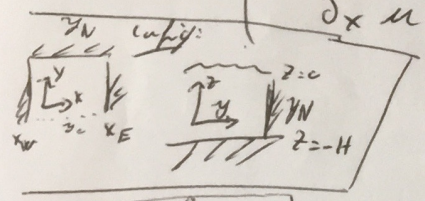


Eqn 1:

$$\partial_t T + u \partial_x T + v \partial_y T + w \partial_z T = \partial_x (K_x \partial_x T) + \partial_y (K_y \partial_y T) + \partial_z (K_z \partial_z T)$$

$$\partial_x u + \partial_y v + \partial_z w = 0$$



$$\hookrightarrow \partial_t T + \partial_x (uT) + \partial_y (vT) + \partial_z (wT) = \dots$$

$$\bar{T} = \frac{1}{H(x_e - x_w)} \int_{x_w}^{x_e} \int_{y_w}^{y_e} \int_{-H}^0 T dx dy dz$$

$$\partial_t \bar{T} = ?$$

$$\partial_t \bar{T} + \frac{1}{x_e - x_w} \left[\left(\overline{uT} \right) \Big|_{x_w}^{x_e} - \left(\overline{uT} \right) \Big|_{x_w}^{x_e} \right] + \partial_y (\overline{vT}) + \partial_z (\overline{wT})$$

$$= \frac{1}{x_e - x_w} \left[\left(\overline{K_x \partial_x T} \right) \Big|_{x_w}^{x_e} - \left(\overline{K_x \partial_x T} \right) \Big|_{x_w}^{x_e} \right] + \partial_y (\overline{K_y \partial_y T}) + \partial_z (\overline{K_z \partial_z T})$$

BC:
(*) Solid: $u=0$
(**) Adiabatic: $K_n \partial_n T = 0$

$$\partial_t \bar{T} + \partial_y (\overline{vT}) + \frac{1}{H} \left[\left(\overline{wT} \right) \Big|_{z=0}^{z=0} - \left(\overline{wT} \right) \Big|_{z=-H}^{z=-H} \right]$$

$$= \partial_y (\overline{K_y \partial_y T}) + \frac{1}{H} \left[\left(\overline{K_z \partial_z T} \right) \Big|_{z=0}^{z=0} - \left(\overline{K_z \partial_z T} \right) \Big|_{z=-H}^{z=-H} \right]$$

Surface B.C:

$$\frac{1}{H} K_z \partial_z T = F(x, y)$$

$$\partial_t \bar{T} + \partial_y (\overline{vT}) = \partial_y (\overline{K_y \partial_y T}) + F$$

$$\partial_t \bar{T} + \frac{1}{y_e - y_w} \left[\left(\overline{vT} \right) \Big|_{y_w}^{y_e} - \left(\overline{vT} \right) \Big|_{y_w}^{y_e} \right] - \frac{1}{y_e - y_w} \left[\left(\overline{K_y \partial_y T} \right) \Big|_{y_w}^{y_e} - \left(\overline{K_y \partial_y T} \right) \Big|_{y_w}^{y_e} \right] = F$$

$$\partial_t \bar{T} = + \frac{1}{y_e - y_w} \left(\overline{vT} \right) \Big|_{y_w}^{y_e} - \frac{1}{y_e - y_w} \left(\overline{K_y \partial_y T} \right) \Big|_{y_w}^{y_e} + F$$

if $Pe \ll 1$, (Adv $\gg$ diff)

$$\partial_t \bar{T} = + \frac{1}{y_e - y_w} \left(\overline{vT} \right) \Big|_{y_w}^{y_e} + F$$

$$\overline{VT}^{x,z} = ?$$

(2/2)

$$V = \overline{V}^x + V' \quad , \quad T = \overline{T}^x + T'$$

$$V = \overline{V}^z + V'' \quad , \quad T = \overline{T}^z + T''$$

$$\overline{VT}^x = \overline{V}^x \overline{T}^x + \overline{V' T'}^x$$

$$\begin{aligned} \overline{VT}^{xz} &= \overline{\overline{V}^x \overline{T}^x}^z + \overline{V' T'}^{xz} \\ &= \overline{\overline{V}^x \overline{T}^x}^z + \overline{V' T'}^{xz} \end{aligned}$$

Mass is conserved

$$\overline{VT}^{xz} = \overline{\overline{V}^{x''} \overline{T}^{x''}}^z + \overline{V' T'}^{xz}$$

Finally

$$\frac{\partial}{\partial t} \overline{VT}^{x,y,z} = + \frac{1}{y_N - y_0} \left[\overline{\overline{V}^{x''} \overline{T}^{x''}}^z + \frac{1}{y_N - y_0} \overline{V' T'}^{xz} + \overline{F}^{xy} \right]$$

AMOC change in a thermally stratified fluid

$\frac{\partial}{\partial t} \overline{VT}^{xyz} \rightarrow 0$

$\overline{VT}^{xyz} \rightarrow$

Hence

$$\overline{VT}^{xyz} \Big|_t = \overline{VT}^{xyz} \Big|_{t=t_0} + \frac{1}{y_N - y_0} \int_{t_0}^t \left[\overline{\overline{V}^{x''} \overline{T}^{x''}}^z + \frac{1}{y_N - y_0} \overline{V' T'}^{xz} + \overline{F}^{xy} \right] dt$$

deficit in the AMOC