

Set of eqns

$\vec{D}u \rightarrow 0$

$$-fv = -\frac{1}{\rho} \partial_x P + \nu \partial_{xx} u \quad (1)$$

$$R_0 \ll 1 \quad \left\{ \begin{array}{l} |f u| \gg \nu \partial_{xx} u \\ |f v| \gg \nu \partial_{xx} v \end{array} \right. \quad \left\{ \begin{array}{l} \partial_x u \\ \partial_x v \end{array} \right.$$

$\vec{D}v \rightarrow 0$

$$fu = -\frac{1}{\rho} \partial_y P + \nu \partial_{yy} v \quad (2)$$

$$\partial_z P = -\rho g \quad (3)$$

$$\partial_x u + \partial_y v + \partial_z w = 0 \quad (4)$$

zonally averaged $\Rightarrow \bar{X} = \frac{1}{W} \int_{x_w}^{x_E} X dx, \forall X$

$\frac{d}{dz} \bar{X} = \frac{1}{W} \left(\bar{X} \frac{dW}{dz} \right)$

$\bar{W} = \int_{x_w}^{x_E} dx = (x_E - x_w)$

$$(4) \Rightarrow \partial_y \bar{v} + \partial_z \bar{w} = 0 \quad \text{with} \quad \bar{u}|_{x_w} = \bar{u}|_{x_E} = 0$$

$$(1) \& (3) \Rightarrow f \partial_z v = -\frac{g}{\rho} \partial_x \rho$$

$$(4) \Rightarrow f \partial_z \bar{v} = -\frac{g}{\rho} \partial_x \bar{\rho}$$

$$f \partial_z \bar{v} = -\frac{g}{\rho W} (\rho|_{x_E} - \rho|_{x_w})$$

$$f \partial_z \bar{v} = \frac{g}{\rho W} (\rho|_{x_w} - \rho|_{x_E})$$

$[\partial_y \bar{v} + \partial_z \bar{w} = 0]$ allows us to define a stream function such as

$$\bar{v} = -\frac{1}{W} \partial_z \psi \quad \& \quad w = \frac{1}{W} \partial_y \psi$$

So, if $f \neq 0$,

$$\partial_z^2 \psi = \frac{g}{f \rho} (\rho|_{x_E} - \rho|_{x_w})$$

$$\partial_z \psi = A + \int_z^0 \frac{g}{f \rho} (\rho|_{x_E} - \rho|_{x_w}) dz$$

(24)

$$\partial_z \psi = A + \int_z^0 \frac{g}{f\ell_0} (\rho|_{z_E} - \rho|_{z_W}) dz'$$

$$\psi = -Az + B + \int_z^0 \int_{z'}^0 \frac{g}{f\ell_0} (\rho|_{z_E} - \rho|_{z_W}) dz'' dz' \quad (*)$$

We need two conditions ...

$$\psi|_{z=-H} = 0 \quad \text{and} \quad \psi|_{z=0} = ?$$

or $\psi|_{z=-H} = \psi_0$ if $\ell_0 > 0$

Now divergence:

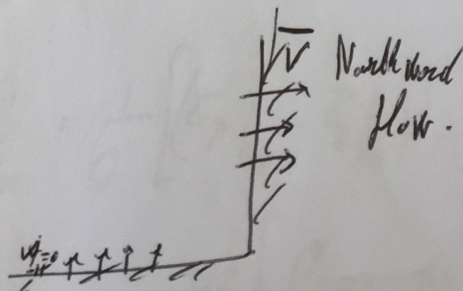
$$\partial_y \bar{v} + \partial_z \bar{w} = 0$$

$$\int_{-H}^0 \partial_y \bar{v} dz + (\bar{w}|_0 - \bar{w}|_{-H}) = 0$$

$$\int_{-H}^0 \bar{v}|_{y_1} dz - \int_{-H}^0 \bar{v} dz + \int_{y_1}^0 \bar{w}|_0 dy = 0$$

$$\int_{-H}^0 \bar{v} dz = \int_{-H}^0 \bar{v}|_{y_1} dz + \int_{y_1}^0 \bar{w}|_0 dy //$$

$$\begin{aligned} \int_{-H}^0 \bar{v} dz &= -\frac{1}{W} \int_{-H}^0 \partial_z \psi dz \\ &= -\frac{1}{W} [\psi|_0 - \psi|_{-H}] \end{aligned}$$



$$\psi|_{z=0} = \psi|_{z=0, y=y_1} - W \int_{y_1}^0 \bar{w}|_0 dy = 0$$

How much flow get out barotropically in the North

What is $\overline{W}|_0$?

Ethman layer!
Ethman pumping
 $\frac{c}{h_0} \left(\frac{1}{2} \frac{1}{h_0} \frac{1}{2} \right)$

In the vicinity
of the surface
visc. is important as
geostrophy

$$\begin{cases} -f \frac{1}{\rho} = -\frac{1}{\rho} \partial_x p + \nu \partial_z^2 u \\ f u = -\frac{1}{\rho} \partial_y p + \nu \partial_z^2 v \\ \partial_x u + \partial_y v + \partial_z w = 0 \end{cases}$$

$$\int_{h_0}^0 (\partial_x u + \partial_y v) dz = -\frac{1}{\rho} \frac{f}{h_0} \frac{1}{2} + \frac{w}{h_0}$$

($f=0$)

$$\frac{w}{h_0} = \frac{1}{f} \int_{h_0}^0 -\frac{1}{\rho} (\partial_x \partial_y p + \partial_y \partial_x p) + \nu \partial_z^2 (\partial_x u + \partial_y v) dz$$

$$f \frac{w}{h_0} = \partial_x \left[\nu \partial_z^2 v \right]_{h_0}^0 - \partial_y \left[\nu \partial_z^2 u \right]_{h_0}^0$$

No momentum transfer at the base of Ethman
layer

$$\nu \partial_z^2 \left(\frac{u}{v} \right)_{h_0} = 0$$

$$\begin{cases} \nu \partial_z^2 v = \frac{1}{\rho} \tau_y \\ \nu \partial_z^2 u = \frac{1}{\rho} \tau_x \end{cases}$$

$$f \frac{w}{h_0} = \frac{1}{\rho} (\partial_x \tau_y - \partial_y \tau_x)$$

$$f \overline{w}|_{h_0} = \frac{1}{\rho} \left[(\overline{\tau_y}|_{x_0} - \overline{\tau_x}|_{y_0}) - \partial_y \overline{\tau_x} \right]$$

$$\psi|_{z=0} = \psi|_{x=y=h_0} - \frac{w}{f \rho} \left[\int_{y_0}^{y_1} (\overline{\tau_y}|_{x_0} - \overline{\tau_x}|_{y_0}) - \overline{\tau_x}|_{y=y_1} + \overline{\tau_x} \right]$$

$$\Rightarrow \psi|_{z=0} = D$$

(4.14)

$$\psi = -Az + \psi|_{z=0} + \int_z^0 \int_{z'}^0 \frac{g}{f\rho_0} (\rho|_{z_E} - \rho|_{z_W}) dz'' dz'$$

$$\psi|_{-H} = AH + \psi|_{z=0} + \int_{-H}^0 \int_{z'}^0 \frac{g}{f} (\rho|_{z_E} - \rho|_{z_W}) dz'' dz'$$

$$\psi|_{-H} = 0$$

$$A = -\frac{1}{H} \psi|_{z=0} - \frac{1}{H} \int_{-H}^0 \int_{z'}^0 \frac{g}{f} (\rho|_{z_E} - \rho|_{z_W}) dz'' dz'$$

$$\psi = \psi|_{z=0} \left(\frac{z+H}{H} \right) + \frac{z}{H} \int_{-H}^0 \int_{z'}^0 \frac{g}{f} (\rho|_{z_E} - \rho|_{z_W}) dz'' dz' + \int_z^0 \int_{z'}^0 \frac{g}{f} (\rho|_{z_E} - \rho|_{z_W}) dz'' dz'$$

$$\hookrightarrow \frac{z}{H} \langle P \rangle + P$$

$$\text{if } z=0$$

$$0 \langle P \rangle + 0 = 0$$

$$\text{if } z=-H$$

$$-\frac{H}{H} \langle P \rangle + P = 0$$

where $\psi|_{z=0}$ is given by wind & barotropic outflow.