

$$\cancel{D_t u}^{(3)} = f v = -\frac{1}{\rho_0} \partial_x P + \cancel{V_{isc}}^{(4)} \quad (1)$$

$$\cancel{D_t v}^{(3)} = f u = -\frac{1}{\rho_0} \partial_y P + \cancel{V_{isc}}^{(4)} \quad (1)$$

$$\partial_z P = -\rho g$$

$$\partial_x u + \partial_y v + \partial_z w = 0$$

$$\cancel{D_t \phi}^{(4)} = \cancel{Diff}^{(2)}$$

low freq

$$Re \gg 1 \quad (1)$$

low shift

$$Pe \gg 1 \quad (2)$$

large scale

$$Ro \ll 1 \quad (3)$$

equilibrium

$$\partial_t \rightarrow 0 \text{ or } t \rightarrow \infty \quad (4)$$

$$-f v = -\frac{1}{\rho_0} \partial_x P$$

$$f u = -\frac{1}{\rho_0} \partial_y P$$

$$\partial_z P = -\rho g$$

$$\partial_x u + \partial_y v + \partial_z w = 0$$

$$u \partial_x \phi + v \partial_y \phi + w \partial_z \phi = 0 \rightarrow \text{two equations relating } w \text{ to the rest.}$$

$$-f \partial_z v = \frac{g}{\rho_0} \partial_x \phi \quad (1)$$

$$f \partial_z u = \frac{g}{\rho_0} \partial_y \phi \quad (2)$$

$$u \partial_x \phi + v \partial_y \phi + w \partial_z \phi = 0 \quad (3)$$

$$w = f \frac{R^2}{N^2} \partial_z \phi$$

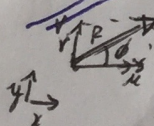
(*) & (**) →

$$-f u \partial_z v + f v \partial_z u - w N^2 = 0$$

$$w = \frac{f}{N^2} (v \partial_z u - u \partial_z v) \quad (*)$$

$$u = R \cos \phi$$

$$v = R \sin \phi$$



$$w = \frac{f}{N^2} \partial_z \phi$$

$$\partial_z (\cos \phi) = -\sin \phi \partial_z \phi$$

$$\partial_z (\sin \phi) = \cos \phi \partial_z \phi$$

$$\begin{aligned} -u \partial_z v &= -R \cos \phi \left[\partial_z R \sin \phi + R \partial_z \sin \phi \right] \\ v \partial_z u &= R \sin \phi \left[\partial_z R \cos \phi - R \partial_z \cos \phi \right] \end{aligned} \Rightarrow v \partial_z u + u \partial_z v = -R \partial_z R \sin \phi \cos \phi - R^2 \partial_z \sin \phi \cos \phi + R \partial_z R \sin \phi \cos \phi - R^2 \partial_z \sin \phi \cos \phi = -R^2 \partial_z (\cos^2 \phi + \sin^2 \phi) = -R^2 \partial_z \phi$$