

Global Observations of Eddy–Mean Flow Interactions at the Ocean Surface and in the Deep Ocean

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ABSTRACT: Mesoscale turbulence is ubiquitous in the ocean and is particularly active in regions of intense mean currents. The description and understanding of ocean interactions between the mesoscale turbulence and the mean flow are thus one of the current timely questions of ocean sciences. Here, we use the surface and deep (~ 1000 -m depth) Argo displacements to characterize the eddy–mean flow interactions and their properties. The surface eddy–mean flow interactions are almost two orders of magnitude higher than its deep counterpart. They are mostly located near western-intensified boundary currents and in the Antarctic Circumpolar Current. Strong interactions also exist along the equator but only at the surface. Following a kinetic energy (KE) framework, we show that the work of the Reynolds stress acting on the mean KE can be decomposed into a local term and a nonlocal term. These two terms mainly compensate each other at 2° scale, both at the surface and at depth. However, the nonlocal term vanishes at large scale (of several 10°) in the deep ocean. The local term corresponds to conversion from mean KE to eddy KE, related to strain instability of the mean flow. The nonlocal term is a redistribution of KE. We show that it can be reinterpreted as a redistribution of eddy KE by the modified mean flow, where the modified mean flow is the mean flow stretched and squashed along the turbulence ellipsoid. These properties provide theoretical and observational ground for a better understanding of the ocean mesoscale turbulence impact on the mean flow.

KEYWORDS: Ocean; Large-scale motions; Mesoscale processes; Momentum; Ocean circulation; In situ oceanic observations

1. Introduction

Because of its relatively large heat capacity, the ocean is a key component of the climate system. It modulates global warming by capturing 90% of heat excess (IPCC 2021; von Schuckmann et al. 2023) and acts on a wide range of time scales, from inertial motions to secular variations (Dijkstra 2000). In particular, ocean mesoscale eddies are ubiquitous in the ocean (Chelton et al. 2007). Hence, in boundary currents and in the Antarctic Circumpolar Current (ACC), permanent large-scale flow coexists with transient mesoscale eddies (e.g., Sévellec et al. 2024). In these regions, eddies contribute to the large-scale circulation of tracers in two folds. First, they advect tracer properties as an eddy-induced advection (Gent and McWilliams 1990) and contribute to the residual circulation (Marshall and Radko 2003) as observed in the Southern Ocean (e.g., Sévellec et al. 2019). Second, they spread tracer through their stirring and act as isopycnal diffusion (Redi 1982), as recently estimated from observations of deep Argo float displacements by Sévellec et al. (2022), for instance.

In parallel to these conceptual and observational progresses, there also exist questions on the role of this mesoscale eddy turbulence on the large-scale ocean momentum dynamics. This follows the idea that eddies can force the mean flow, the so-called eddy-driven mean flow or negative viscosity

phenomena (Starr 1968). This denotes the action of eddies as a Reynolds stress potentially forcing the mean flow, as classically derived in Davidson (2004), in the recent review of Nakamura (2024), or consistently with the vast literature on Eliassen–Palm fluxes (e.g., Eliassen and Palm 1961; Aiki et al. 2015; Matsuta and Masumoto 2021, for instance). This subject originates from atmospheric observations of the mid-latitude westerlies (see Starr 1968). This observational hypothesis was later confirmed by laboratory experiments (Whitehead 1975; Colin de Verdière 1979). This mechanism also exists in the ocean and explains a significant part of the inertial gyres in western boundary currents (Holland 1978; Rhines and Holland 1979). An emblematic example is the Zapiola Gyre (Saunders and King 1995) which exhibits an eddy-driven character, as suggested by both theory and numerical simulations (de Miranda et al. 1999), as well as a recent observational analysis (Sévellec et al. 2024). We could also cite the Azores Front Current (Alves and Colin de Verdière 1999).

Beyond this mean-flow “forcing” by the mesoscale eddy turbulence, it is widely accepted that eddies are also “fed” by barotropic and baroclinic mean flow instabilities (Pedlosky 1979). Together, these two processes lead to the concept of eddy–mean flow interactions. To quantitatively estimate how the eddy and mean flow exchange energy, Lorenz (1955) developed the now widely used energy cycle. This allows for a consistency between global gain and loss from mean kinetic energy (MKE) and loss and gain from eddy kinetic energy (EKE), respectively. However, this consistency breaks down when looking at local scale, where the kinetic energy (KE)

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can also be transferred from other locations. This implies that the eddy–mean flow interactions have different forms and values in the MKE evolution equation and in the EKE evolution equation (Murakami 2011; Murakami et al. 2011). Chen et al. (2014, 2016) re-examined this issue and rationalized the nonlocal interactions through the definition of a ghost energy reservoir, which is zero on time average (but not at all times). It corresponds to the residual kinetic energy, also called interaction term (e.g., Murakami 2011) or cross kinetic energy (e.g., Kang and Curchitser 2015; Jamet et al. 2022). It is the correlation between the mean and the turbulent velocities and can be described as the first-order KE anomaly (with MKE and EKE being the zeroth and second-order anomalies, respectively). Despite recent global observations of the local eddy–mean flow interactions (Sévellec et al. 2024), observations of the total and nonlocal interactions and their relation to the local interactions remain unknown.

To compute the surface and deep eddy–mean flow interactions based on observations, we use the ANDRO dataset that gives us access to the surface and 1000-m depth velocity recorded by Argo floats. Here, beyond the assessment of total interactions, we also focus on the decomposition in terms of local and nonlocal interactions (Chen et al. 2014, 2016). In particular, we suggest alternative formulations for these two terms that allow us to physical reinterpret the local interactions as the alignment of the turbulence with the mean strain (Sévellec et al. 2024) and of the nonlocal interactions as a modified advection of EKE. Finally, we show that these local and nonlocal terms mainly compensate each other at 2° scale. Since the nonlocal term vanishes when globally averaged, this implies the existence of an effective scale over which the local terms dominates the eddy–mean flow interactions. Testing these ideas in five key regions (i.e., Gulf Stream, Zapiola Gyre, Agulhas Current, Kuroshio, and equatorial Pacific), we show that the effective scale corresponds to several tens of degrees.

The manuscript is organized as follows. In section 2, the dataset is described. In section 3, the theoretical framework is given. In section 4, observed eddy–mean flow interactions are shown, local and nonlocal terms are described, and the spatial range of the nonlocal term in key regions is assessed. In section 5, properties of local and nonlocal terms are derived and discussed. Conclusions are given in section 6.

2. The ANDRO dataset

To compute the eddy–mean flow interactions, we use the ANDRO dataset (109566; Ollitrault et al. 2022). This dataset uses observed Argo drifter surface and deep displacements to track two surface velocities and one deep velocities per Argo float cycle (Ollitrault and Rannou 2013). The two surface velocities are estimated through the first 6-h and the last 6-h surface drift. These are computed by successive and successful positioning at the surface after the ascent and before the descent. (If the surface drift is less than 6 h, the two surface velocities are equal, and one of the two velocities will be disregarded in the rest of the study.) The deep velocities are computed using the deep drift through the last successful

positioning before the descent and the first successful positioning after the ascent. Hence, this dataset provides both surface and deep velocities from Argo (Fig. 1). The particular ANDRO dataset used in this study contains all the deep and surface drifts from 2000 to 2009, but is only partially complete for 2010–23.

Whereas all surface displacements and their corresponding velocities are used (leading to 2 387 280 displacements representative of a surface layer equal to the height of a typical Argo float, i.e., ~ 1.5 m), we have selected velocities at depth to displacements occurring between 950 and 1150 dbar for depth consistency (Fig. 1a). This selection leads to 1 177 974 deep displacements. In the rest of the study, the deep velocities will be considered to be at ~ 1000 -m depth.

The mean displacement distribution is 313.6 and 139.1 per $2^\circ \times 2^\circ$ (after removing 0 values) with a maximum value reaching 6349 and 754 per $2^\circ \times 2^\circ$ for the surface and deep observations, respectively. The distribution of the individual velocity observations shows a relatively high distribution in the Mediterranean Sea, the northeast Atlantic, equatorial Pacific, and Kuroshio regions for the surface (Fig. 1b) and in the equatorial region, north subtropical Atlantic, and south subtropical Pacific for the deep velocities (Fig. 1c).

At each integer of longitudes and latitudes (i.e., following a $1^\circ \times 1^\circ$ resolution grid), we define the zonal and meridional surface and deep mean velocities:

$$\bar{u} = \frac{1}{n} \sum_{i=1}^n u_i, \quad (1a)$$

$$\bar{v} = \frac{1}{n} \sum_{i=1}^n v_i, \quad (1b)$$

where u_i and v_i are the zonal and meridional surface and deep velocities estimated from individual Argo surface and deep displacements, respectively, n is the total number of these individual velocities found within a $2^\circ \times 2^\circ$ cell, and i is the index of these individual velocities. Equivalently, their unbiased variance and covariance can be defined as

$$\overline{u'^2} = \frac{1}{n-1} \sum_{i=1}^n (u_i - \bar{u})^2, \quad (2a)$$

$$\overline{v'^2} = \frac{1}{n-1} \sum_{i=1}^n (v_i - \bar{v})^2, \quad (2b)$$

$$\overline{u'v'} = \frac{1}{n-1} \sum_{i=1}^n (u_i - \bar{u})(v_i - \bar{v}). \quad (2c)$$

Here, these deviations from the mean flow could be due to (permanent) spatial differences of the flow within the $2^\circ \times 2^\circ$ grid cell, to time variations of the flow, or both. These spatial or time variations are aggregated together to give the local velocity statistics. Given the observational constraints, the mean corresponds to permanent features larger than $2^\circ \times 2^\circ$, whereas the turbulence corresponds to spatiotemporally variable features smaller than 2° but longer than ~ 6 h or ~ 10 days for surface or deep observations, respectively. Hence, there is a

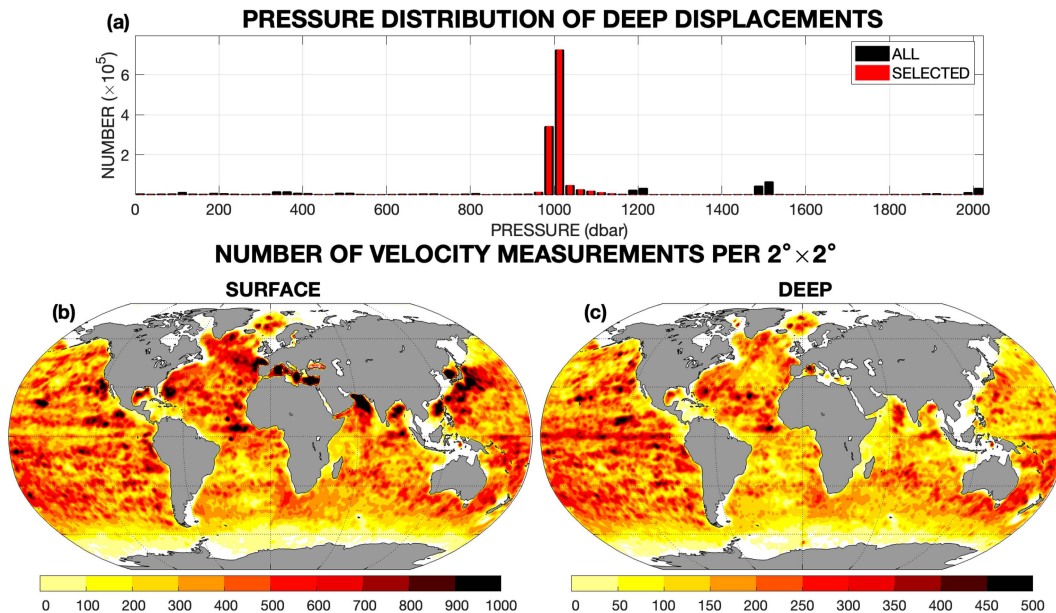


FIG. 1. Distribution of velocities observations from the ANDRO dataset. (a) Distributions of the parking depth pressure of the Argo float deep displacements from the ANDRO dataset (109566). (b),(c) Number of surface and deep velocities observations per $2^\circ \times 2^\circ$, respectively. For the surface velocities, the full ANDRO dataset of surface displacements is used. For the deep velocities in (a), deep parking depths from 950 to 1150 dbar are selected from the full ANDRO dataset of deep displacements.

difference of nature between the surface and deep velocities and interactions derived from them. Indeed, the fastest dynamics sampled at the surface includes inertial motions, whereas it is mainly restricted to mesoscale dynamics at depth.

A full description and discussion of error sources and the limits of the dataset can be found in Sévellec et al. (2024). The authors show the robustness of the mean, variance, and covariance of the deep velocities. In particular, they show their robustness to compute deep KE transfers. Regarding the surface velocity error, the main error source is on the positioning of floats. The ANDRO dataset provides full error estimations. This corresponds to a random velocity error of $\sim 2.5 \text{ cm s}^{-1}$ (or $6.4 \times 10^{-4} \text{ m}^2 \text{ s}^{-2}$ of velocity variance). This is less than 1% of the KE values in key regions of the rest of the study (intense boundary currents, equatorial region, and the Antarctic Circumpolar Current). Hence, these errors will be neglected and not further discussed.

Beyond this error estimation, the representativeness of ANDRO dataset can also be estimated. Indeed, the observed velocities correspond to sparse and nonuniform sampling of the true velocity field. To test the robustness of the analysis, Sévellec et al. (2024) used older versions of the datasets to mimic subsampling. This led to the same quantitative results. In this study, we have further tested this idea by sampling velocities from Global Ocean Reanalysis and Simulations (GLORYS12V1) [the Copernicus Marine Environment Monitoring Service (CMEMS) global reanalysis]. The data were selected from 1 January to 30 June 2010 in the Gulf Stream region ($20^\circ\text{--}55^\circ\text{N}$ and $90^\circ\text{--}30^\circ\text{E}$). We choose the Gulf Stream as a possible pathologic region because of its high level of turbulence.

To reproduce nonuniform sparse observations, we have sampled (with replacement) velocities within $2^\circ \times 2^\circ$ cells (consistently with our study) over the entire period. For the surface and deep velocities, it corresponds to 400 and 200 samples of $24 \times 24 \times 181 = 104\,256$ and of $24 \times 24 \times 18 = 10\,368$ total daily and 10-day velocities, respectively. The 400 and 200 values were set as conservative values for number of observations representative of ANDRO (Fig. 1). We find that the sub-sampled result is extremely accurate in computing MKE and EKE with a skill score (R^2) above 96%.

Another test was to subsample the ANDRO dataset in regions of high turbulence to estimate the convergence of MKE and EKE computations. This was done in four key regions of our analysis (i.e., where the transfers are important): $60^\circ\text{W}\text{--}40^\circ\text{N}$ for the Gulf Stream, $50^\circ\text{W}\text{--}50^\circ\text{S}$ for the Zapiola Gyre, $150^\circ\text{E}\text{--}35^\circ\text{N}$ for Kuroshio, and $150^\circ\text{E}\text{--}35^\circ\text{S}$ for the Agulhas Current. The subsampling corresponds to using sampling with the replacement method and repeated it 1000 times to provide uncertainty. This analysis reveals the fast convergence of the computation. At the surface, for a number of samples between 150 and 250, we have 100% and above 98% mean convergence for EKE and MKE, respectively. At 1000-m depth, for a number of samples between 75 and 125, we have 100% and above 96% mean convergence for EKE and MKE, respectively. These values are robust over the 1000 experiments with negligible error within the 99% confidence interval. In comparison, the actual number of samples available in the ANDRO dataset is higher than 200 and 100 for the surface and 1000-m depth, respectively. This is true almost everywhere and in particular in key turbulent regions, critical for our study (Fig. 1).

Despite the different sources of errors (sampling and measurements), we rationalize these good results by the robustness of the mean and the standard deviation of the velocities (leading to MKE and EKE, respectively), given the relatively high number of observations (>100). As shown later, the accurate mean and standard deviation are needed for an accurate estimation of the eddy–mean flow interactions.

3. Theoretical framework

There exists a large literature on the derivation of kinetic energy together with Reynolds decomposition (e.g., [Davidson 2004](#)). Hence, the [section 3a](#) and the derivations until (10) are not fully original but are provided to the reader for completeness, with specific reference to the relevant literature.

a. General derivations

To derive the ocean-scale interactions between the mean flow and its anomaly, we start from Boussinesq, incompressible, nonhydrostatic, primitive equations appropriate for high Reynolds number ($\text{Re} \gg 1$ i.e., viscous forces are negligible) together with the nondivergence. This reads

$$D_t \mathbf{u} + f \mathbf{k} \times \mathbf{u} = -\frac{1}{\rho_0} \nabla p - \frac{g}{\rho_0} \rho \mathbf{k} + \mathbf{F}, \quad (3a)$$

$$\nabla \cdot \mathbf{u}^\dagger = 0, \quad (3b)$$

where $\cdot$ and $\times$ are the three-dimensional scalar and vector products, respectively; $\dagger$ denotes the transpose operator; t is time; $D_t = \partial_t + (\mathbf{u} \cdot \nabla)$ is the material derivative; $\nabla = (\partial_x, \partial_y, \partial_z)$ is the three-dimensional derivative operator, with x, y , and z being the longitude, latitude, and depth, respectively; $\mathbf{u} = (u, v, w)$ is the three-dimensional velocity vector, with u, v , and w being the zonal, meridional, and vertical velocities, respectively; $\rho_{(0)}$ is the (reference) density for seawater; p is the pressure; f is the Coriolis parameter; g is the acceleration due to gravity; $\mathbf{F} = (F_x, F_y, F_z)$ is the vector of surface momentum forcing, with F_x, F_y , and F_z being the zonal, meridional, and vertical forcing, respectively; and $\mathbf{k} = (0, 0, 1)$ is the vertical unit vector.

Splitting the velocities, density, and pressure between a (spatiotemporal) mean ($\bar{\cdot}$) and an anomaly ($\cdot'$) from this mean, such as $\mathbf{u} = \bar{\mathbf{u}} + \mathbf{u}'$, $\rho = \bar{\rho} + \rho'$, $p = \bar{p} + p'$, and $\mathbf{F} = \bar{\mathbf{F}} + \mathbf{F}'$ [with $\bar{\mathbf{u}}' = (0, 0, 0)$, $\bar{\rho}' = 0$, $\bar{p}' = 0$, and $\bar{\mathbf{F}}' = (0, 0, 0)$], we obtain a set of equations for the mean and a set of equations for the anomaly. We can define KE as well as its decomposition in the MKE, residual KE (RKE), and EKE as zeroth-, first-, and second-order reservoirs of kinetic energy, respectively:

$$\text{KE} = \frac{1}{2} \mathbf{u} \cdot \mathbf{u}^\dagger = \text{MKE} + \text{RKE} + \text{EKE}, \quad (4a)$$

$$\text{where MKE} = \frac{1}{2} \bar{\mathbf{u}} \cdot \bar{\mathbf{u}}^\dagger, \text{RKE} = \mathbf{u}' \cdot \bar{\mathbf{u}}^\dagger, \text{and EKE} = \frac{1}{2} \mathbf{u}' \cdot \mathbf{u}'^\dagger. \quad (4b)$$

RKE is not as used as MKE and EKE, but as it will be shown later is crucial to assess the nonlocal component of ocean-scale interactions. It measures the alignment of the

turbulence with the mean flow. Note that the absence of RKE at all times is identical to the turbulence being orthogonal to the mean flow at all times.

From these definitions and (3), and consistently with [Chen et al. \(2014, 2016\)](#), we can derive the evolution of the MKE and the mean evolution of RKE and of EKE:

$$\underbrace{\partial_t \text{MKE}}_{\sim 0} = -\nabla \cdot (\bar{\mathbf{u}}^\dagger \text{MKE}) - \underbrace{\bar{\mathbf{u}} \cdot (\nabla \cdot \mathbf{R})^\dagger}_{\mathcal{R}} - \frac{1}{\rho_0} \nabla \cdot (\bar{\mathbf{u}}^\dagger \bar{p}) - \frac{g}{\rho_0} \bar{w} \bar{\rho} + \bar{\mathbf{u}} \cdot \bar{\mathbf{F}}^\dagger, \quad (5a)$$

$$\partial_t \text{RKE} = 0, \quad (5b)$$

$$\underbrace{\partial_t \text{EKE}}_{\sim 0} = -\nabla \cdot (\bar{\mathbf{u}}^\dagger \text{EKE}) - \underbrace{\mathbf{u}' \cdot \mathbf{G} \cdot \mathbf{u}'^\dagger}_{-T} - \frac{1}{\rho_0} \nabla \cdot (\bar{\mathbf{u}}^\dagger p') - \frac{g}{\rho_0} \bar{w}' \bar{\rho}' + \bar{\mathbf{u}}' \cdot \bar{\mathbf{F}}'^\dagger, \quad (5c)$$

where $\mathcal{R}$ is associated with eddy–mean flow interactions corresponding to the mean work of the Reynolds stress,¹ $\mathbf{R}$ is the Reynolds stress tensor, $-T$ is associated with eddy–mean flow interactions corresponding to the KE transfers between MKE and EKE (as demonstrated below), and $\mathbf{G}$ is the mean velocity gradient tensor ([Eckart 1948](#); [Okubo 1970](#); [Weiss 1991](#)), generalized for three-dimensional dynamics. The Reynolds stress tensor reads

$$\mathbf{R} = \begin{pmatrix} \overline{u'^2} & \overline{u'v'} & \overline{u'w'} \\ \overline{u'v'} & \overline{v'^2} & \overline{v'w'} \\ \overline{u'w'} & \overline{v'w'} & \overline{w'^2} \end{pmatrix}. \quad (6)$$

The mean velocity gradient tensor reads

$$\mathbf{G} = \begin{pmatrix} \partial_x \bar{u} & \partial_x \bar{v} & \partial_x \bar{w} \\ \partial_y \bar{u} & \partial_y \bar{v} & \partial_y \bar{w} \\ \partial_z \bar{u} & \partial_z \bar{v} & \partial_z \bar{w} \end{pmatrix}. \quad (7)$$

With this formulation, (5a), we see that MKE is modified by mean advection of MKE, by the mean work of the Reynolds stress ($\mathcal{R}$), by the mean work of the mean pressure gradient, by the mean baroclinic transfer from potential energy, and by the mean work of the mean momentum forcing ([Fig. 2](#)). From (5b), we find that RKE does not change on average. Whereas, the EKE mean evolution, (5c), is driven by total advection of mean EKE, by the transfer between MKE and EKE ($-T$), by the anomalous work of the anomalous pressure gradient, by the anomalous baroclinic transfer from potential energy, and by the anomalous work of the anomalous momentum forcing ([Fig. 2](#)).

¹ The notation $\nabla \cdot \mathbf{R}$ can be confusing, but respects the algebraic product between matrices. It should not be interpreted as the divergence of a tensor, though. It is simply the matrix form of the divergence of the Eliassen–Palm flux vector.

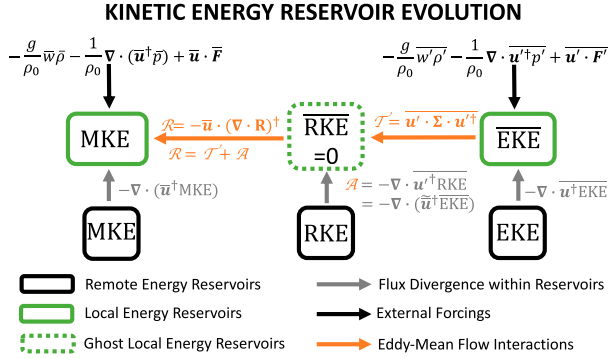


FIG. 2. Schematic of the KE reservoir budget. KE budget showing the action of external forcings (mean and anomalous pressure work, baroclinic potential energy transfer, and forcing work) and (mean and total) divergence of MKE and EKE fluxes within reservoirs, as well as the eddy-mean flow interactions acting between reservoirs and corresponding to the mean work of the Reynolds stress ($\mathcal{R}$) and the KE transfers ($\mathcal{T}$), respectively, following (5a). The eddy-mean flow interactions go through the RKE ghost reservoir (i.e., $\text{RKE} = 0$). There, the eddy advection of RKE ($\mathcal{A}$, nonlocal term) is combined with the KE transfers ($\mathcal{T}$, local term) from EKE to give rise to the work of the Reynolds stress acting on MKE ($\mathcal{R} = \mathcal{T} + \mathcal{A}$). There are a number of possible combinations to determine how the unique decomposition $\mathcal{R} = \mathcal{T} + \mathcal{A}$ acts between MKE, RKE, and EKE reservoirs. Here, we follow the neat argument of Chen et al. (2016), suggesting that the instantaneous KE components must be advected by the total velocities. Adding this constraint allows for a unique solution and leads to RKE reservoir acting as a buffer between MKE and EKE reservoirs.

It is worth noting that the KE transfer term ($\mathcal{T}$) is a scalar product so that only the symmetric component of the mean velocity gradient tensor will act. Hence, we have

$$\mathcal{T} = \overline{\mathbf{u}' \cdot \mathbf{G} \cdot \mathbf{u}'^{\dagger}} = \overline{\mathbf{u}' \cdot \mathbf{\Sigma} \cdot \mathbf{u}'^{\dagger}}, \quad (8)$$

where the mean flow deformation tensor ($\mathbf{\Sigma}$), identified by Lapeyre et al. (1999) and Balwada et al. (2021) in the context of tracer gradient magnitude dynamics, reads

$$\mathbf{\Sigma} = \frac{\mathbf{G} + \mathbf{G}^{\dagger}}{2} = \begin{bmatrix} \partial_x \bar{u} & \frac{1}{2}(\partial_x \bar{c} + \partial_y \bar{u}) & \frac{1}{2}(\partial_z \bar{u} + \partial_x \bar{w}) \\ \frac{1}{2}(\partial_x \bar{v} + \partial_y \bar{u}) & \partial_y \bar{v} & \frac{1}{2}(\partial_z \bar{v} + \partial_y \bar{w}) \\ \frac{1}{2}(\partial_z \bar{u} + \partial_x \bar{w}) & \frac{1}{2}(\partial_z \bar{v} + \partial_y \bar{w}) & \partial_z \bar{w} \end{bmatrix}. \quad (9)$$

b. Physical interpretations

Here, $\mathcal{R}$ is an eddy-mean flow interaction term and would correspond to the eddy-scale closure for MKE. This term can be reformulated following Chen et al. (2014, 2016) as

$$\mathcal{R} = \underbrace{+\overline{\mathbf{u}' \cdot \mathbf{\Sigma} \cdot \mathbf{u}'^{\dagger}}}_{\mathcal{T}} - \underbrace{\nabla \cdot \overline{\mathbf{u}'^{\dagger} \text{RKE}}}_{\mathcal{A}}. \quad (10)$$

This leads to an alternative interpretation of the mean work of the Reynold stress acting on MKE, where eddy-

mean interactions come in two forms of fundamentally different physical meaning (Fig. 2):

- 1) a KE transfer between MKE and EKE ($\mathcal{T}$), which becomes explicit when compared to (5c) as shown by Sévellec et al. (2024);
- 2) an eddy advection of RKE ($\mathcal{A}$), corresponding to an anomalous flux convergence of RKE.

KE transfer, sometimes defined as local eddy-mean flow interactions, could have a net global effect; whereas, eddy advection of RKE, sometimes defined as nonlocal eddy-mean flow interactions, is a divergence of a flux and has no net global effect. In this context, RKE becomes a “ghost” reservoir able to contribute to MKE change but always in equilibrium on average (5b). Consistently with Chen et al. (2014, 2016), since the mean RKE cannot change, the eddy advection of RKE ($\mathcal{A}$) can be reinterpreted as an instantaneous spatial redistribution of RKE by the turbulent flow to compensate the local mismatch between Reynolds stress ($\mathcal{R}$) and KE transfers ($\mathcal{T}$). As described above and used in the rest of the study, the chosen symbols for the three key terms are $\mathcal{R}$ which stands for Reynolds stress, $\mathcal{T}$ which stands for transfers of KE, and $\mathcal{A}$ which stands for (eddy) advection of RKE.

Another interesting property related to the mean eddy RKE advection is

$$\mathcal{A} = -\nabla \cdot \overline{\mathbf{u}'^{\dagger} \text{RKE}}, \quad (11)$$

$$= -\nabla \cdot (\mathbf{R} \cdot \bar{\mathbf{u}}^{\dagger}), \quad (12)$$

$$= -\nabla \cdot (\tilde{\mathbf{u}}^{\dagger} \overline{\text{EKE}}), \quad (13)$$

where $\tilde{\mathbf{u}}^{\dagger} \stackrel{\text{def}}{=} (\mathbf{R} \cdot \bar{\mathbf{u}}^{\dagger}) / \overline{\text{EKE}}$ is the modified mean velocity corresponding to the mean velocity stretched or squashed by the local turbulence. This modified mean velocity is introduced as a definition, but it remains based on physical principle since the denominator ($\overline{\text{EKE}}$) is the trace of the tensor in the numerator. This means that this definition is nothing else than a normalization of the Reynolds stress, allowing for geometrical interpretation of the action of $\mathbf{R}$ as a normalized transformation of the mean velocities. As shown in the next paragraph, this also allows us to disentangle the role of the isotropic and anisotropic components of the turbulence through a decomposition of $\tilde{\mathbf{u}}$. With this form, the eddy advection of RKE can be reinterpreted as a divergence of a modified mean flux of EKE (Fig. 3a).

From this expression, we can split the modified velocity in a mean velocity and a modification velocity to reveal the two components of the eddy RKE advection. This is obtained by splitting the Reynolds stress tensor into its isotropic and anisotropic components (Hoskins et al. 1983). This reads

$$\mathcal{A} = \underbrace{-\nabla \cdot (\bar{\mathbf{u}}^{\dagger} \overline{\text{EKE}})}_{\bar{\mathcal{A}}} - \underbrace{\nabla \cdot (\tilde{\mathbf{u}}^{\dagger} \overline{\text{EKE}})}_{\tilde{\mathcal{A}}}, \quad (14)$$

where $\tilde{\mathbf{u}} = \bar{\mathbf{u}} + \tilde{\mathbf{u}}$ and $\tilde{\mathbf{u}}$ are the modification velocities corresponding to the stretching and squashing effect of the turbulence on the mean velocities (i.e., linked to the anisotropy of the turbulence). Hence, $\bar{\mathcal{A}}$ and $\tilde{\mathcal{A}}$ are the isotropic and anisotropic

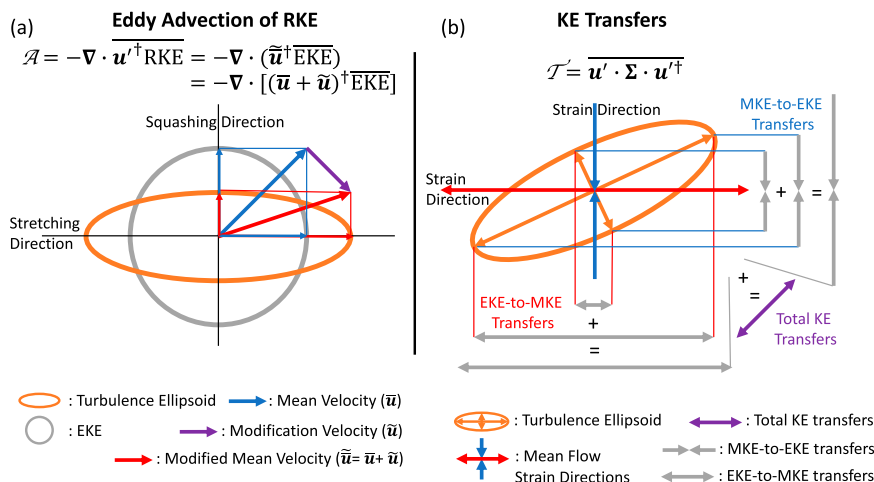


FIG. 3. Mechanisms of eddy-mean flow interactions. The total eddy-mean flow interactions (i.e., the mean work of the Reynolds stress $\mathcal{R}$) can be split into two components with specific physical mechanisms: (a) the eddy-induced advection of RKE ($\mathcal{A}$, nonlocal term) and (b) the KE transfers ($\mathcal{T}$, local term). The eddy-induced advection of RKE can be reinterpreted as an advection of EKE (gray circle) by the modified mean velocities (red arrow), following (13). The modified mean velocities correspond to the mean velocities (blue arrow) stretched and squashed by the local turbulence (orange ellipse) corresponding to the addition of the modification velocities (purple arrow). The KE transfers (purple arrow) is the sum of the natural turbulence directions (orange ellipse with arrows) projected on the mean flow strain directions (red and blue crosses), leading to EKE-to-MKE or MKE-to-EKE transfers (gray arrows) depending on the mean flow strain directions (red and blue, respectively), following (8) and Sévellec et al. (2024).

components of the nonlocal interactions, respectively. In this framework, $\tilde{\mathcal{A}}$ is nothing else than the mean velocity projection onto the Eliassen-Palm flux tensor (Trenberth 1986).

This expression suggests that MKE is somehow affected by the mean advection of EKE. Hence, the eddy-mean flow interactions reveal that MKE and EKE are affected by the same terms but with similar or opposing actions. Indeed, we find that MKE is affected by $\mathcal{T} + \overline{\mathcal{A}} + \tilde{\mathcal{A}}$ and, following (5c), EKE is affected by $-\mathcal{T} + \overline{\mathcal{A}}$, so that MKE and EKE are anti-phase synchronized (anticorrelated) by the KE transfers ($\mathcal{T}$), are synchronized (correlated) by the mean convergence of EKE ($\overline{\mathcal{A}}$), and are desynchronized (decorrelated) by the modification convergence of EKE ($\tilde{\mathcal{A}}$). The desynchronization, associated with the modification convergence of EKE, is controlled by the turbulence anisotropy. It could also be associated with wave activity, as recently proposed by Matsuta and Masumoto (2021, see their appendix). It is important to acknowledge, though, that there are other terms (e.g., “external forcings” in Fig. 2), which also affect the respective evolution of MKE and mean-EKE.

4. Observations of horizontal eddy-mean flow interactions

We now apply the theory described in section 3 to the ANDRO dataset described in section 2. However, given the dataset, which only gives access to the horizontal velocities, we will concentrate on horizontal eddy-mean flow interactions. (Note that away from the equator and for small Rossby number, there are scaling arguments to not primarily consider the contributions which include vertical velocities.)

Following (4), the horizontal kinetic energy reservoirs read

$$\text{MKE}_H = \frac{1}{2} \overline{\mathbf{u}_H \cdot \mathbf{u}_H^{\dagger}}, \quad (15a)$$

$$\text{RKE}_H = \overline{\mathbf{u}_H' \cdot \mathbf{u}_H'^{\dagger}}, \quad (15b)$$

$$\text{EKE}_H = \frac{1}{2} \overline{\mathbf{u}_H' \cdot \mathbf{u}_H'^{\dagger}}, \quad (15c)$$

where $\mathbf{u}_H = (u, v)$ is the two dimensional horizontal velocity vector and MKE_H , RKE_H , and EKE_H are the horizontal MKE, RKE, EKE, respectively. At the surface and at depth (~ 1000 m), the horizontal kinetic energy is particularly intense along western boundary currents, within the equatorial band, and in the Southern Ocean (Figs. 4a,e). At both depths, mean EKE dominates over MKE, whereas the typical amplitude of RKE fluctuations lies between MKE and mean EKE (Figs. 4b-d,f-h). They all share similar geographical patterns.

This simplification to horizontal flow also suggests that three key values need to be re-expressed to assess the effect of “eddies” on the mean flow:

$$\mathcal{R}|_{\text{hor}} = -\overline{\mathbf{u}_H \cdot (\nabla_H \cdot \mathbf{R}_H)^{\dagger}}, \quad (16a)$$

$$\mathcal{T}|_{\text{hor}} = \overline{\mathbf{u}_H' \cdot \Sigma_H \cdot \mathbf{u}_H'^{\dagger}}, \quad (16b)$$

$$\begin{aligned} \mathcal{A}|_{\text{hor}} &= -\nabla_H \cdot \overline{\mathbf{u}_H'^{\dagger} \mathbf{RKE}_H} \\ &= -\nabla_H \cdot (\mathbf{R}_H \cdot \overline{\mathbf{u}_H^{\dagger}}), \end{aligned} \quad (16c)$$

where $\nabla_H = (\partial_x, \partial_y)$ is the two-dimensional horizontal derivative operator, $\mathbf{R}_H$ is the horizontal Reynolds stress tensor, and

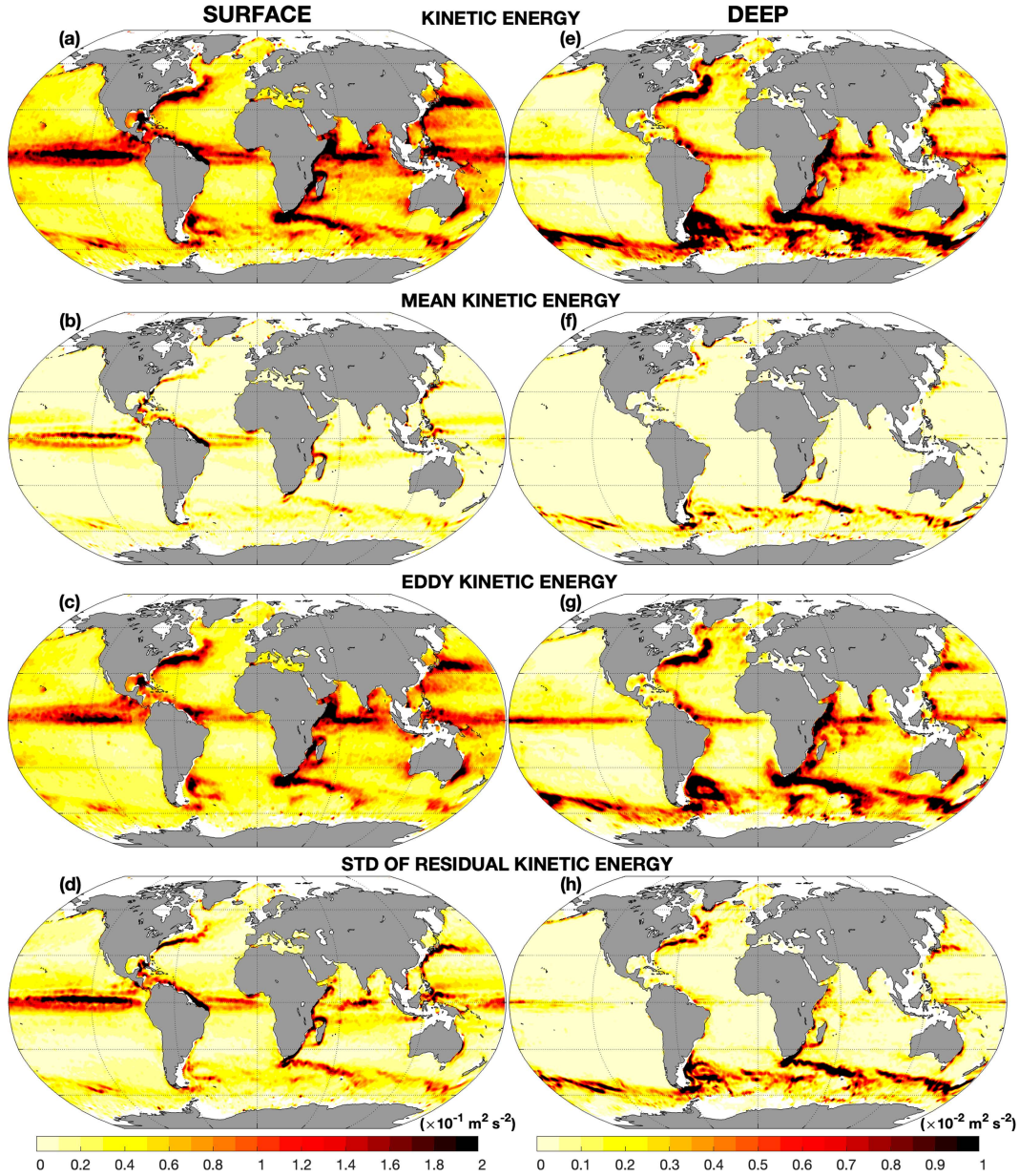


FIG. 4. Kinetic energy inferred from the ANDRO dataset. (a)–(d) Surface and (e)–(h) deep total KE, MKE, EKE, and standard deviation of RKE, following (15).

Σ_H is the horizontal mean flow deformation tensor. [Note that when focusing on horizontal components, we cannot ensure that the flow is nondivergent ($\nabla_H \cdot \mathbf{u}_H^i \neq 0$). Hence, we chose to use the formulation in terms of divergence of energy flux (e.g., $\nabla_H \cdot \mathbf{u}_H^i \text{RKE}_H$), rather than advection of energy (e.g., $\mathbf{u}_H^i \cdot \nabla_H \text{RKE}_H$), which explicitly acknowledges the divergence of the horizontal flow in the advection terms.] The horizontal Reynolds stress tensor reads

$$\mathbf{R}_H = \begin{pmatrix} \overline{u'^2} & \overline{u'v'} \\ \overline{u'v'} & \overline{v'^2} \end{pmatrix}. \quad (17)$$

The mean horizontal flow deformation tensor reads

$$\Sigma_H = \begin{bmatrix} \partial_x \bar{u} & \frac{1}{2}(\partial_x \bar{v} + \partial_y \bar{u}) \\ \frac{1}{2}(\partial_x \bar{v} + \partial_y \bar{u}) & \partial_y \bar{v} \end{bmatrix}. \quad (18)$$

This can be further simplified by defining the notations:

$$\Delta = \partial_x \bar{u} + \partial_y \bar{v}, \quad (19a)$$

$$\sigma_n = \partial_x \bar{u} - \partial_y \bar{v}, \quad (19b)$$

$$\sigma_s = \partial_x \bar{v} + \partial_y \bar{u}, \quad (19c)$$

where Δ is the horizontal divergence and σ_n and σ_s are the horizontal normal and shear strain (or stretching and shearing),

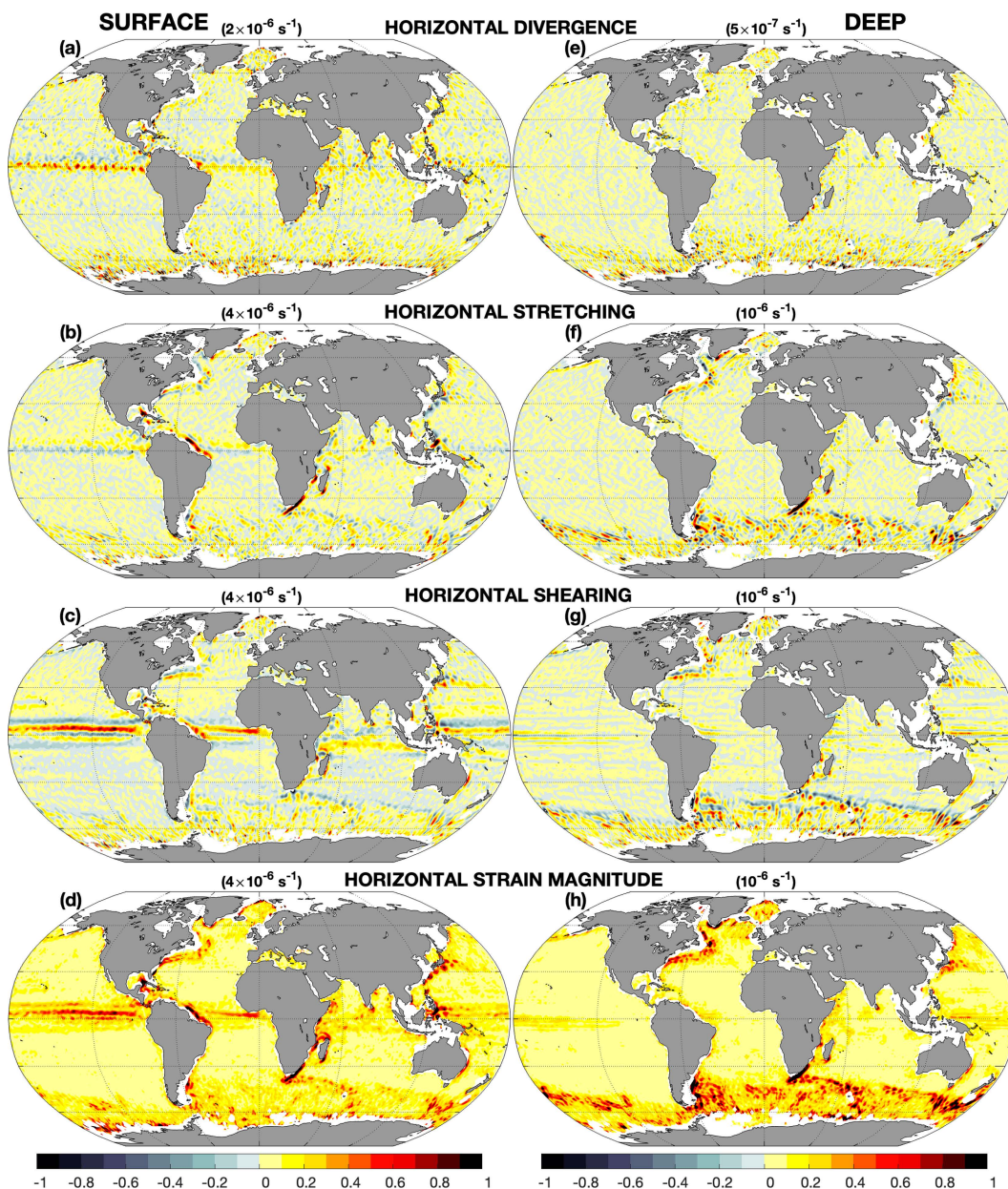


FIG. 5. Horizontal gradient properties of the horizontal velocities. (a)–(d) Surface and (e)–(h) deep horizontal divergence, stretching (or normal strain), shearing (or shear strain), and strain magnitude of horizontal velocities, following (19). Note the small-scale factor for divergence in (a) and (e) compared to strain properties in (b)–(d) and (f)–(h), respectively.

respectively. With these notations, the mean horizontal flow deformation tensor reads

$$\Sigma_H = \frac{1}{2} \begin{pmatrix} \Delta + \sigma_n & \sigma_s \\ \sigma_s & \Delta - \sigma_n \end{pmatrix}. \quad (20)$$

We find that the mean flow is dominated by the stretching and shearing over the divergence for both surface and deep observations (Fig. 5). It is only at the surface in a narrow band along the equator that the divergence can be important (because of wind).

a. Total interactions

The computation of the total horizontal eddy–mean flow interaction terms, from (16a), for the surface and deep ocean reveals that most of the interactions occur in regions of intense KE (Fig. 6a). The work of the Reynolds stress shows an intensification in western boundary currents and in the ACC. Unlike their deep counterparts, the surface total interactions are also important in the surface equatorial band. Interactions are 100 times as intense at the surface as at depth, with absolute values reaching up to $5.7 \times 10^{-6} \text{ m}^2 \text{ s}^{-3}$ at the surface,

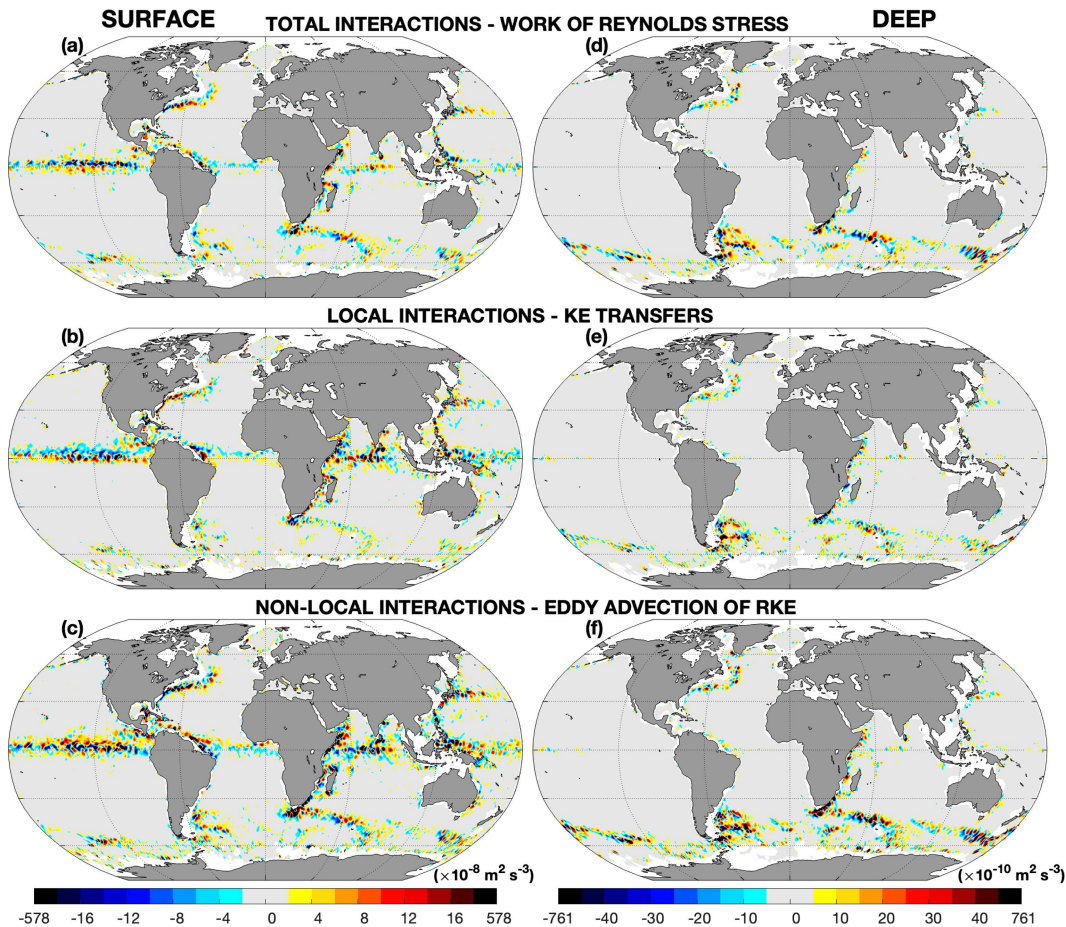


FIG. 6. Horizontal surface and deep eddy-mean flow interactions. Horizontal (a)–(c) surface and (d)–(f) deep total, local, and nonlocal eddy-mean flow interactions, following (16). (a),(d) The total term corresponds to the horizontal component of the mean work of the Reynolds stress ($\mathcal{R}|_{\text{hor}}$). (b),(e) The local term corresponds to horizontal KE transfers between MKE and EKE ($\mathcal{T}|_{\text{hor}}$). (c),(f) The nonlocal term corresponds to the horizontal component of the eddy advection of RKE ($\mathcal{A}|_{\text{hor}}$). For the KE transfers, positive and negative values correspond to transfers from EKE to MKE and from MKE to EKE, respectively. Note that $\mathcal{R} = \mathcal{T} + \mathcal{A}$ but $\mathcal{R}|_{\text{hor}} \neq \mathcal{T}|_{\text{hor}} + \mathcal{A}|_{\text{hor}}$ due to the absence of the vertical terms.

whereas it only reaches $7.6 \times 10^{-8} \text{ m}^2 \text{ s}^{-3}$ at 1000-m depth. (However, it is worth reminding that surface and deep velocities are qualitatively different due to their widely different sampling time scales.)

Whereas regions of important interactions are well defined (as described above), interactions are characterized by their relative small-scale patterns within these regions. Indeed, positive and negative values follow each other along intense mean currents (Fig. 7). This occurs at a horizontal scale that varies from 10° at the largest to the scale of the computation grid cell (i.e., 2°) at the smallest.

In the regions of the boundary currents, the total interactions term shows a few specific properties (Fig. 7). Even if not a perfect match, the total interaction term shows a good consistency between the surface and the deep ocean (Figs. 7a–d vs Figs. 7e–h). This consistency between two depths leads for the robustness of the analysis and can be seen beyond the boundary current regions (e.g., west Indian sector of the ACC; Figs. 7d,h).

Also, and as a consequence, the following description is valid for both the surface and deep ocean. For the Gulf Stream, Kuroshio, and Agulhas Current, a negative signature is visible along the coast where MKE is higher, whereas positive values occur slightly off the coast where MKE is lower. This means that the Reynolds stress slows down the fastest, upstream part of the current and accelerates its downstream part. In the Zapiola Gyre, a similar mechanism exists, but across the mean flow, with negative and positive values along and inside the gyre, respectively. This corresponds to a sink and a source of MKE related to strong and weak values of MKE, respectively. Overall, these observations are consistent with a homogenization or smoothing of the mean current (MKE) by the mesoscale eddy turbulence (Reynolds stresses). This is evidence that the eddy turbulence is forcing the large-scale flow toward a more laminar regime. Along the westward Antarctic Circumpolar Current, a few regions of positive and negative interactions are also visible upstream of topographic barriers: Crozet Plateau, Kerguelen

TOTAL INTERACTIONS

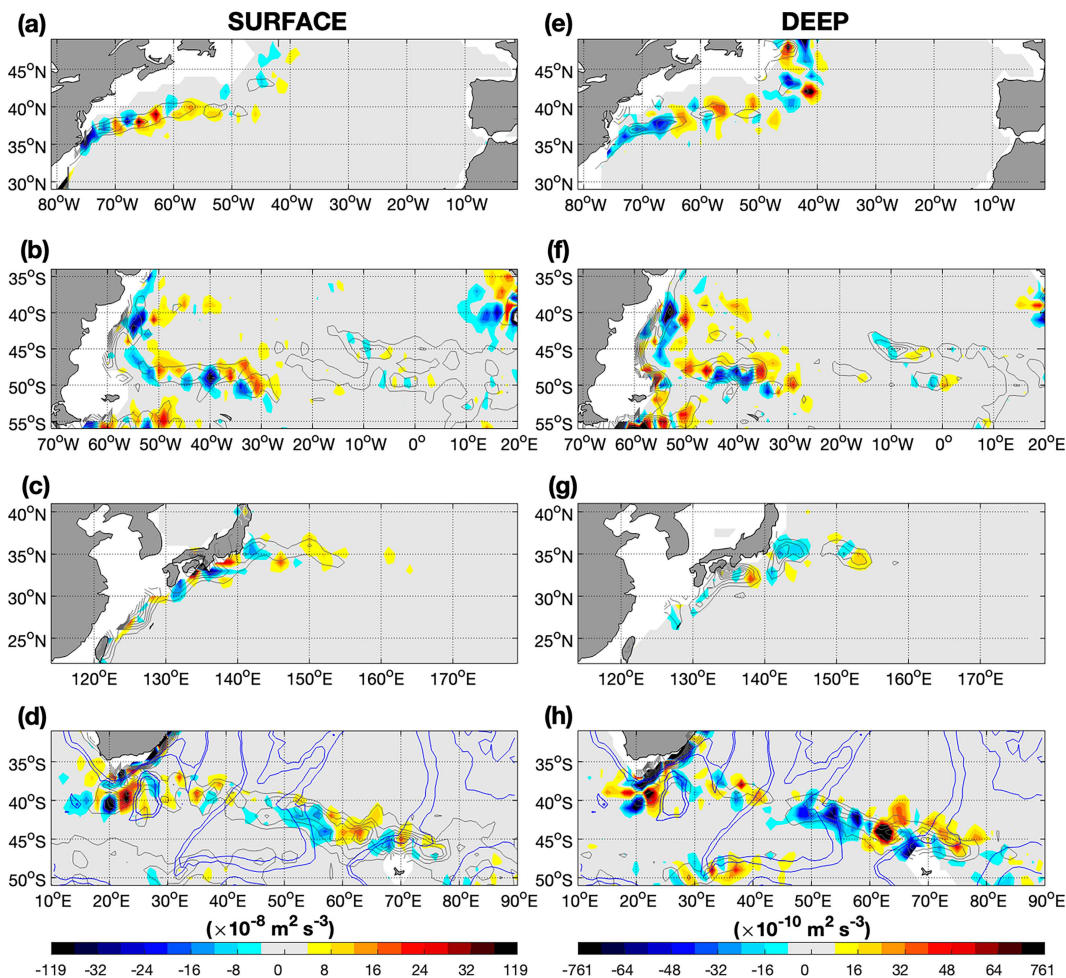


FIG. 7. Regional total horizontal surface and deep eddy-mean flow interactions. Horizontal (a)–(d) surface and (e)–(h) deep total eddy-mean flow interactions, following (16a) and as in Figs. 6a and 6d, but for four specific western boundary regions: (a),(e) Gulf Stream, (b),(f) Zapiola Gyre, (c),(g) Kuroshio, and (d),(h) Agulhas Current. This term corresponds to the horizontal component of the mean work of the Reynolds stress ($\mathcal{R}|_{\text{hor}}$). Gray contours show the surface and deep MKE with contour intervals of 4×10^{-2} and $2 \times 10^{-3} \text{ m}^2 \text{ s}^{-2}$, 2×10^{-2} and $3 \times 10^{-3} \text{ m}^2 \text{ s}^{-2}$, 4×10^{-2} and $1 \times 10^{-3} \text{ m}^2 \text{ s}^{-2}$, and 2×10^{-2} and $4 \times 10^{-3} \text{ m}^2 \text{ s}^{-2}$, for the Gulf Stream, Zapiola Gyre, Kuroshio, and Agulhas Current, respectively. Blue contours show the 4000- and 3500-m depth isobaths.

Plateau, Campbell Plateau, and Pacific–Antarctic Ridge. Focusing on the Crozet Plateau, downstream to the plateau, a region of negative total interactions exists (with slight reduction of MKE). This observational signature is consistent with topographic modulation predicted by idealized modeling studies (Youngs et al. 2017; Cai et al. 2022). Following the flow, upstream of Kerguelen Plateau, a sudden increase in MKE occurs together with a positive total interaction term. This is an observational evidence of upstream topographic effects [as described by Youngs et al. (2017), in an idealized modeling study].

In the surface equatorial Pacific, this total interaction term ($\mathcal{R}|_{\text{hor}}$) is intensified in the band 5°S – 10°N (Fig. 8a). The total interactions display a negative peak located slightly off the equator ($\sim 2^{\circ}\text{N}$) and a weaker positive peak located south

of the equator ($\sim 2^{\circ}\text{S}$). The zonal alignment of the flow properties at the equator allows us to decompose the mean and turbulent flow in an along and an across mean flow component corresponding to the zonal and meridional components. This shows the dominance of the mean zonal flow, which peaks with a negative (i.e., westward) velocity at $\sim 2^{\circ}\text{N}$, whereas the meridional flow reveals velocities almost perfectly skew-symmetric with the equator (Fig. 9a). The turbulence is dominated by the zonal component with a peak at the equator. The meridional turbulent component is roughly half the zonal value. The zonal-meridional covariance is significantly weaker than the two other turbulent flow components. Because of the strong zonal consistency of the interactions (Fig. 8a), we have also computed the components of the total scale interactions in terms of the zonal averages to reflect the

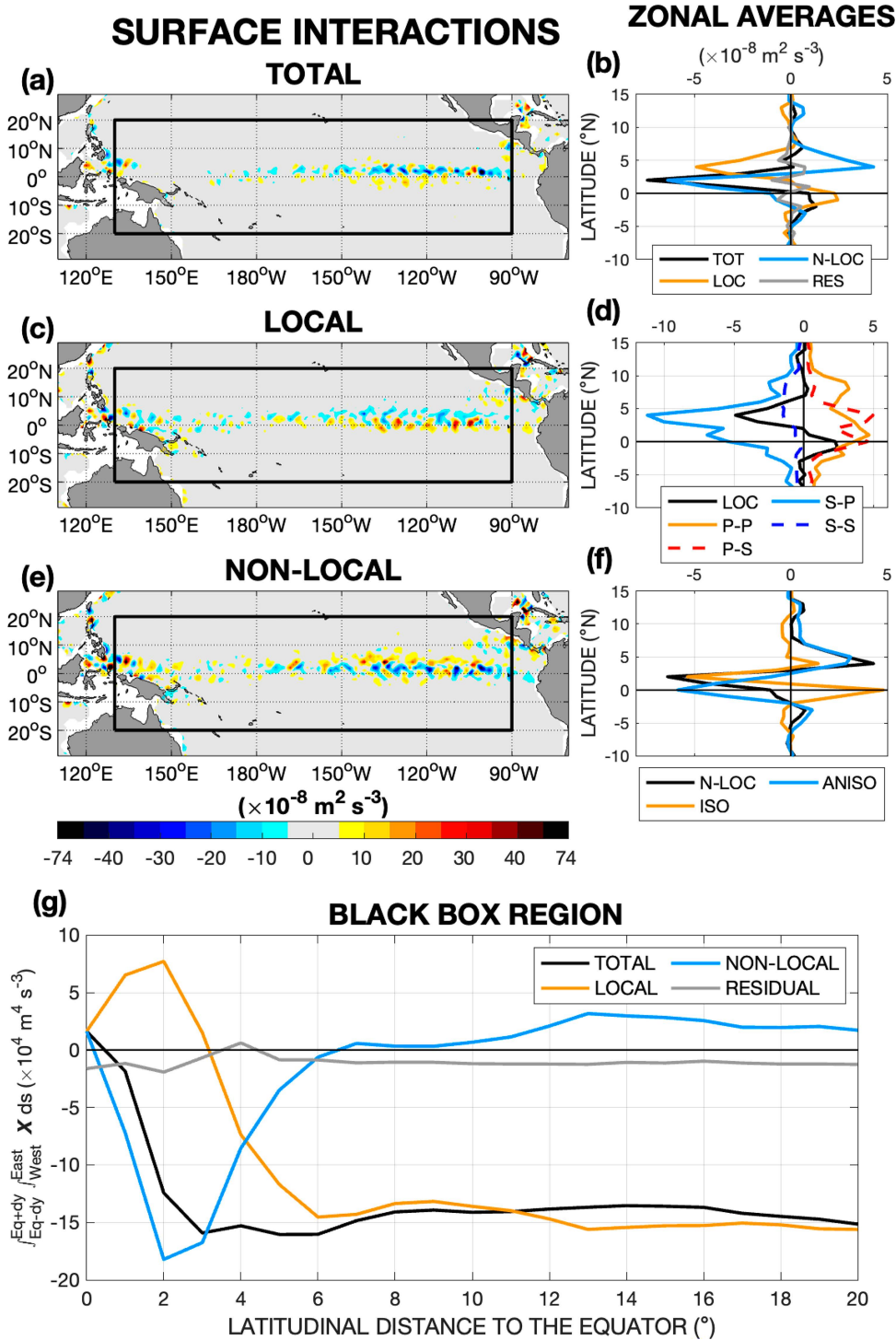


FIG. 8. Components of the surface and deep ocean-scale interactions for the equatorial Pacific. Surface (a) total ($\mathcal{R}_{\text{hor}}$), (c) local ($\mathcal{T}_{\text{hor}}$), and (e) nonlocal ($\mathcal{A}_{\text{hor}}$) horizontal eddy-mean flow interactions, following (4). Zonal averages within the black box regions of the (b) total, (d) local, and (f) nonlocal horizontal-mean flow interactions and their components. (b) The components are the local, nonlocal, and residual, defined as $\mathcal{R}_{\text{hor}} = (\mathcal{T}_{\text{hor}} + \mathcal{A}_{\text{hor}})$. (d) The components are the primary-primary (P-P), primary-secondary (P-S), secondary-primary (S-P), and secondary-secondary (S-S), following (25) and Fig. 14. (f) The components are the mean and modification divergence of EKE flux, following (28) and Fig. 16. (g) Regional meridional cumulative integrals in the black box region follows (22). The black box region goes from 130°E to 90°W and from 20°S to 20°N.

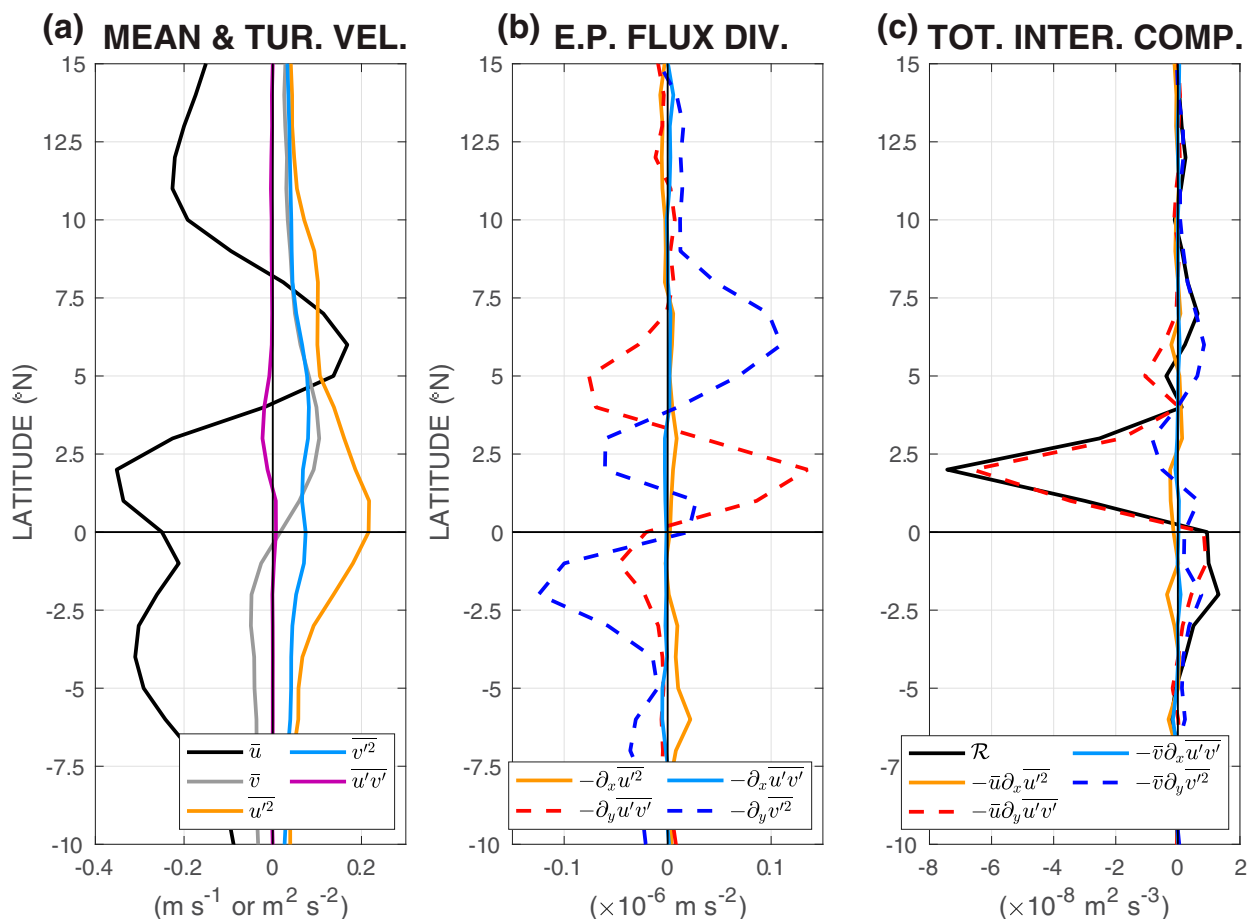


FIG. 9. Total components of the surface ocean-scale interactions for the equatorial Pacific. (a) Zonal average of the mean flow and of the turbulence. The components of the mean flow are the zonal and meridional velocities ($\bar{u}$ and $\bar{v}$ with black and gray lines, respectively). The components of the turbulent flow are the zonal and meridional velocity variances ($\overline{u'^2}$ and $\overline{v'^2}$ with orange and cyan lines, respectively), as well as the zonal and meridional velocity covariance ($\overline{u'v'}$ with purple line). (b) The components of the Eliassen–Palm flux divergence are the zonal gradient of the zonal velocity variance ($-\partial_x \overline{u'^2}$; solid orange line), the zonal and meridional gradients of the velocity covariance ($-\partial_x \overline{u'v'}$ and $-\partial_y \overline{u'v'}$; solid cyan and dashed red lines, respectively), and the meridional gradient of the meridional velocity variance ($-\partial_y \overline{v'^2}$; dashed blue line). (c) Zonal average of the components of the total ocean-scale interactions ($\mathcal{R}$ with black line), following (16a), with its four components: the zonal work of the Reynolds normal stress ($-\bar{u} \partial_x \overline{u'^2}$; orange solid line), the zonal work of the Reynolds shear stress ($-\bar{u} \partial_y \overline{u'v'}$; red dashed line), the meridional work of the Reynolds shear stress ($-\bar{v} \partial_x \overline{u'v'}$; red dashed line), and the meridional work of the Reynolds normal stress ($-\bar{v} \partial_y \overline{v'^2}$; orange solid line). Zonal average are done within the black box indicated in Fig. 8 and going from 130°E to 90°W and from 20°S to 20°N.

properties of the along- and across-mean flow properties (Figs. 9b,c). We find that the Reynolds stresses (or Eliassen–Palm flux divergence, Eliassen and Palm 1961) are dominated by the meridional gradient of the velocity covariance and by the meridional gradient of the meridional velocity variance (Fig. 9b). The former acts on the mean zonal velocity, whereas the latter acts on the mean meridional velocity. This leads to the strong dominance of the zonal work of the Reynolds shear stress (i.e., $-\bar{u} \partial_y \overline{u'v'}$). This is an expected result for a zonally dominated flow (Starr 1968; Nakamura 2024). Here, the main jet ($\sim 2^\circ\text{N}$) is slowed down by the turbulence, whereas the local minimum ($\sim 1^\circ\text{S}$) is energized by the turbulence (Fig. 9c). [Note the slight slowdown of the westward Equatorial Counter Current by the same process at 5°N

(Fig. 9c).] This corresponds to a homogenization of MKE, consistently with the description for boundary currents. Analysis of the surface equatorial Atlantic region (not shown) reveals the exact same properties and leads to strictly identical conclusions.

b. Local and nonlocal interactions

To go beyond the analysis of the work of the Reynolds stress (i.e., the total interaction term), we compute the horizontal local and nonlocal eddy–mean flow interaction terms, derived from (16). They show similar patterns than the total interactions (Figs. 6b,c,e,f): intensification in western boundary currents and the ACC. Again, the local and nonlocal interactions are 100 times as intense at the surface as at depth, with

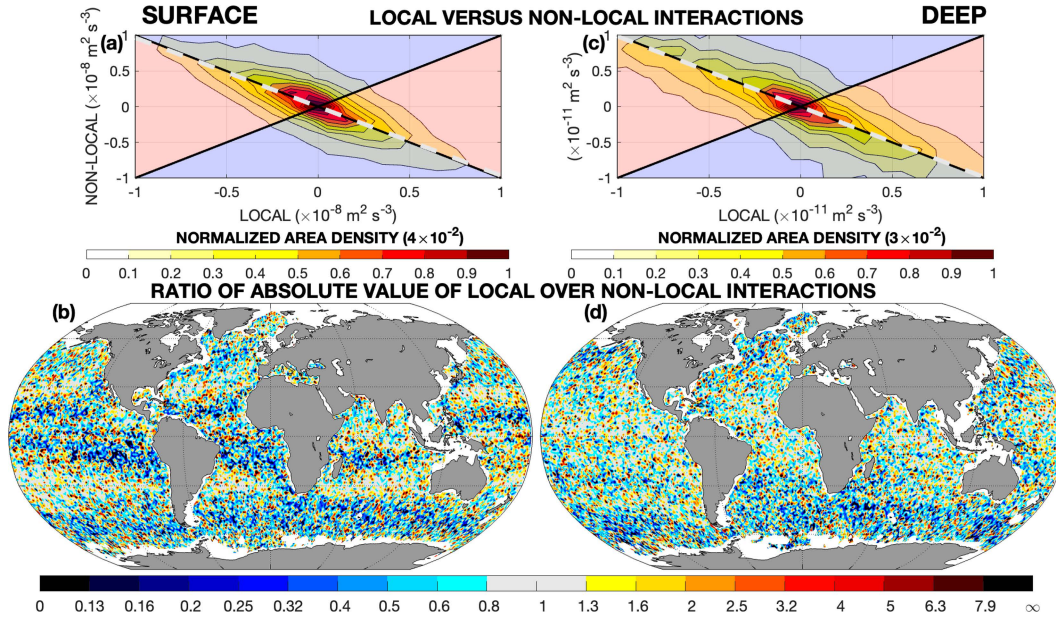


FIG. 10. Horizontal surface and deep local vs nonlocal interactions. (a),(c) Normalized area density of horizontal local vs nonlocal interactions (T_{hor} and A_{hor} , respectively), following (16b) and (16c), for the surface and deep observations, respectively. The gray dashed lines correspond to the best robust linear fits and lead to linear regression coefficients of -0.97 and -0.99 for surface and deep observations, respectively. The red and blue shaded regions correspond to local and nonlocal dominated regions, respectively. The area densities are computed using bin ranging over $\pm 1.1 \times 10^{-8}$ and $\pm 1.1 \times 10^{-11} \text{ m}^2 \text{ s}^{-1}$ for surface and deep observations, respectively; the bin widths are 0.1×10^{-8} and $0.2 \times 10^{-8} \text{ m}^2 \text{ s}^{-1}$ for surface local and nonlocal interactions, respectively, and 0.1×10^{-11} and $0.2 \times 10^{-11} \text{ m}^2 \text{ s}^{-1}$ for deep local and nonlocal interactions, respectively. Densities are normalized so their integral over the computed ranges are 1. (b),(d) Ratio of the absolute value of the local over nonlocal horizontal interactions (i.e., $|T_{\text{hor}}|/|A_{\text{hor}}|$) for the surface and deep observations, respectively. Value higher and lower than 1 corresponds to local and nonlocal dominance, respectively.

absolute values reaching up to 5.8×10^{-6} and $5.1 \times 10^{-8} \text{ m}^2 \text{ s}^{-3}$ at the surface and at 1000-m depth, respectively. Overall, the total interaction term is evenly split between its local and its nonlocal components (KE transfers and eddy RKE advection, respectively).

In the equatorial Pacific, given the relative zonal consistency of the surface interaction terms (Figs. 8a,c,e), we focus on zonal averages to disentangle the specific role of the local and nonlocal interactions and their components. Here, the total interactions come from the interplay between local and nonlocal interactions. The positive and negative peaks (located at $\sim 1^\circ\text{S}$ and at 2°N , respectively) are being controlled by the former and the latter, respectively (Fig. 8b). Hence, the slowdown of the flow maximum is due to nonlocal interactions, whereas the acceleration of the local minimum is due to local interactions. The local and nonlocal interactions also display a strong negative and positive signature at $\sim 3^\circ\text{--}8^\circ\text{N}$ with similar amplitude, respectively. This leads to no signature in the total interactions (analysis for the equatorial Atlantic reveals identical results, not shown).

Looking at the statistics of the local and nonlocal interactions and in particular at their relation, we find that these two terms have opposite effects for both surface and deep observations (Figs. 10a,c, respectively). This is particularly striking through the computation of the linear regression coefficient

between local and nonlocal interactions: -0.97 and -0.99 for surface and deep observations, respectively. This suggests that at the grid scale of the computation, the local and nonlocal interactions have a compensating effect on the total interactions. Physically, this means that RKE is advected to compensate the effect of instabilities and limit their impact on the work of the Reynolds stress so that changes in EKE due to KE transfers are compensated to avoid a resulting effect on MKE. Further investigating the dominant factor between local and nonlocal interactions, we have computed the ratio of their absolute value ($|T_{\text{hor}}|/|A_{\text{hor}}|$). This shows a dominance of the nonlocal term (Figs. 10b,d): 39% and 61% of the global area are dominated by local and nonlocal interactions, respectively, for both surface and deep observations. These results suffer from an apparent paradox, though. Indeed, unlike the statistical analysis, the geographical maps do not show such a clear compensating effect of the local and nonlocal terms and the total term is of the same order of magnitude than its two components (Fig. 6). This can be rationalized by acknowledging that the geographical maps highlights extreme values (of absolute values higher than 2×10^{-7} and $5 \times 10^{-9} \text{ m}^2 \text{ s}^{-3}$ for surface and deep observations, respectively), whereas the statistical description are strongly impacted by weak values (of values ranging within $\pm 10^{-8}$ and $\pm 10^{-11} \text{ m}^2 \text{ s}^{-3}$ for surface and deep observations, respectively). Also, we know by

the formulation of the two interaction terms, (10), that the nonlocal interaction term vanishes when globally average (i.e., no net global effect of eddy advection of RKE), whereas the local interaction term does not. We conclude that at grid scale, the two terms mainly statistically compensate each other, with a larger area of nonlocal interaction dominance, but at global scale, the local term is the only contributing factor.

c. Spatial-mean convergence of eddy–mean flow interactions

As described above, one of the key properties of the eddy–mean flow interactions is that the local and nonlocal interactions largely compensate each other at the computational grid scale (i.e., $\sim 2^\circ \times 2^\circ$), whereas the local interactions dominate at global scale (since the nonlocal interactions vanish—being the divergence of a flux with no-net flux at the boundaries). This raises the question of an effective scale: a spatial scale between the computational grid scale and the global scale at which local interactions become dominant over nonlocal interactions. This suggests the existence of a “sweet spot,” where the Reynolds stress could be parameterized solely through local interactions. In this context, [Jamet et al. \(2022\)](#) show that a spatial integral of $\sim 4^\circ \times 4^\circ$ is enough to remove the impact of nonlocal interactions in a $1/60^\circ$ -resolution forced ocean model analysis of the west Mediterranean Sea, close to the Gibraltar Strait. We have diagnosed the existence of this effective scale for four boundary current regions and for the equatorial Pacific region.

To determine this effective scale in western boundary currents, we have computed the horizontal integrals of the horizontal Reynolds stress, local interactions, and nonlocal interactions for a range of spatial integrals. This integral reads

$$\bar{\mathcal{X}}^x = \int_{x_{\text{West}}}^x \int_{y_{\text{South}}}^{y_{\text{North}}} \mathcal{X} ds, \quad (21)$$

where $\mathcal{X}$ could be the horizontal Reynold stress ($\mathcal{R}|_{\text{hor}}$), the horizontal local interactions ($\mathcal{T}|_{\text{hor}}$), the horizontal nonlocal interactions ($\mathcal{A}|_{\text{hor}}$), and the residual (i.e., the vertical term); $\bar{\mathcal{X}}^x$ represents its spatial integral with dependency to x ; ds is the unit spatial variable; x is the variable eastern boundary of the integral; and x_{West} , y_{South} , and y_{North} are the western, southern, and northern boundaries of the integral. Because the horizontal terms do not perfectly close the budget, we have also defined a residual as $\mathcal{R}|_{\text{hor}} - (\mathcal{T}|_{\text{hor}} + \mathcal{A}|_{\text{hor}})$, representing the importance of the vertical term. For all regions, different boundaries have been tested and led to the same qualitative and quantitative results. These computations were done for both surface and deep observations; however, significant results (i.e., integrals more important than their residual) were only visible for the deep observations. Hence, we restrict this part of the analysis to the deep displacements.

The four boundary regions ([Fig. 11a](#)) are defined as follows:

- Gulf Stream: from -80° to -30°E and from 33° to 43°N ;
- Zapiola Gyre: from -70° to -25°E and from 52° to 37°S ;
- Kuroshio: from 120° to 175°E and from 25°S to 40°N ;
- Agulhas Current: from 12° to 70°E and from 46° to 34°S .

Our diagnostics reveal that the horizontal Reynolds stress is dominated by the nonlocal interactions for integrals from a few degrees to a few tens of degrees ([Fig. 11](#)). However, as expected, these nonlocal interactions vanish for large enough integrals. Hence, we find an effective scale on the order of several tens of degrees. Beyond this scale, the horizontal Reynolds stress is dominated by the local interactions.

Beyond these general results, there exist specific properties for each region. For the Zapiola Gyre, the larger scale (i.e., several tens of degrees) shows dominance of the local interactions with a KE transfers toward MKE [as discussed in [Sévellec et al. \(2024\)](#)]. For Kuroshio, despite exhibiting the same convergence, the robustness is less clear, given the importance of the residual term. For the Agulhas Current, the local interactions largely control the Reynolds stress after only several degrees of average.

For the equatorial region, that is defined by the region going from 130°E to 90°W and from 20°S to 20°N , we apply a slightly different strategy. Here, the variable boundary is meridional and acknowledges the symmetry with the equator. Hence, the integral reads

$$\bar{\mathcal{X}}^{dy} = \int_{x_{\text{West}}}^{x_{\text{East}}} \int_{y_{\text{Eq}}-dy}^{y_{\text{Eq}}+dy} \mathcal{X} ds, \quad (22)$$

where $\bar{\mathcal{X}}^{dy}$ represents the spatial integral of $\mathcal{X}$ with dependency to dy and dy is the variable boundary distance to the equator of the integral.

To determine the convergence of the total, local, and nonlocal interactions at large scale, we apply the formulation above. We find that, for scales of a few degrees, the Reynolds stress is controlled by the nonlocal interactions ([Fig. 8g](#)). However, for scales from several degrees to a few tens of degrees, the local interactions dominate. The residual remains small for all integral extensions. This allows the definition of the effective scale around tens of degrees (above $2 \times 6^\circ$). For integral extensions of $\pm 6^\circ$ (integral from 6°S to 6°N) and above, we find that local interactions correspond to KE transfers from MKE to EKE, controlling a Reynolds stress slowing down MKE.

5. Decomposition of the local and nonlocal interactions from observations

To further characterize the horizontal eddy–mean flow interactions, the two key tensors [i.e., the horizontal component of the Reynolds stress tensor ($\mathbf{R}_H$) and of the mean flow deformation tensor ($\mathbf{\Sigma}_H$)] can be diagonalized to show their natural decomposition. The symmetric nature of the two tensors is particularly interesting since it leads to eigenvectors that are orthogonal to each other, precluding from the need to compute biorthogonal eigenvectors (e.g., [Farrell and Ioannou 1996a,b](#)). Hence, without loss of generality, we have

$$\mathbf{R}_H = \tilde{\lambda}_p \tilde{\mathbf{p}}^\dagger \cdot \tilde{\mathbf{p}} + \tilde{\lambda}_s \tilde{\mathbf{s}}^\dagger \cdot \tilde{\mathbf{s}}, = \begin{pmatrix} \tilde{p}_x & \tilde{s}_x \\ \tilde{p}_y & \tilde{s}_y \end{pmatrix} \begin{pmatrix} \tilde{\lambda}_p & 0 \\ 0 & \tilde{\lambda}_s \end{pmatrix} \begin{pmatrix} \tilde{p}_x & \tilde{p}_y \\ \tilde{s}_x & \tilde{s}_y \end{pmatrix}, \quad (23)$$

where $\tilde{\mathbf{p}} = (\tilde{p}_x, \tilde{p}_y)$ and $\tilde{\mathbf{s}} = (\tilde{s}_x, \tilde{s}_y)$ are the eigenvectors associated with the eigenvalues $\tilde{\lambda}_p$ and $\tilde{\lambda}_s$, respectively, of the

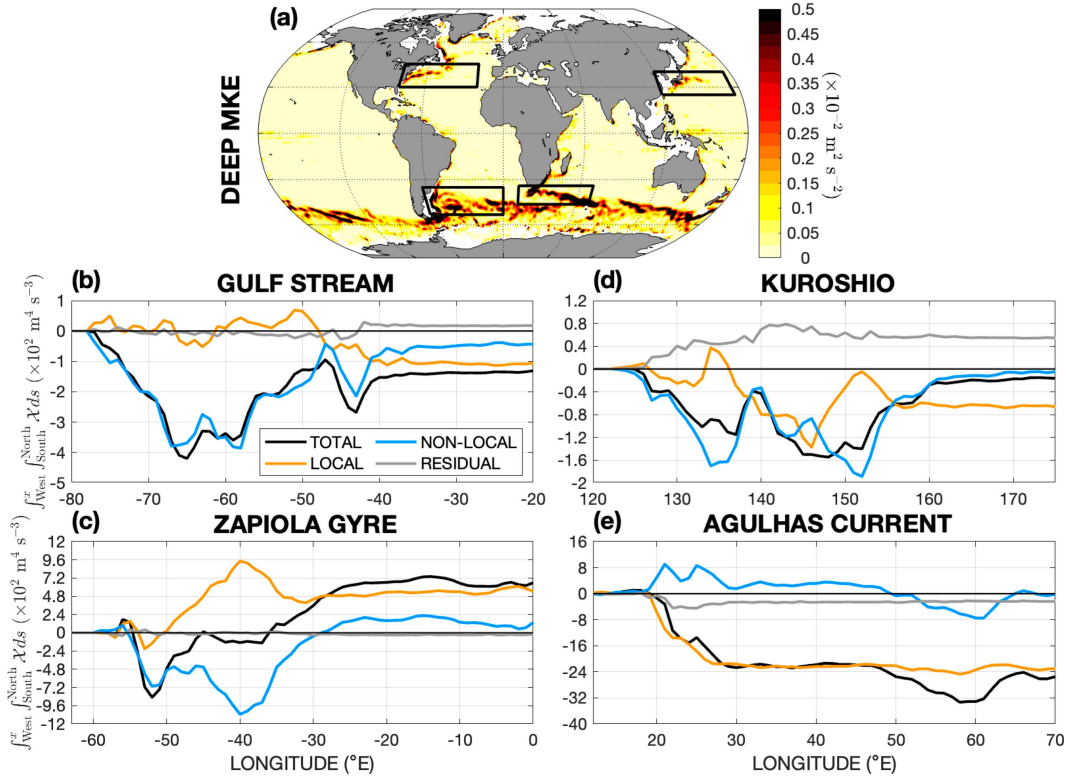


FIG. 11. Convergence of deep eddy-mean flow interactions in western boundary regions. (a) MKE and black boxes depicting western boundary region definition. (b)–(e) Convergence computation for the Gulf Stream, Zapiola Gyre, Kuroshio regions, and Agulhas Current, respectively. Zonally cumulative averages of the components follow (21), where the components are the total ($\mathcal{R}|_{\text{hor}}$; black lines), local ($\mathcal{T}|_{\text{hor}}$; orange lines), nonlocal ($\mathcal{A}|_{\text{hor}}$; cyan lines), and residual [$\mathcal{R}|_{\text{hor}} - (\mathcal{T}|_{\text{hor}} + \mathcal{A}|_{\text{hor}})$; gray lines] terms.

Reynolds stress tensor, so that $\tilde{\mathbf{p}} \cdot \tilde{\mathbf{p}}^\dagger = \tilde{\mathbf{s}} \cdot \tilde{\mathbf{s}}^\dagger = 1$ and $\tilde{\mathbf{p}} \cdot \tilde{\mathbf{s}}^\dagger = 0$. The eigenvalues correspond to $\tilde{\lambda}_{p/s} = \text{EKE}_H \pm \tau/2$, where $\tau^2 = (\overline{u'^2} - \overline{v'^2})^2 + 4\overline{u'v'^2}$. Note that this is an alternative way to decompose the turbulent ellipsoid (Waterman and Lilly 2015).

The decomposition shows regions of high turbulence mainly located within intense western boundary currents, the Antarctic Circumpolar Current, and the equator for both surface and deep observations (Fig. 12). The amplitude is on the order of 0.1 and $10^{-2} \text{ m}^2 \text{ s}^{-2}$ for surface and deep observations, respectively. Primary and secondary eigenvalues have similar patterns, with the primary amplitude being twice as big as the secondary amplitude.

Equivalently to the Reynolds stress tensor eigen-decomposition, we can decompose the mean flow deformation tensor. We obtain

$$\Sigma_H = \hat{\lambda}_p \hat{\mathbf{p}}^\dagger \cdot \hat{\mathbf{p}} + \hat{\lambda}_s \hat{\mathbf{s}}^\dagger \cdot \hat{\mathbf{s}}, = \begin{pmatrix} \hat{p}_x & \hat{s}_x \\ \hat{p}_y & \hat{s}_y \end{pmatrix} \begin{pmatrix} \hat{\lambda}_p & 0 \\ 0 & \hat{\lambda}_s \end{pmatrix} \begin{pmatrix} \hat{p}_x & \hat{p}_y \\ \hat{s}_x & \hat{s}_y \end{pmatrix}, \quad (24)$$

where $\hat{\mathbf{p}} = (\hat{p}_x, \hat{p}_y)$ and $\hat{\mathbf{s}} = (\hat{s}_x, \hat{s}_y)$ are the eigenvectors associated with the eigenvalues $\hat{\lambda}_p$ and $\hat{\lambda}_s$, respectively, of the mean flow deformation tensor, so that $\hat{\mathbf{p}} \cdot \hat{\mathbf{p}}^\dagger = \hat{\mathbf{s}} \cdot \hat{\mathbf{s}}^\dagger = 1$ and

$\hat{\mathbf{p}} \cdot \hat{\mathbf{s}}^\dagger = 0$. The eigenvalues correspond to $\hat{\lambda}_{p/s} = (\Delta \pm \sigma)/2$, where $\sigma^2 = \sigma_n^2 + \sigma_s^2$ is the strain magnitude.

As expected by the dominance of the strain over the divergence, the decomposition reveals two almost opposing eigenvalues that are mainly located within western boundaries, with significant values along intensified western boundary currents, and in the Southern Ocean (Fig. 13). For the surface observations, a strong signature also exists along the equator. The surface eigenvalue amplitudes are twice as strong as the deep ones. They correspond to growing or decaying e -folding times on the order of 10 days depending on their sign.

a. Local interaction decomposition

Following Sévellec et al. (2024), we can revisit the expression of KE transfers. This reads

$$\begin{aligned} \mathcal{T}|_{\text{hor}} &= \overline{\mathbf{u}'_H \cdot \Sigma_H \cdot \mathbf{u}'_H^\dagger} \\ &= \hat{\lambda}_p \tilde{\lambda}_p (\hat{\mathbf{p}} \cdot \hat{\mathbf{p}}^\dagger)^2 + \hat{\lambda}_p \tilde{\lambda}_s (\hat{\mathbf{p}} \cdot \hat{\mathbf{s}}^\dagger)^2 \\ &\quad + \hat{\lambda}_s \tilde{\lambda}_p (\hat{\mathbf{s}} \cdot \hat{\mathbf{p}}^\dagger)^2 + \hat{\lambda}_s \tilde{\lambda}_s (\hat{\mathbf{s}} \cdot \hat{\mathbf{s}}^\dagger)^2. \end{aligned} \quad (25)$$

This suggests that the KE transfers occur through the alignment of the natural direction of turbulence with the natural direction of the mean flow deformation (Fig. 3b). Since the

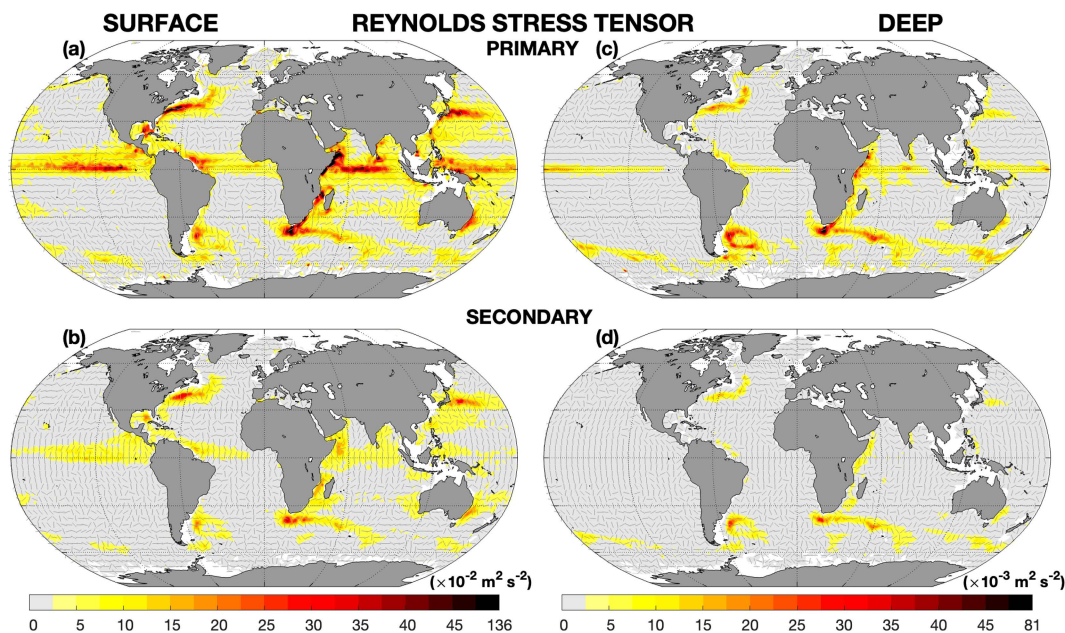


FIG. 12. Eigen-decomposition of the horizontal Reynolds stress tensor. (a),(b) Surface and (c),(d) deep eigen-decomposition, following (23); the eigenvalues are shown with color contours and eigenvectors are shown with gray segments (only on $4^\circ \times 4^\circ$ grid for legibility). For each depth, (a),(c) primary and (b),(d) secondary eigenvectors and eigenvalues are shown.

mean flow is mainly, or even very, strain dominated ($\sigma > |\Delta|$ or $\sigma \gg |\Delta|$, Fig. 5), the eigenvalues of the mean flow deformation tensor are positive and negative, whereas the eigenvalues of the Reynolds stress tensor are both positive. This means that the transfers from MKE to EKE or from EKE to MKE are

controlled by the direction of the eigenvectors of the mean flow deformation tensor. Sévellec et al. (2024) show that these directions correspond to local momentum instability leading to the growth (along $\hat{s}$) and the decay (along $\hat{p}$) of momentum velocity anomalies [consistently with the description of Kundu (1990), for

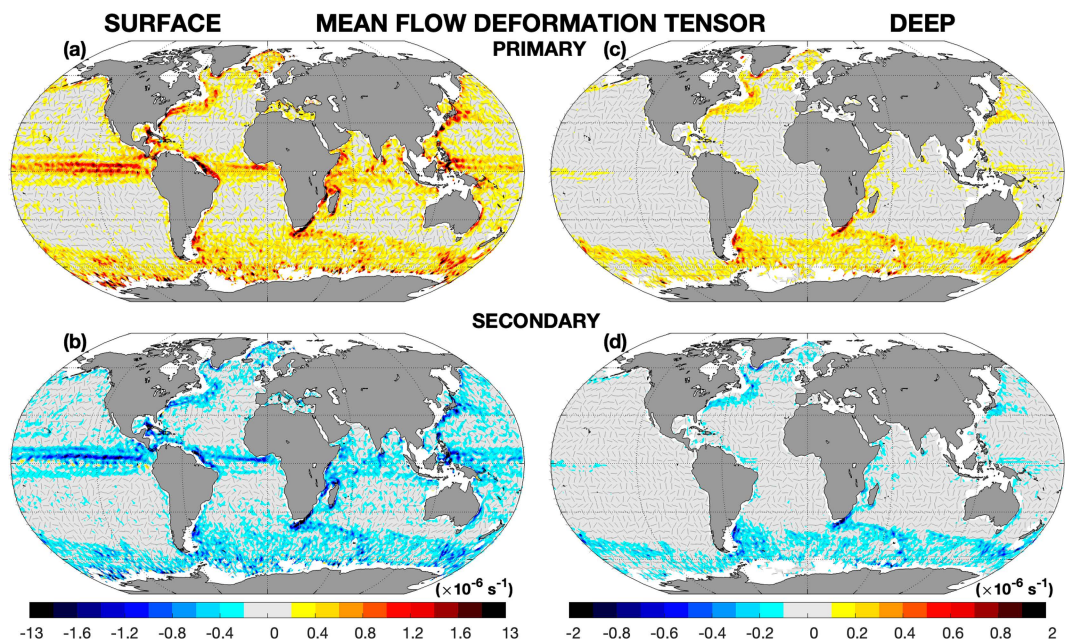


FIG. 13. Eigen-decomposition of the horizontal mean flow deformation tensor. (a),(b) Surface and (c),(d) deep eigen-decomposition, following (24); the eigenvalues are shown with color contours and eigenvectors are shown with gray segments (only on $4^\circ \times 4^\circ$ grid for legibility). For each depth, (a),(c) primary and (b),(d) secondary eigenvectors and eigenvalues are shown.

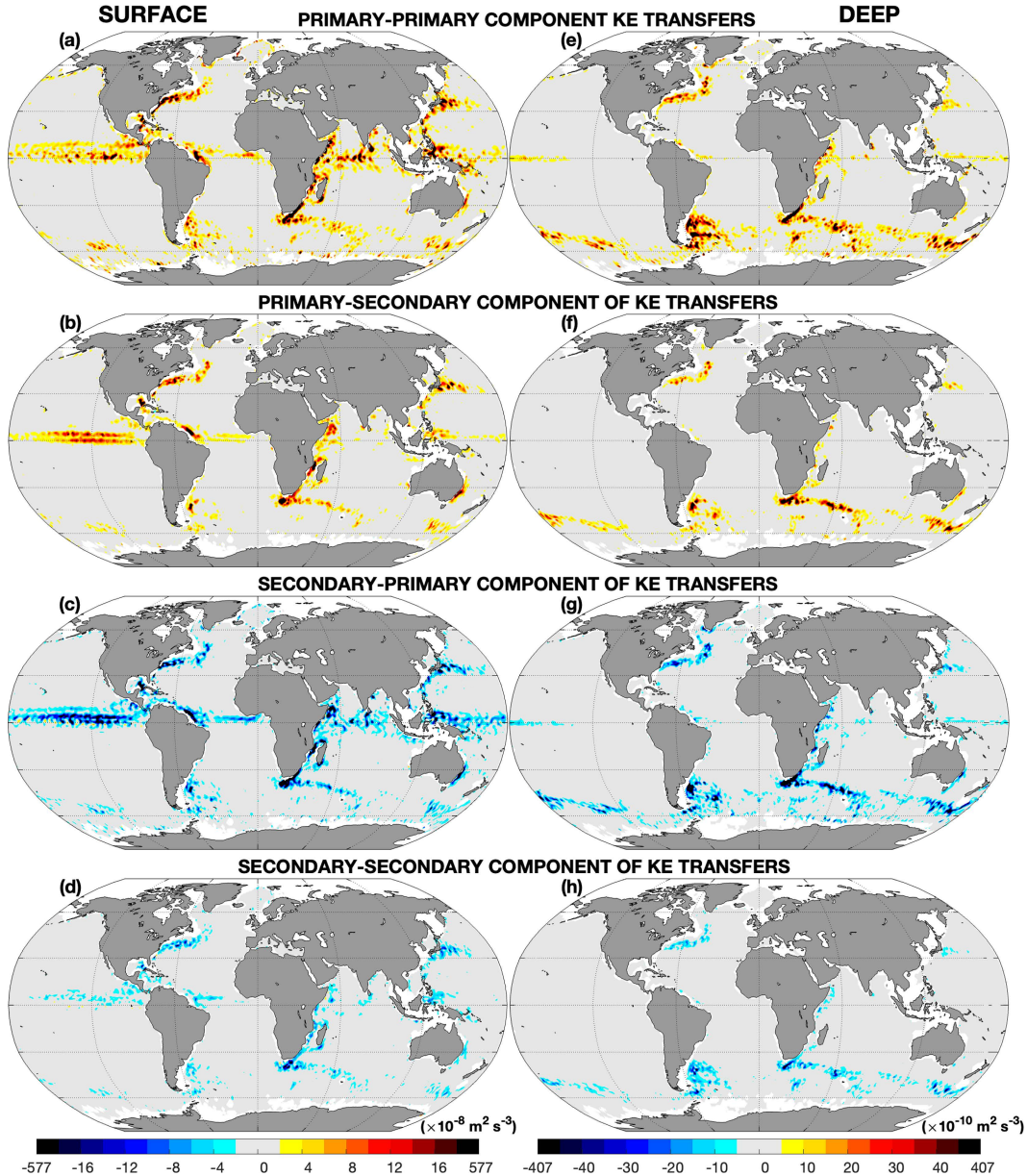


FIG. 14. Components of the KE transfers. (a)–(d) Surface and (e)–(h) deep components of the KE transfers (T_{hor}), following (25). The components correspond to the action of (a),(e) the primary mode of the mean flow deformation with the primary mode of the turbulence $[\tilde{\lambda}_p \tilde{\lambda}_p (\tilde{\mathbf{p}} \cdot \tilde{\mathbf{p}}^\dagger)^2]$, of (b),(f) the primary mode of the mean flow deformation with the secondary mode of the turbulence $[\tilde{\lambda}_p \tilde{\lambda}_s (\tilde{\mathbf{p}} \cdot \tilde{\mathbf{s}}^\dagger)^2]$, of (c),(g) the secondary mode of the mean flow deformation with the primary mode of the turbulence $[\tilde{\lambda}_s \tilde{\lambda}_p (\tilde{\mathbf{s}} \cdot \tilde{\mathbf{p}}^\dagger)^2]$, and of (d),(h) the secondary mode of the mean flow deformation with the secondary mode of the turbulence $[\tilde{\lambda}_s \tilde{\lambda}_s (\tilde{\mathbf{s}} \cdot \tilde{\mathbf{s}}^\dagger)^2]$. The sum of these four components leads to total KE transfers, as described in Fig. 3b. Positive and negative values correspond to transfers from EKE to MKE and from MKE to EKE, respectively.

instance]. In terms of KE transfers, this leads to KE transfers from MKE to EKE or from EKE to MKE, respectively.

The decomposition reveals patterns consistent with the overall location of the interactions: the western boundary currents and the Antarctic Circumpolar Current, as well as the equator for the surface (Fig. 14). The amplitude is forty times

as strong for the surface as for the deep ocean, with values ranging from a few to tens of $10^{-8} \text{ m}^2 \text{ s}^{-3}$ and from tens to hundreds of $10^{-10} \text{ m}^2 \text{ s}^{-3}$ for the surface and deep observations, respectively. Overall, the decomposition in observations confirms that the direction of the transfers is controlled by the strain direction (i.e., eigenvectors of the mean flow deformation

tensor). The alignment of the turbulence with these directions selects the amplitudes of the transfers (consistently with Fig. 3b and Sévellec et al. 2024). It should be noted that both surface primary and secondary eigenvalues are positive in a few locations exactly along the equator (Fig. 13). These peculiar locations lead to positive transfers regardless of the components (Fig. 14). This is due to the dominance of the horizontal divergence over the strain at these locations.

In the equatorial Pacific, the four components of the local interactions display relatively broad shape, spreading from 5°S to 10°N, with maximum absolute amplitudes slightly north of the equator (Fig. 8d). The main negative peak, corresponding to KE transfers toward EKE, derives from the alignment of the primary direction of the turbulence with the negative strain direction. On the other hand, the positive peak, corresponding to eddy rectification at the equator (transfers from EKE to MKE), is well spread between the two natural turbulent directions and their alignment with the positive strain direction. (Again, results are equivalent in the equatorial Atlantic, not shown.)

b. Nonlocal interaction decomposition

Aiming for the decomposition of the nonlocal interactions following (14), we can first express the horizontal eddy advection of RKE as

$$\begin{aligned}\mathcal{A}|_{\text{hor}} &= -\nabla_H \cdot (\mathbf{R}_H \cdot \tilde{\mathbf{u}}_H^\dagger), \\ &= -\nabla_H \cdot (\tilde{\mathbf{u}}_H^\dagger \overline{\text{EKE}}_H),\end{aligned}\quad (26)$$

where $\tilde{\mathbf{u}}_H \stackrel{\text{def}}{=} (\mathbf{R}_H \cdot \tilde{\mathbf{u}}_H^\dagger) / \overline{\text{EKE}}_H$ is the horizontal modified mean velocity corresponding to the horizontal mean velocities stretched or squashed by the horizontal turbulence along its natural directions ($\tilde{\mathbf{p}}$ and $\tilde{\mathbf{s}}$, respectively). This potentially highly divergent velocity reads

$$\tilde{\mathbf{u}}_H = \frac{\tilde{\lambda}_p}{\overline{\text{EKE}}_H} (\tilde{\mathbf{p}} \cdot \tilde{\mathbf{u}}_H^\dagger) \tilde{\mathbf{p}} + \frac{\tilde{\lambda}_s}{\overline{\text{EKE}}_H} (\tilde{\mathbf{s}} \cdot \tilde{\mathbf{u}}_H^\dagger) \tilde{\mathbf{s}}. \quad (27)$$

These last expressions show that the eddy advection of RKE can be reinterpreted as an advection of mean-EKE by the modified mean velocity (Fig. 3a). The stretching and squashing factors are $\tilde{\lambda}_p / \overline{\text{EKE}}_H$ and $\tilde{\lambda}_s / \overline{\text{EKE}}_H$, respectively. Since $\tilde{\lambda}_p + \tilde{\lambda}_s = 2\overline{\text{EKE}}_H$ (i.e., the trace of $\mathbf{R}_H$), we can see that the stretching and squashing are conservative in the sense that it is stretching along the $\tilde{\mathbf{p}}$ direction as much as it is squashing along the $\tilde{\mathbf{s}}$ direction.

The horizontal modification velocities (i.e., $\tilde{\mathbf{u}}_H = \tilde{\mathbf{u}}_H - \bar{\mathbf{u}}_H$, which encompassed the stretching and squashing of the mean velocity by the turbulence) have a strong zonal component (Figs. 15a,b,d,e). At both the surface and depth, it corresponds to a signature along the equator. At the surface, both zonal and meridional velocities are also present in the western boundaries and in the Southern Ocean. At depth, the signature has a similar geographical location, but is relatively more intense in the Southern Ocean. These modification velocities induce an overall more intense flow than the mean velocities along the equator, especially important for the surface velocities, and along the western boundaries (Figs. 15c,f). In the

Southern Ocean, both slowdown and intensification are visible, despite being dominated by the latter, and are particularly important for the deep velocities. Overall, these modification velocities are consistent with regions which have both intense mean current and strong turbulence [as expected from (27) and Fig. 3a]. These velocities lead to a horizontal divergence that is particularly important at the equator for the surface and in the Southern Ocean at depth.

We can then express the two components of the eddy advection of RKE as

$$\mathcal{A}|_{\text{hor}} = -\nabla_H \cdot (\tilde{\mathbf{u}}_H^\dagger \overline{\text{EKE}}_H) - \nabla_H \cdot (\tilde{\mathbf{u}}_H^\dagger \text{EKE}_H), \quad (28)$$

The two components are then the divergence of EKE flux induced by the mean and modification flow, respectively. As mentioned in section 3, the former being the isotropic component and the latter being the anisotropic component of the nonlocal interactions.

Applying this decomposition to our observations suggests that the isotropic component is more important than the anisotropic component for both the surface and deep observations (Fig. 16). This suggests that the nonlocal interactions are primarily captured by a mean advection of mean EKE, albeit a nonnegligible impact of the anisotropic component. The signature of these two components is intense in similar regions: along the western boundaries, especially where intense currents exist, the equatorial region, and the Southern Ocean. It is interesting to note that, despite the signature of zonal modification velocities in the deep equatorial region (related to the important stretching by the zonal turbulence), the anisotropic eddy advection of RKE is weak (because of the weak zonal gradient of EKE along the equator).

Focusing on the equatorial Pacific, where the modification velocities are particularly intense, the main negative peaks of the nonlocal interactions (Fig. 8f) are controlled by the isotropic component (being equivalent to a mean advection of EKE), whereas the positive secondary peaks are controlled by the anisotropic components (suggesting the importance of modification velocities despite weak mean velocities). At the equator, an almost perfect compensation of the isotropic and anisotropic components (suggesting a modification velocities perfectly compensating the mean velocities, equivalently to a non-Doppler effect of EKE) leads to weak nonlocal interactions. (Note that results for the equatorial Atlantic are equivalent, not shown.)

6. Conclusions

Large-scale mean ocean circulation contributes to the redistribution of heat within the ocean. However, in ocean observations, mesoscale eddies are ubiquitous and are often dominating the signal over the large-scale ocean circulation. Hence, the role of the mesoscale eddy turbulence in setting the mean circulation is a timely question of ocean physics. Eddies contribute to the mean circulation through eddy-mean flow interactions that take the form of the mean work of the Reynolds stress. Despite advancements with eddy-resolving numerical models, there remain questions on how these eddy-mean flow interactions act. Beyond that, this

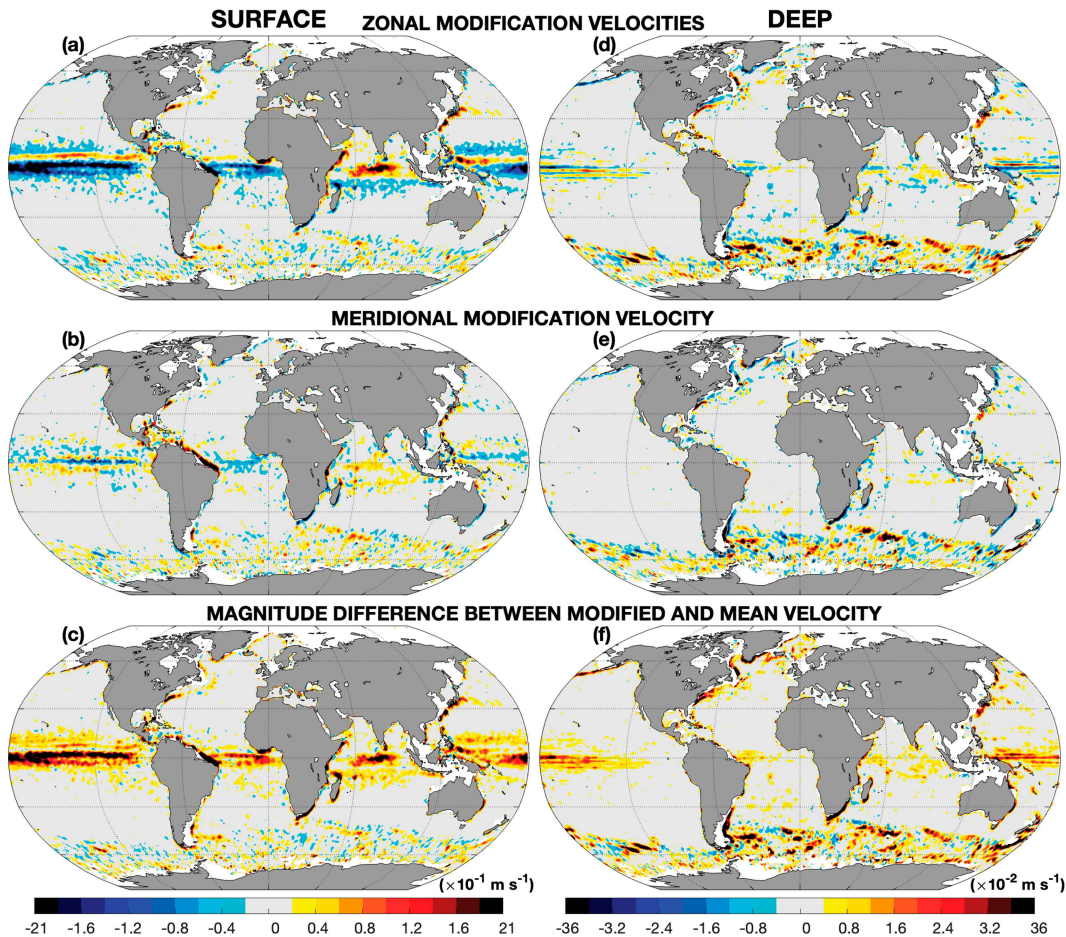


FIG. 15. Modification velocities. (a),(b) Surface and (d),(e) deep modification velocities ($\bar{\mathbf{u}}_H$), in terms of its zonal and meridional component, respectively, as well as (c) surface and (f) deep difference in horizontal magnitude of the modified and of the mean velocities. The modified velocities are stretched and squashed by the turbulence ($\tilde{\mathbf{u}}_H = \bar{\mathbf{u}}_H + \bar{\mathbf{u}}_H$), leading to the addition of the modification velocities ($\bar{\mathbf{u}}_H$) to the mean velocities ($\bar{\mathbf{u}}_H$), as described in Fig. 3a.

mechanism is also poorly observed, which further obscures our understanding of the actual links between eddies and mean circulation and more broadly ocean-scale interactions.

In this study, using a unique observation dataset of Argo float drifts (i.e., the ANDRO dataset), we show for the first time the action of eddy on the magnitude of the mean flow at the surface and at 1000-m depth. We find that the eddy–mean flow interactions are particularly important within intense boundary currents and in the Southern Ocean for both surface and deep observations. These interactions are also important along the equator but only for the surface observations. Values are on the order of 10^{-7} or $10^{-9} \text{ m}^2 \text{ s}^{-3}$, corresponding to a factor of 100 between the action of the surface and 1000-m depth eddies. (Note that the MKE changes only by a factor of 10 between surface and depth.) This suggests that the work of the Reynolds stress is relatively more important for the surface dynamics. Although, this conclusion has to be taken with care since our observed surface and deep velocities are different by nature. Indeed, the former acknowledges processes up to inertial scales, whereas the latter only acknowledges processes up to the mesoscale dynamics.

In the Gulf Stream, Zapiola Gyre, Agulhas Current, and Kuroshio, the eddy–mean flow interactions induce a loss of MKE along the coast and a gain of MKE away from the coast, where the flow is faster and slower, respectively. This is consistent between the surface and the 1000-m depth analyses. For the surface and deep Zapiola Gyre region, a sink and source of MKE is visible along and inside the gyre, respectively. In both the surface equatorial Pacific and Atlantic, the eddy–mean flow interactions lead to a slowdown of the South Equatorial Current. This slowdown occurs through the zonal work of the Reynolds shear stress, consistently with previous analyses of zonally dominated flow (Starr 1968; Nakamura 2024). In all these regions, our observations are evidence of the mesoscale eddy turbulence acting to homogenize the mean flow, forcing it toward a more laminar regime. Our analysis also provides observational evidences of topographic modulation of the total eddy–mean flow interactions in the region of the Crozet and Kerguelen Plateaus (consistently with idealized numerical modeling studies, Youngs et al. 2017; Cai et al. 2022).

Following Chen et al. (2014, 2016), the eddy–mean flow interactions can be decomposed into local and nonlocal eddy–

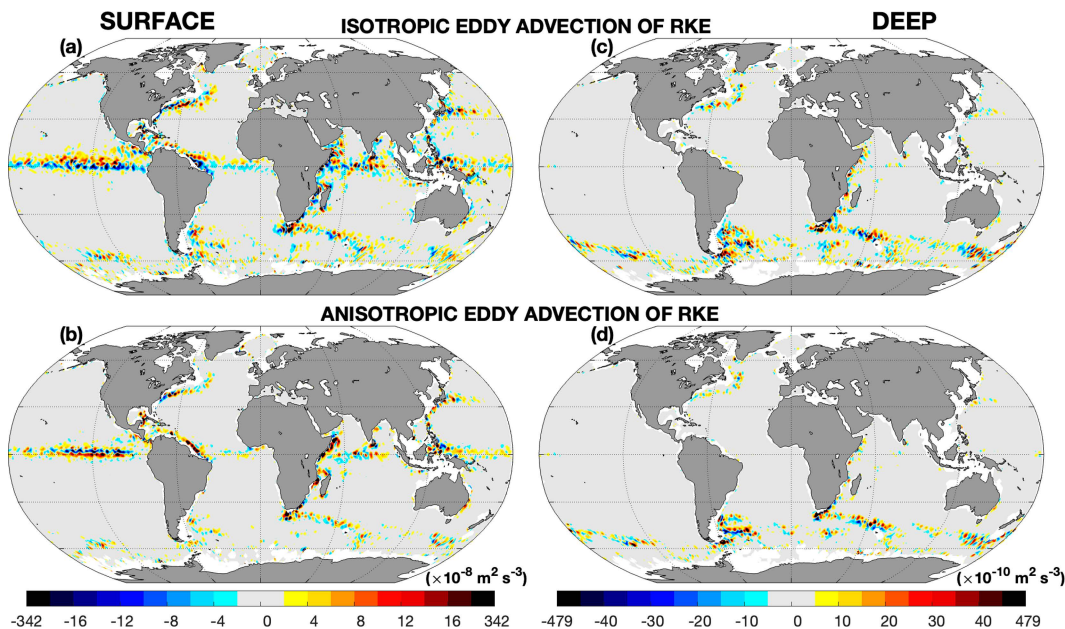


FIG. 16. Components of the eddy advection of RKE. (a),(b) Surface and (c),(d) deep components of the nonlocal horizontal eddy–mean flow interactions, corresponding to the eddy advection of RKE ($A|_{hor}$), following (28). The two components correspond to the divergence of EKE flux induced by the mean and the modification flow ($\bar{u}_H$ and $\tilde{u}_H$), respectively. This also corresponds to the components induced by the isotropic and anisotropic components of the turbulence.

mean flow interactions. These two interactions have fundamentally different physical meaning. The local interactions correspond to KE transfers between MKE and EKE. Consistently with Sévellec et al. (2024), we show that it corresponds to the projection of the turbulence directions on the mean strain directions. This leads to two competing effects depending on the alignment of the turbulence along the positive or negative strain directions, allowing for a dynamical attribution of EKE-to-MKE and MKE-to-EKE transfers, respectively. The nonlocal interactions correspond to an eddy advection of RKE, which only acts to redistribute KE and can be reinterpreted as the advection of EKE by modified velocities. The velocity “modification” is related to the anisotropic component of the turbulence (associated with the Eliassen–Palm flux tensor; Trenberth 1986). It is particularly important at the equator and, to a lesser degree, along boundary currents and the Southern Ocean.

Our analysis also reveals an important compensation at the $2^\circ \times 2^\circ$ scale (i.e., grid scale of our analysis) between local and nonlocal interactions. This suggests that, at this scale, the total eddy–mean flow interactions are weak and that the energization of eddies does not induce a similar mean flow energy loss: KE can be reorganized through eddy advection of RKE to compensate for KE transfers. However, this reorganization implies that the mean flow will have to compensate for these KE transfers somewhere else or at larger scale, since the nonlocal interactions cannot have a global net effect. Studying several regions of intense interactions, the Gulf Stream, the Zapiola Gyre, the Agulhas Current, the Kuroshio, and the equatorial Pacific, we find that the compensation occurs over

a typical effective scale of several tens of degrees, except at the equator where the compensation occurs over several degrees. Despite providing observational evidences of the existence of an effective scale, we have not characterized its physical constraints. We hypothesize that this scale is controlled by the typical length scale of the dominant turbulence of the studied region (i.e., equatorial instability waves, mesoscale turbulence). However, further studies are needed to test our proposed hypothesis or alternative ones (e.g., a control by the typical mean flow length scale).

Our analysis suffers from several inherent limits that pave the way for future work directions. Our analysis is limited to the study of horizontal flow at the surface and at 1000-m depth. The velocities derived for these two depths are different in nature: sampling processes up to inertial scales and up to mesoscale dynamics for the surface and depth, respectively. Also, having only two depths limits the three-dimensional interpretation of the eddy–mean flow interactions. Reproducing this analysis with a state-of-the-art eddy-resolving ocean model will be the goal of future analyses. Beyond that, the eddy–mean flow interactions at the surface, by impacting the amplitude and geographical shape of the surface velocities, suggest potential impacts on the air–sea momentum exchanges. Attributing the role of the different components diagnosed in this study in an ocean–atmosphere coupled model should help us better understand the role of eddies in the climate.

Despite progresses in ocean modeling and in particular in modeling of ocean mesoscale to submesoscale processes, the explicit simulation of these features for global, long climate

simulations remains challenging. Our observations and analyses provide a step forward characterizing the role of eddy–mean flow interactions. In particular, we have shown specific key properties:

- the compensation between the local and the nonlocal interactions at $2^\circ \times 2^\circ$ scale,
- the control of the local interactions by the mean flow deformation tensor,
- the formulation of the nonlocal interactions in terms of a modified advection of EKE.

The results of this observational study can be used as a “ground truth” over which eddy-resolving ocean numerical simulations can be compared and validated.

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Data availability statement. The ANDRO dataset used in this study corresponds to the 2024 Release (key: “109566”) and is available through the www.seanoe.org platform (<https://doi.org/10.17882/47077>). The CMEMS global reanalysis data (GLORYS12V1) used in this study are available on the Copernicus website (<https://doi.org/10.48670/moi-00021>).

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