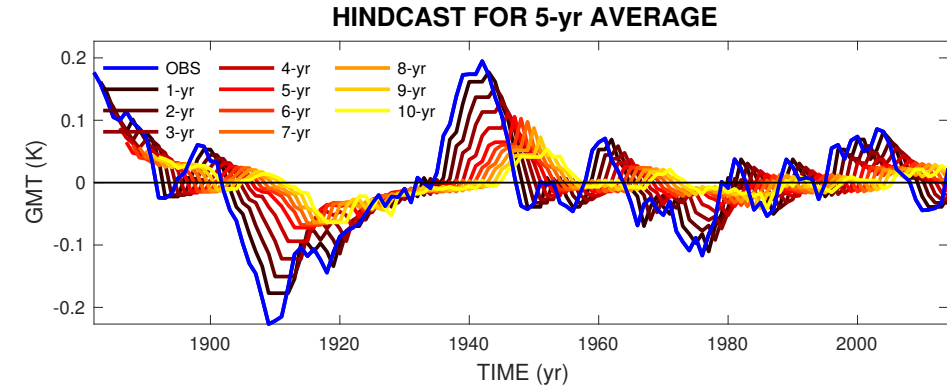
Acceptable skill

Prediction Reliability:

$$\text{Fiabilité} = \sqrt{\left\{ \frac{\overline{\left(\overline{x_i p_i(t)}^i - o(t) \right)^2}^t}{\left[\overline{x_i - \overline{x_i p_i(t)}^i \right]^2 \overline{p_i(t)}^i} \right\}^t}.$$

1

Good statistical prediction of events



Transfer Operators and Analog Methods

Skill Comparison: PROCAST vs DePreSys3 (UK)

Skill on GMT prediction

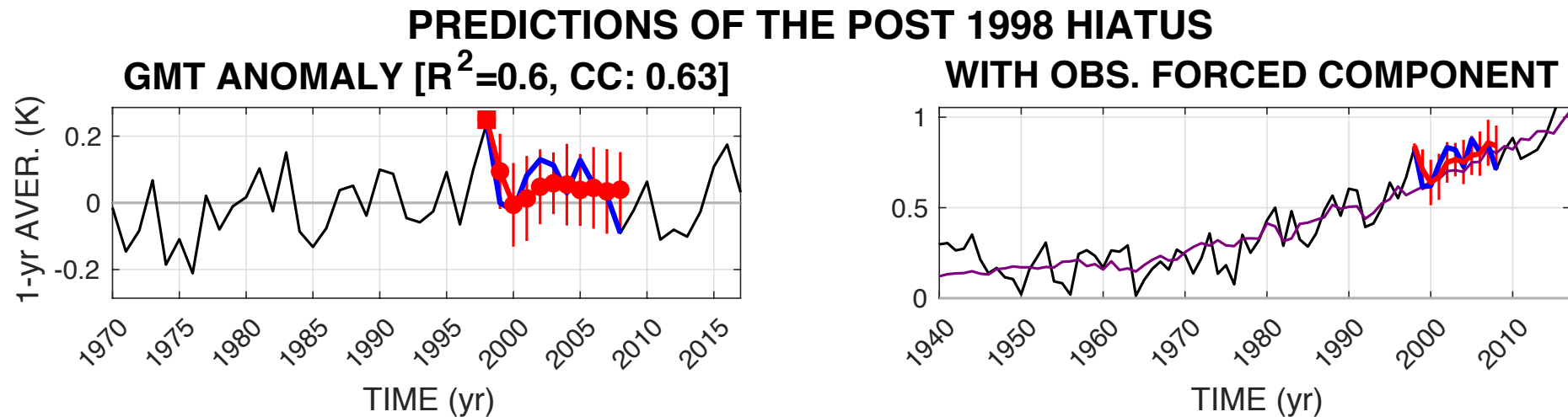
- 1-5-yr error
 - PROCAST – 0.104 K
 - DePreSys – 0.151 K
- Reliability
 - PROCAST – 1
 - DePreSys – 2.3
- Computational Performance
 - PROCAST – 22 ms on a laptop
 - DePreSys – 1 week on a supercomputer

Intrinsic Limits

PROCAST do not yet predict where (region) and when (season) predicted GMT changes will manifest...

Transfer Operators and Analog Methods

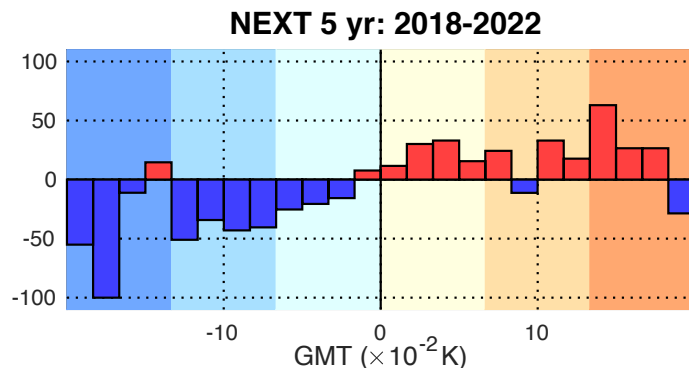
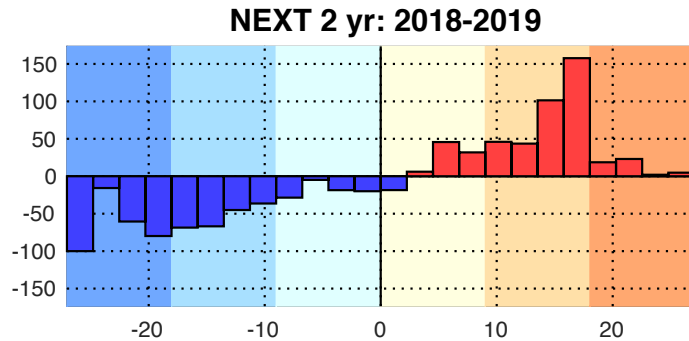
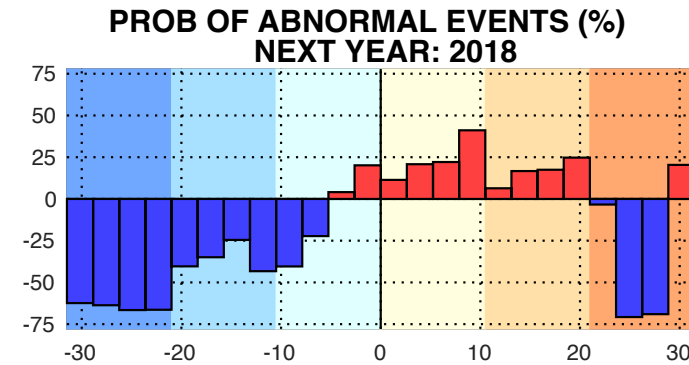
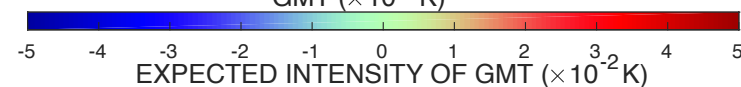
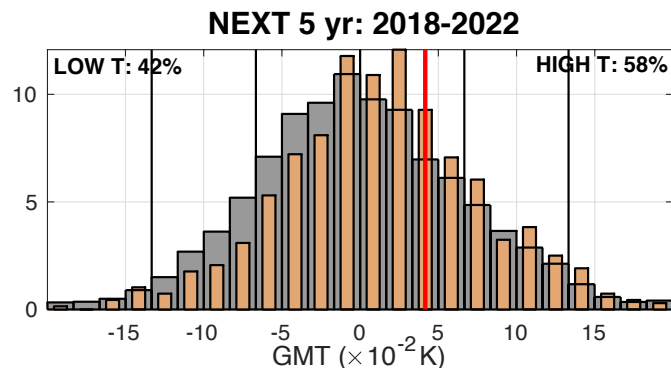
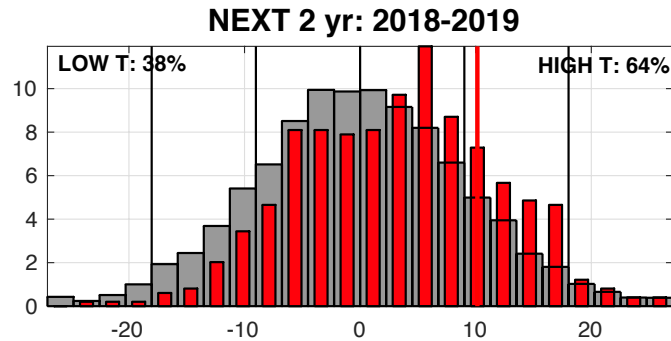
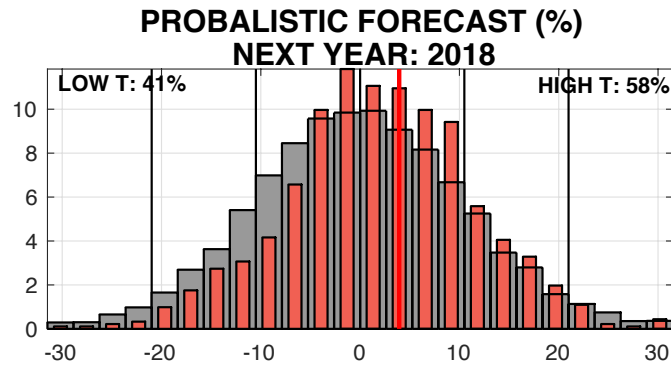
Skill Example: The global warming Hiatus



- Good qualitative and quantitative prediction of the post-1998 hiatus of global warming.
- In hindsight, the post-1998 hiatus of global warming was predictable by PROCAST.

Transfer Operators and Analog Methods

*Prediction
for 2018-
2022*



Transfer Operators and Analog Methods

Final Conclusions

- Method
 - Fast and efficient method to predict GMT based on a range of Transfer Operators or Analog based on CMIP5 database
 - The dramatically reduced phase-space does not allow for Markov Chain.
- Result

Toward anomalously warm years beyond the forced global warming

Literature

Transfer Operators:

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