



Decadal variability of the Antarctic Circumpolar Current in an idealized chaotic ocean–atmosphere coupled model

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ARTICLE INFO

Communicated by Dmitry Pelinovsky

Keywords:

Synchrony
Chaos
Idealized modelling
Extreme events
Ocean circulation

ABSTRACT

An idealized ocean–atmosphere coupled model exhibiting chaotic behaviour is developed to represent the dynamics of the Antarctic Circumpolar Current, one of the most intense ocean currents on Earth. On a meridional-depth plane, the model is driven by the competition between eddy-induced advection and wind-induced advection, where a feedback of meridional ocean temperature contrast on the wind intensity is allowed. Along the zonal direction, the lattice of individual meridional-depth dynamics are zonally coupled through zonal turbulent diffusivity. We show that ocean–atmosphere coupling is crucial to set the chaotic dynamics, whereas the zonal turbulent diffusivity is crucial for setting a realistic intensity of the Antarctic Circumpolar Current. For a value of zonal turbulent diffusivity between 200 and 10 000 m² s^{−1} the zonal mean transport is 121 Sv (1 Sv ≡ 10⁶ m³ s^{−1}) and exhibits a main spectral signature at 8.1 years. The diffusivity coefficient controls the level of zonal synchronicity of the Antarctic Circumpolar Current going from a fully zonally-independent system to a fully synchronized lattice with no zonal variation. The realistic regime corresponds to an intermediate level of synchronicity, where the footprint length scale measured the zonal distance over which local meridional-depth dynamics are impacting each other. This zonal length scale footprint is set by the competition between the zonal turbulent diffusivity, zonally coupling the dynamics, and the extreme event (>82 percentile) return period (~48.5 years), resetting the local meridional-depth dynamics.

1. Introduction

Since the pre-industrial era the Earth climate has changed, is still changing, and is projected to further change in the next century at an unrecorded rate [1]. This disequibrated view can be seen through the global warming of the Earth surface over the last century [1], for instance. Given its ability to store heat, 90% of the heat excess is stored in the ocean [2], the ocean is a modulator of climate variations. Hence it becomes more and more timely to better understand the dynamics of the ocean.

The Antarctic Circumpolar Current (ACC) is a unique ocean current as it can travel eastward throughout the Southern Ocean without continent restriction. Hence it is one of the most intense ocean current on Earth. This large scale circulation is classically measured in Sverdrup (1 Sv ≡ 10⁶ m³ s^{−1}) and is estimated between 125 Sv and 150 Sv from observations [e.g., 3–7]. This circulation contributes to the transport of heat, freshwater, and other ocean properties. In particular, it acts as a mediator between three ocean basins, connecting the Atlantic Ocean, the Indian Ocean, and the Pacific Ocean. Numerical studies of

the ACC, in an ocean–atmosphere coupled context, have suggested the interannual to decadal variability of the ACC [8–10].

Traditionally the ocean circulation in Southern Ocean has been described through the competition of wind driven circulation and eddy-induced circulation in a zonally averaged framework [e.g., 11,12]. The former is induced by the atmospheric wind which, through a surface momentum stress, accelerate the ocean surface leading to an Ekman transport [13] and a wind-induced circulation [the Deacon cell, 14,15]. The latter acknowledges the role of mesoscale eddies [16], that are ubiquitous in the ocean [17]. This mesoscale turbulence induces an eddy-driven circulation [18]. Hence these two circulations largely compensate each other giving rise to the residual circulation [19,20].

Despite being particularly revealing, this traditional view suffers from several shortcomings. Indeed, within the traditional view the wind-induced circulation is only seen as a forcing of the atmosphere onto the ocean. However, this hypothesis is questionable and a range of studies suggests that the Southern Ocean dynamics is better described within an ocean–atmosphere coupled framework [8–10]. For instance, the ocean is able to exchange heat with the atmosphere, modifying

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the circulation of the latter and so the wind-stress perceived by the ocean [21,22]. Also a range of more recent studies has highlighted the importance of the zonal disparity for representing the ACC [23,24]. This questions the validity of the zonally averaged framework used in the traditional view of the ACC.

The zonal disparity, or, more accurately, the zonal coherence over a restricted length scale, can be reinterpreted as local synchronization. This means that the ACC can be described as the sum of local phase coherent jets. However the global phase coherence (i.e., full synchrony), allowing for an accurate zonally averaged framework used in the traditional ACC description, seems unlikely and has been contradicted by observations [23,24]. The topic of synchronicity and spontaneous order have crossed over a large range sciences [25]. Hence, self-organization framework provides quantitative tools to assess and interpret the zonal coherence (partial synchronicity) of the ACC [26]. Local synchronicity and intermittency have been studied over a large range of context [e.g., 27]. In particular, as it will be explicitly shown later in this study, the occurrence of phase coherence and intermittency in a one-dimensional lattice of equivalent chaotic oscillators appear relevant for the study of the ACC [e.g., 28–30].

In this study, we have developed an idealized model of the ACC and of the dynamics of the Southern Ocean. This idealized model follows the residual circulation framework but includes two key elements: the wind feedback and the zonal disparity. The former leads to an instability and a chaotic behaviour of the system with decadal variations, whereas the latter corresponds to building a lattice of these chaotic oscillators that are zonally coupled through turbulent diffusion. Studying the impact of the coupling factor on the lattice behaviour, we show that realistic ACC transport is only achieved by intermediate coupling leading to local, partial synchronicity. The local synchronicity is rationalized by the competition between the zonal diffusion, coupling the lattice, and the occurrence of extreme events, decoupling the lattice.

The manuscript is organized as follows: the studied set of equations and its validity for the ACC dynamics are presented in Section 2; the dynamics of a single chaotic oscillator and the role of the wind feedback is given in Section 3; in Section 4, the full locally-coupled lattice dynamics is presented and the role of zonal diffusion in coupling and inducing synchrony is studied; discussion and conclusions are provided in Section 5.

2. Set of equations

To represent the ACC we start from the most complex set of equations of our study. (Specific simplified version will be studied later.) These equations correspond to temperature and momentum evolution along an ocean streamline, within the depth-latitude space, and projected onto Fourier modes as fully described in the two-part appendix. [Also full derivation can be found in 31–33, for instance.] It follows a long history of loop model derived to study ocean dynamics [32–42]. This set of equations represents temperature evolution set by advection and diffusion together with an anomalous momentum equation sets by wind stress and friction. This latter mimics an anomalous Ekman transport [13]. As described below it is valid for the large-scale ocean circulation and in particular for the ACC dynamics. This reads:

$$\dot{\omega}_i = -\gamma(\omega_i + \epsilon y_i), \quad (1a)$$

$$\dot{y}_i = \Omega_i z_i - K y_i + \tilde{\kappa} (-2y_i + y_{i-1} + y_{i+1}), \quad (1b)$$

$$\dot{z}_i = -\Omega_i y_i - K z_i + H + \tilde{\kappa} (-2z_i + z_{i-1} + z_{i+1}), \quad (1c)$$

where i the loop index (from 1 to N , N being the total number of loops), $\Omega_i = \Omega_0 + \omega_i + k y_i$ is the residual circulation (i.e., the total circulation), $\Omega_0 = -0.18 \text{ yr}^{-1}$ ($1 \text{ yr} \equiv 1 \text{ year}$) is the mean circulation representative of mean wind forcing effect [i.e., the Deacon cell, 14,15, set to $\Omega_0 \times (W h_{\text{ACC}} L) / (2\pi \times 86400 \times 365.25) \simeq -30 \text{ Sv}$], ω_i is the

anomalous circulation set by the wind anomaly, y_i and z_i are the North-South and bottom-top temperature differences, $K = 0.1 \text{ yr}^{-1}$ is the along the loop diffusivity coefficient, $\tilde{\kappa} = \kappa \times (N^2 / W^2) \times (86400 \times 365.25)$ is the inter-loop or zonal re-scaled diffusivity coefficient (and κ is the zonal diffusivity coefficient), $k = 0.1 \text{ yr}^{-1} \text{ K}^{-1}$ is the Gent–McWilliams coefficient [18], $H = -1 \text{ K yr}^{-1}$ is bottom-top heating difference, $\gamma = 1 \text{ yr}^{-1}$ is the inverse of the atmospheric relaxation timescale, $\epsilon = 0.6 \text{ yr}^{-1} \text{ K}^{-1}$ is the wind feedback coefficient, $W = 21 \times 10^6 \text{ m}$ is the zonal extent of the Southern Ocean, $L = 800 \times 10^3 \text{ m}$ is the meridional extent of the ACC, $h_{\text{ACC}} = 2000 \text{ m}$ is the vertical extent of the ACC (i.e., $\sim$ Drake passage), and the factor 86400×365.25 converts a year in seconds. All values are set as realistic values for the ocean circulation. (Note that the sensitivity of a range of values of H and ϵ will be tested and value of κ is not yet given since it is our control parameter.)

In this framework, the ocean temperature (y_i and z_i) of a single loop is affected by advection by the residual circulation (Ω_i), diffusion within the loop and between loops. The bottom-top temperature difference is also driven by the heating ($H < 0$ corresponding to heating at the surface and cooling at the bottom). The residual circulation is controlled by three terms: the mean circulation (set by the mean wind), the anomalous circulation (set by the feedback of the North-South temperature difference on the circulation), and the Gent–McWilliams eddy velocity (proportional to the North-South temperature difference). Here all the loops have the same parameters and are locally coupled through the zonal diffusion with a periodic boundary condition (Fig. 1). Note that all diffusion terms (i.e., along-loop diffusion and zonal diffusion) are representative of turbulent closure parameterizing the (deformation) effect of small-scale turbulence as small-scale turbulent diffusion [43,44].

Here we did not impose a meridional temperature contrast. This follows a range of studies showing that in the Southern Ocean the meridional temperature contrast is a consequence of the ocean dynamic adjustment through wind-upwelling of the cold deep water [e.g., 19,20, 45–47]. Hence the existence meridional temperature contrast could be used to validate the good behaviour of the idealized model (as it will be shown in Section 3.1), rather than being imposed by external forcing.

A key aspect of the idealized model is how the flow is established given the intensity of the meridional temperature contrast. Here we follows the principle of turbulent closure suggested by Gent and McWilliams [18]. However unlike the classical expression, which relies on the ratio of horizontal to vertical density gradients, we take the flow only proportional to the meridional density gradient [consistently with baroclinic instability condition, 48]. This assumptions is based on the fact that in the ACC the dynamics is primary dominated by the horizontal flow in comparison to the vertical flow [49]. Under constant stratification, or a linearization around a reference stratification, the proposed approach and the classical Gent–McWilliams mesoscale-eddy-closure are equivalent.

One key assumption, that leads to (1a), is that the ocean temperature difference is impacting the wind, hence the wind stress, and hence the wind driven circulation. Here the rationale is that large-scale meridional ocean temperature difference warms and cools correspondingly the meridional atmospheric temperature difference, which by geostrophy increase or decrease the zonal wind over the Southern ocean. This wind drives a wind-stress accelerating/decelerating the circulation through an Ekman transport. Assuming that the wind anomalies induced by the surface ocean temperature contrast remains small compared to the mean atmospheric wind, we can applied a linear proportionality coefficient between the meridional temperature contrast (y_i) and the induced circulation (ω_i). This large-scale ocean feedback on wind is consistent with previous analyses [21,22].

One blind-spot of our description is the zonal transport and its impact on the temperature field. Because of the geostrophic balance on the large-scale considered the meridional temperature difference induced a zonal transport [50,51]. Following [52] and assuming weak velocity

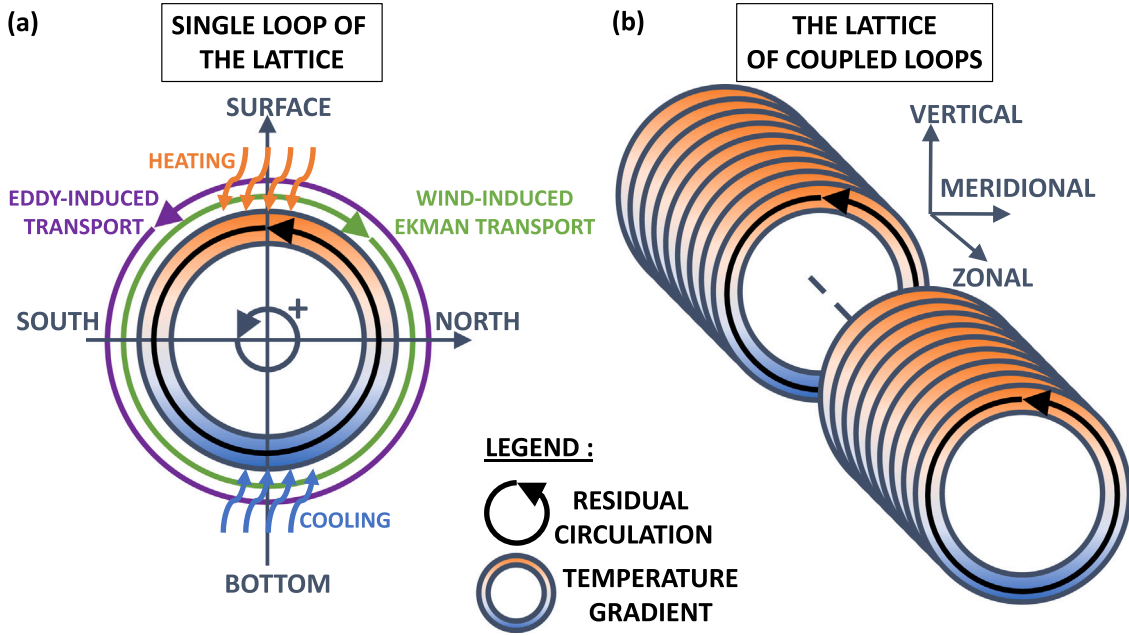


Fig. 1. Schematic of (a) a single loop and (b) the lattice of locally coupled loops. A single loop corresponds to three variables: meridional and vertical temperature differences (y_i and z_i , respectively) and a residual circulation (Ω_i). These variables are forced by wind-induced Ekman transport (both mean and anomaly: Ω_0 and ω_i , respectively) and heat flux (H). The lattice corresponds to the local coupling of 200 loops through zonal diffusion with a periodic boundary condition.

at the ocean bottom, This geostrophic transport can be estimated with the zonal temperature difference: $\psi_i = \iint_{\text{ACC}} u_i dv \approx -\frac{\alpha g h h_{\text{ACC}}}{f} y_i$, where u_i is the zonal velocity, v is the unit surface of the depth-latitude plane, $\alpha = 2 \times 10^{-4} \text{ K}^{-1}$ is the thermal contraction coefficient, g is the Earth gravity acceleration, $h = 4500 \text{ m}$ is the ocean depth of, $h_{\text{ACC}} = 2000 \text{ m}$ is the depth of ACC, and $f = -10^{-4} \text{ s}^{-1}$ is the Coriolis parameter. Physically this transport is composed by two terms, a barotropic and a baroclinic component (given our idealized model, the latter being restricted to the first baroclinic model equivalently to a $1\frac{1}{2}$ -layer model). The barotropic one advects the full water column consistently over the vertical, whereas the baroclinic component leads to a shear with opposite direction of advection between the surface layer and the bottom layer. However because of the non-Doppler effect [53–55] the temperature anomaly described by the loop are not transported through the baroclinic component [perfect compensation of the mean and anomalous advection, 52]. Hence only remains the barotropic zonal transport. It induces a vertically-consistent zonal transport. Hence loops are all travelling zonally through advection by this transport, but there is no net zonal transport between loops. In this study we will disregard this barotropic transport, by acknowledging that loops are not spatially set, but their location changes in time accordingly with the barotropic transport. Also because of the curvature of the Earth there exists Rossby wave propagation [53]. This term will induce an equivalent velocity that can be added to the barotropic flow. Alternatively, one can assume the validity of the f -plane approximation [48], where these waves are absent.

In this type of model, there is no explicit convective process that remove static instability (preventing $z > 0$). However $z > 0$ is still an unstable state for the loop and advection will reestablish a stable stratification [34,38,39].

Finally salinity is neglected in our study for simplicity purpose. It is possible to expand the loop equations to include salinity differences. However salinity does not feedback on the atmospheric temperature. Also idealized studies suggest the possible representation of a realistic ACC when using only temperature [56,57].

Our set-up is to use $N = 200$ loops so that each of them represents a zonal extension of 1.8° of longitude. This value is set so that it is equivalent to eddy-less ocean model configuration. This allows the validity

regime of the Gent–McWilliams mesoscale-eddy-closure [18] and the comparison with typical state-of-the-art climate models [i.e., horizontal resolution comparable to Coupled Model Intercomparison Project Phase 6 – CMIP6 – exercise, 58].

3. A single ACC loop model

Before describing the dynamics of the full lattice, we investigate the behaviour of a single loop. A single loop can be physically interpreted as the behaviour under the absence of coupling ($\kappa \rightarrow 0$) or under a fully locked coupling ($\kappa \rightarrow +\infty$). The former corresponds to the absence of turbulent mixing, which is somehow unrealistic, whereas the latter corresponds to the absence of zonal disparity, where all the loops are fully synchronized. An alternative rationale for the last case is that it corresponds to zonal mean, where zonal disparities are ignored. This zonally-averaged view is often used in the description of the Southern Ocean dynamics and the ACC in particular [e.g., 11,12,19,20], but more recently questioned [e.g., 23,59].

The description of the single loop will be done in two steps. First we will describe the dynamics in the absence of wind feedback ($\epsilon = 0$), which is the most traditional theoretical view [19,20]. Then we will incorporate the wind feedback ($\epsilon \neq 0$) and show the occurrence of chaotic dynamics.

3.1. No wind feedback ($\epsilon = 0$) and no zonal diffusion ($\kappa = 0$)

As suggested above, the absence of zonal diffusion makes the set of equations independent of the index i . After acknowledging the absence of wind feedback, this leads to the new set of equations:

$$\dot{\omega} = -\gamma\omega, \quad (2a)$$

$$\dot{y} = (\Omega_0 + \omega + ky)z - Ky, \quad (2b)$$

$$\dot{z} = -(\Omega_0 + \omega + ky)y + H - Kz. \quad (2c)$$

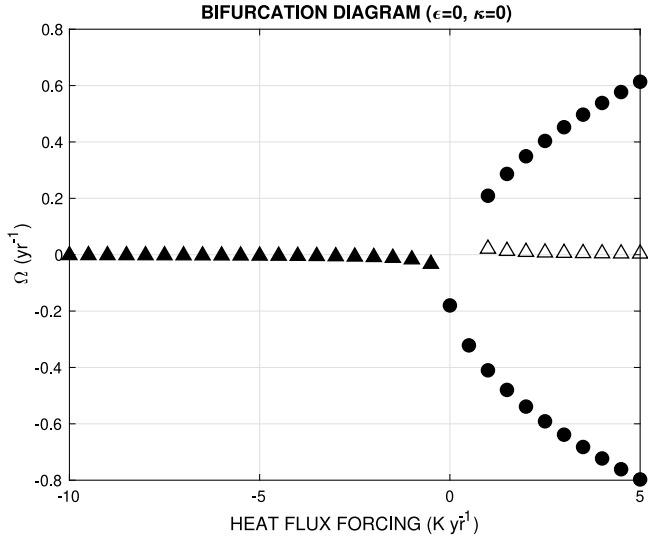


Fig. 2. Bifurcation diagram of a single ACC loop model without wind ($\epsilon = 0$). The circles or triangles denote steady states with or without an oscillatory least damped eigenvalue, respectively. Plain or empty marker denotes a stable or an unstable steady state.

Here the momentum equation is trivial, and meridional and vertical temperature differences are advected and diffused. At the steady state ($\dot{\omega} = \dot{y} = \dot{z} = 0$), we obtain that $\bar{\omega} = 0$ (no anomaly of advection) and

$$\bar{z} = -\frac{\Omega_0 + k\bar{y}}{K}\bar{y} + \frac{H}{K}, \quad (3a)$$

$$k^2\bar{y}^3 + 2\Omega_0 k\bar{y}^2 + (\Omega_0^2 + K^2 - kH)\bar{y} - H\Omega_0 = 0, \quad (3b)$$

where $\bar{\cdot}$ denotes the values at steady state.

A linear stability analysis of the steady states leads to

$$(\lambda + \gamma) [\lambda^2 + (2K - k\bar{z})\lambda + K^2 - Kk\bar{z} + (\Omega_0 + \bar{\omega} + 2k\bar{y})(\Omega_0 + \bar{\omega} + k\bar{y})] = 0, \quad (4)$$

where λ represents the eigenvalues. If the real part of any solution is positive it shows that the steady state is asymptotically unstable. Alternatively, if the real part of all the solutions is negative it shows that the steady state is asymptotically stable.

This analysis shows the existence of a single stable steady state for realistic value of the heat flux parameter (i.e., $H < 0$ — cooling at the bottom and warming at the top). This steady state corresponds to a weak residual circulation. The eddy flux ($k\bar{y}$) compensates a large part of the mean circulation: $\bar{\Omega} \ll \Omega_0$. An imperfect pitchfork bifurcation appears at $H = 0 \text{ K yr}^{-1}$ leading to an unstable steady state and two stable steady state for $H > 0$ (Fig. 2). The imperfection parameter is the asymmetric term of Eq. (2), namely the mean advection (Ω_0). Setting Ω_0 to zero, corresponding to the absence of the westerlies over the Southern Ocean, leads to a supercritical pitchfork bifurcation.

Setting the heat flux to $H = -1 \text{ K yr}^{-1}$, we obtain realistic North-South and bottom-top temperature differences of the order of a few kelvins. With this reference parameter value, we find an eastward flow of 281 Sv. This is above observational estimates [e.g., 3–6], but overall in a realistic range given the idealization of the model and further approximations (i.e., no wind feedback — $\epsilon = 0$ — and no zonal disparity — $\kappa = 0$).

It is worth noting that an even simpler set of equations can be derived assuming the absence of diffusion ($K = 0$). In this case the equilibrium equation becomes simply $\Omega_0 + k\bar{y} = 0$, which implies a residual circulation equal to zero (i.e., no temperature advection at equilibrium). This is the typical first-order traditional explanation of

the Southern Ocean dynamics [19,20]. This suggests that the meridional temperature difference is set so that the eddy-induced transport exactly compensate the Deacon cell, so that no residual flow exists [46, 47]. This would lead to an eastward flow of 310 Sv (also above observational estimates). This approximation also implies that the vertical temperature difference increases to infinity, since with the absence of advection and diffusion, the vertical heat forcing will accumulate in the vertical temperature difference.

A rationale of this steady-state, and in particular of its stability, can be described through the action of the Gent–McWilliams component of the circulation. Indeed this term acts as a stabilizing factor. If a positive anomaly of the North-South temperature difference occurs ($y' > 0$, where $'$ indicates an anomaly), this leads to a positive circulation ($ky' > 0$), counterbalancing the negative mean flow ($\Omega_0 < 0$). Then, given the vertical heat flux, it advects a cold water from the bottom to the North and warm water from the top to the South. This damps the initial anomalous North-South temperature difference. Hence the Gent–McWilliams component of the circulation, which mimics the large-scale effect of baroclinic instability [60–63], acts as a damping mechanism on temperature anomaly.

3.2. Impact of the wind feedback

Now that we have estimated the dynamics in the absence of large-scale wind feedback, we add it to the set of equations, but we remain within a single loop framework. This new set of equations reads:

$$\dot{\omega} = -\gamma(\omega + \epsilon y), \quad (5a)$$

$$\dot{y} = (\Omega_0 + \omega + ky)z - Ky, \quad (5b)$$

$$\dot{z} = -(\Omega_0 + \omega + ky)y + H - Kz. \quad (5c)$$

The main difference comes from the momentum equation when the wind feedback accelerate the anomalous circulation.

We now compute the steady state, following:

$$(k - \epsilon)^2 \bar{y}^3 + 2\Omega_0(k - \epsilon)\bar{y}^2 + [\Omega_0 + K^2 - H(k - \epsilon)]\bar{y} - H\Omega_0 = 0$$

$$\text{with } \bar{\omega} = -\epsilon\bar{y}, \text{ and } \bar{z} = -\frac{\Omega_0 + \bar{\omega} + k\bar{y}}{K}\bar{y} + \frac{H}{K}.$$

The stability of this steady state is given by the sign of the real part of the least damped eigenvalue. The eigenvalues are given by the roots of

$$\begin{aligned} &\lambda^3 + (\gamma + 2K - k\bar{z})\lambda^2 + [K\gamma + (K + \gamma)(K - k\bar{z}) \\ &+ \gamma\epsilon\bar{z} + (\Omega_0 + \bar{\omega} + k\bar{y})(\Omega_0 + \bar{\omega} + 2k\bar{y})]\lambda \\ &+ K\gamma(K - k\bar{z}) + \gamma(k - \epsilon)\bar{y}(\Omega_0 + \bar{\omega} + k\bar{y}) + K\gamma\epsilon\bar{z} + \gamma(\Omega_0 + \bar{\omega} + k\bar{y})^2 = 0, \end{aligned}$$

This analysis reveals the occurrence of an imperfect pitchfork bifurcation when the wind feedback parameter (ϵ) is increased (Fig. 3). First there only exists a stable branch, from $\epsilon = 0 \text{ yr}^{-1} \text{ K}^{-1}$ until $\epsilon = 0.175 \text{ yr}^{-1} \text{ K}^{-1}$. Two other unstable branches occur from $\epsilon = 0.175 \text{ yr}^{-1} \text{ K}^{-1}$. For higher value, from $0.25 \text{ yr}^{-1} \text{ K}^{-1}$, the stable branch becomes unstable (through a Hopf bifurcation). From there and for higher values tested (up to $1 \text{ yr}^{-1} \text{ K}^{-1}$), there is three unstable steady states. This is typical of chaotic system such as the idealized atmospheric convective model [64], the rotating water wheel [65], or the idealized model of the large-scale ocean circulation of the North Atlantic [32].

To further clarify the dynamics of a single loop we performed numerical time integration of (5). These numerical simulations have been done using a 4th order Runge–Kutta method with a time stepping of 0.1 yr. We did 4 simulations using 0.25, 0.35, 0.4, and $0.45 \text{ yr}^{-1} \text{ K}^{-1}$ as parameter values for the wind feedback coefficient (Fig. 4). These values correspond to the regime where there exists three unstable steady states (Fig. 3). The simulations show that the chaotic behaviour

is visible from values of $0.25 \text{ yr}^{-1} \text{ K}^{-1}$. For higher values of the coefficient the trajectories describe a more and more complex/intertwined attractor typical of chaotic dynamics (i.e., strange attractor). Here the trajectories evolve as “a ball in a pinball machine” [65]. As mentioned previously this behaviour is equivalent to atmospheric convection described by Lorenz [64] or to the rotating water wheel described by Strogatz [65].

In this context we set the wind feedback coefficient to $0.6 \text{ yr}^{-1} \text{ K}^{-1}$. This, at steady state ($\bar{\omega} = -\epsilon \bar{y}$), leads to an anomalous circulation of -2 Sv ($\approx -\epsilon \bar{y} \times (Wh_{\text{ACC}}L)/(2\pi N \times 86400 \times 365.25)$) for a meridional temperature difference of $\bar{y} = 4 \text{ K}$. This corresponds to the regime where the chaotic attractor is fully developed. In this regime the trajectories described a strange attractor in the phase space, their time evolution reveals switch between positive and negative values of zonal transport with a main spectral signature in the range of 35–40 yr (Fig. 5). Here, the mean zonal transport is -39 Sv , suggesting an unrealistic westward mean flow of the ACC.

Since in absence of wind feedback the transport is eastward, the occurrence of this unrealistic transport is a consequence of the wind feedback. Indeed the wind feedback leads to an instability of the system, pushing the system away from the traditional equilibrate view of the residual circulation, described in previous subsection [19,20]. The instability can be described in 3 steps. (1) A positive anomalous zonal temperature contrast ($y' > 0$) leads to a decrease of the circulation ($\dot{\omega} < 0$), and so intensifying the mean circulation ($\omega < 0$). (2) Given the vertical heat flux (warming the top and cooling the bottom), this intensified circulation brings warm water North and cold water South. (3) This reinforces the initial positive anomalous zonal temperature contrast.

Overall we have shown that the wind feedback leads to the destabilization of the loop model, leading to chaotic behaviour. This instability also implies an unrealistic westward mean transport. Whereas the addition of large-scale wind feedback is, a priori, wishable since it exists, it appears incompatible with realistic ACC transport within our zonally-averaged framework encompassed by a single loop. Hence in the rest of the study we will remove the assumption of the absence of zonal disparity in the dynamics of the ACC. We will study a lattice of coupled loops fully acknowledging the zonal disparity of the dynamics.

4. A one-dimensional locally-coupled lattice

As described above the lattice corresponds to the full complexity of the set of equations defined above [i.e., (1) with $\epsilon \neq 0$ and $\kappa \neq 0$]. Here the wind feedback is considered and individual loops are locally coupled through a Laplacian diffusion of temperature. To study the lattice behaviour we made several numerical simulations of 200 000 yr and used a range of diffusion coefficients from 0 (no-coupling) to $100\,000 \text{ m}^2 \text{ s}^{-1}$. This range is overall consistent with observational values, suggesting a range between a few hundreds to a couple of ten-thousands metre squared per second [24,66–69].

The time stepping of the 4th order Runge and Kutta method has been adjusted for stability reason for high zonal diffusivity coefficients. For 5000, 10 000, and 20 000 $\text{m}^2 \text{ s}^{-1}$ we used 0.01 yr; for 50 000 $\text{m}^2 \text{ s}^{-1}$ we used 0.004 yr; and for 100 000 $\text{m}^2 \text{ s}^{-1}$ we used 0.002 yr.

4.1. Synchronicity and lattice phase transition

To estimate changes in the ACC we first concentrate on local value of the zonal transport (ψ_i). Numerical experiments show that locally the individual loop oscillates between eastward and westward flow. Also the lattice changes its behaviour when increasing the diffusivity (Fig. 6). Qualitatively this corresponds to a zonal widening of typical structure of variability. The variability timescale appears more robust to change in diffusivity.

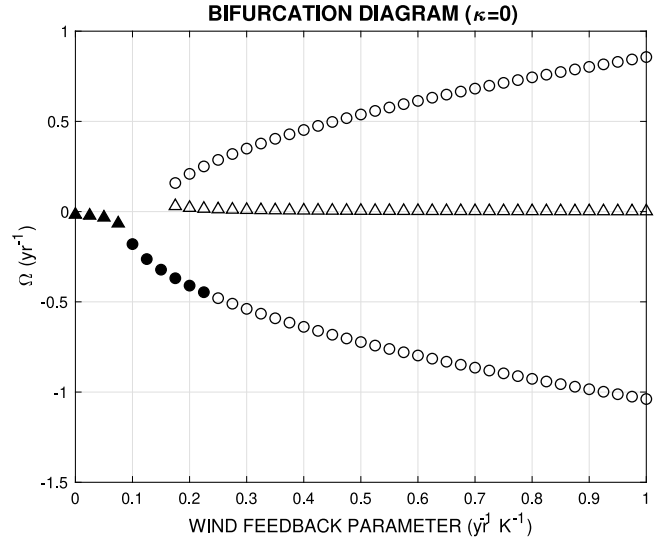


Fig. 3. Bifurcation diagram of a single ACC loop model using the wind feedback parameter (ϵ) as the control parameter. The circles or triangles denote steady states with or without an oscillatory least damped eigenvalue. Plain or empty marker denotes a stable or an unstable steady state, respectively.

To have a global diagnostic of the lattice we define the zonal average (or inter-loop average) of the local zonal ACC transport. This reads:

$$\psi = \frac{1}{N} \sum_{i=1}^N \psi_i, \quad (6)$$

where ψ is the zonal ACC transport. This shows periods alternating between strong and weak eastward transport (Fig. 6). Qualitatively, as for the local diagnostic, the variability is not particularly affected by the change in diffusion coefficient.

To further clarify through a single diagnostic the behaviour of the lattice we compute the transport as the time-mean of the zonal ACC transport: $\Psi = \langle \psi \rangle_t$, where

$$\langle \cdot \rangle_t = \frac{1}{T_m} \int_0^{T_m} \cdot dt \quad (7)$$

and T_m is a sufficiently long time scale. This shows a change in the overall strength of the transport with diffusion coefficient changes (Fig. 6). In particular, this leads to a realistic eastward flow, unlike with the single loop.

Together with the transport (Ψ), to be more quantitative we define two other indices: the period and the synchronicity index. The period (Π) is the maximum peak of the power spectral density of ψ . To compute the synchronicity index [following 26], we first define $V_i = y_i^2 + z_i^2$, which measures an amplitude of temperature differences. Then we define

$$V(t) = \frac{1}{N} \sum_{i=1}^N V_i, \quad (8)$$

where $V(t)$ is the inter-loop mean (or zonally averaged) of the temperature difference amplitude. From there, two variances can be computed:

$$\sigma_V^2 = \langle V^2(t) \rangle_t - \langle V(t) \rangle_t^2, \quad (9)$$

and

$$\sigma_{V_i}^2 = \langle V_i^2(t) \rangle_t - \langle V_i(t) \rangle_t^2, \quad (10)$$

where σ_V^2 is time-variance of the inter-loop mean and $\sigma_{V_i}^2$ is the individual loop time-variance. From these variances we can write the

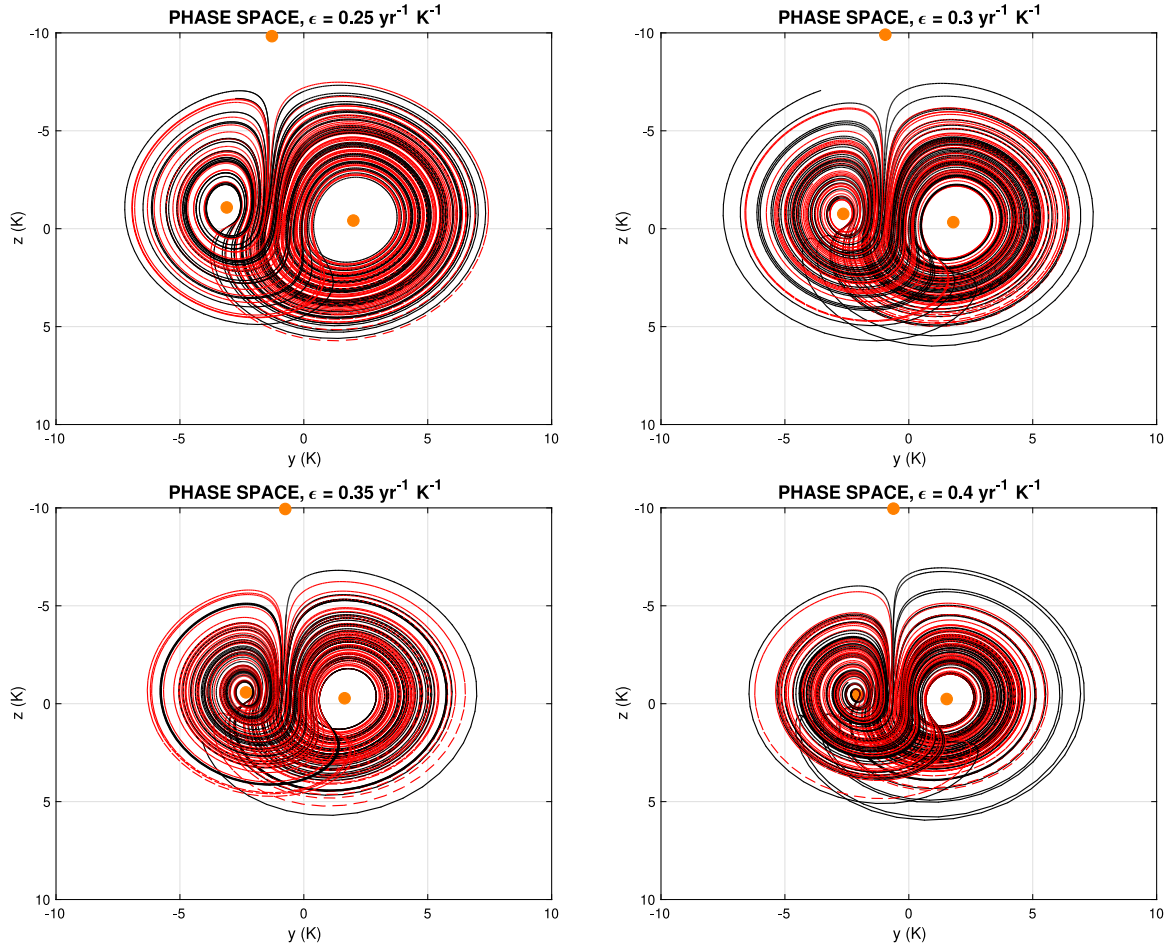


Fig. 4. Trajectories in the phase space for the single ACC loop model and for four different values of the wind feedback parameter where there is three unstable steady states: $\epsilon = 0.25, 0.35, 0.4$, and $0.45 \text{ yr}^{-1} \text{ K}^{-1}$. For each panel, the orange spheres indicate the three unstable steady states and the black and red lines show two examples of trajectories starting with different initial conditions. Despite, the chaotic behaviour being visible since $\epsilon = 0.25 \text{ yr}^{-1} \text{ K}^{-1}$, the increase of the wind feedback parameter modified the shape of the strange attractor.

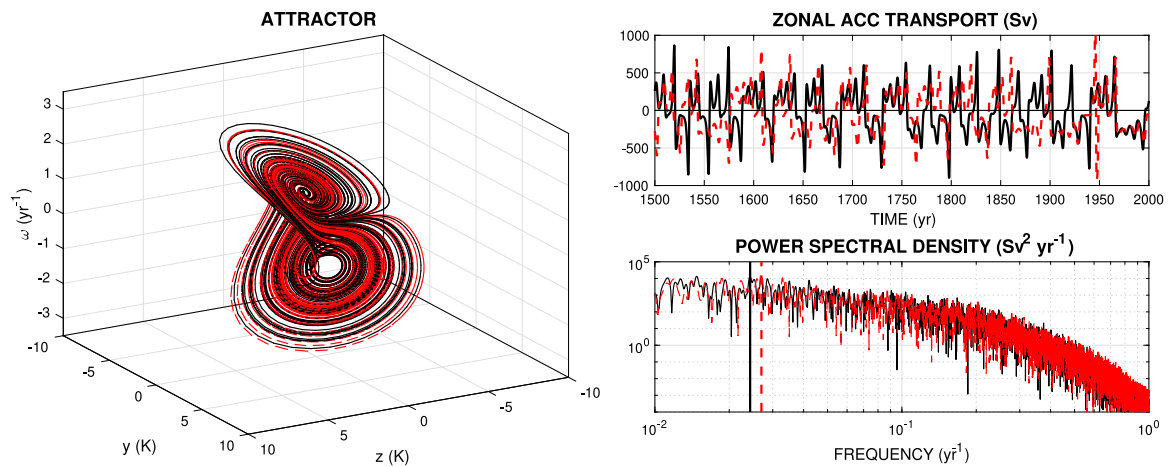


Fig. 5. (left) Two trajectories in the phase space for the fully developed chaotic attractor, corresponding to the use of the reference value of the wind feedback parameter: $\epsilon = 0.6 \text{ yr}^{-1} \text{ K}^{-1}$. The two trajectories differ by their initial conditions. (top right) time evolution of the two trajectories. (bottom right) spectrum of the two trajectories. The vertical lines show the corresponding power spectral density maximum, highlighting a period in the range of 35–40 yr. As expected, despite their different time evolution (i.e., different phasing), the trajectories follow the same attractor and have common spectral properties.

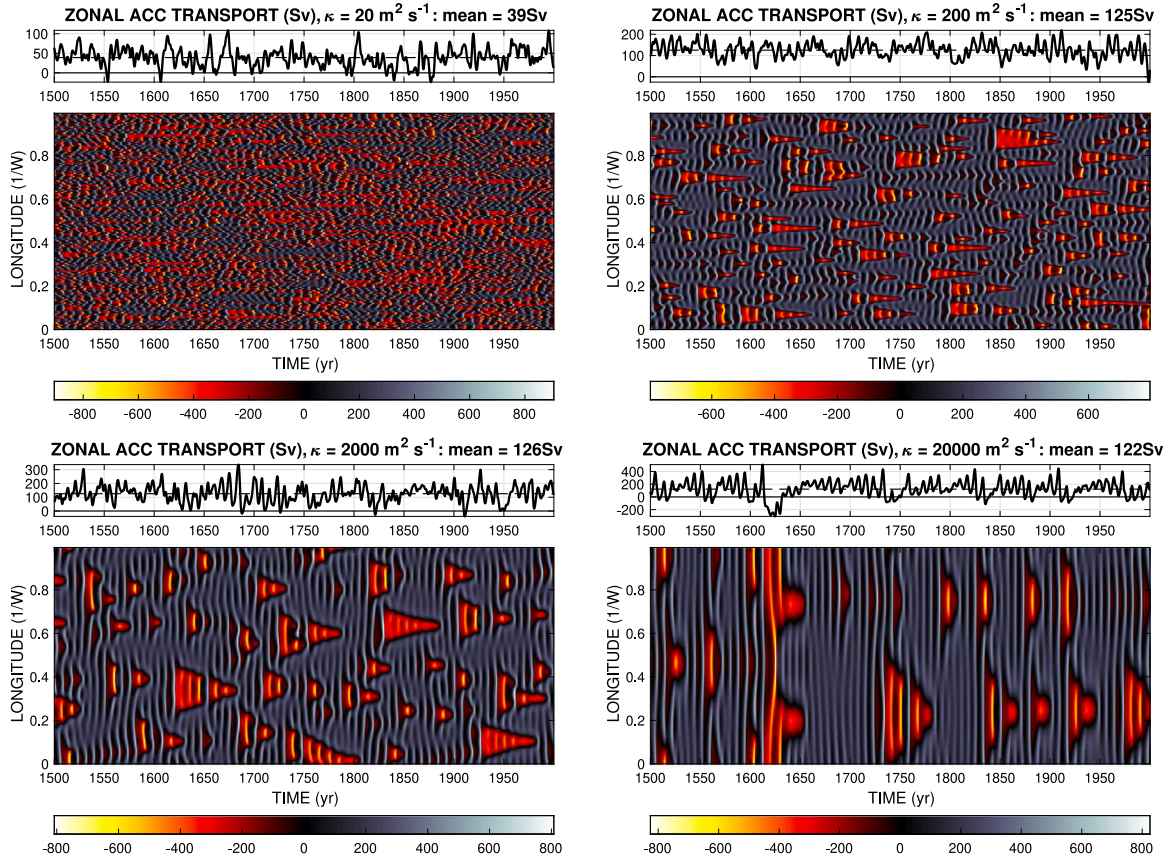


Fig. 6. Zonal ACC transport (ψ) and local zonal ACC transport (ψ_i) as a function of time for four zonal diffusion coefficients (as indicated in the title): (top left) $20 \text{ m}^2 \text{ s}^{-1}$, (top right) $200 \text{ m}^2 \text{ s}^{-1}$, (bottom left) $2000 \text{ m}^2 \text{ s}^{-1}$, and (bottom right) $20000 \text{ m}^2 \text{ s}^{-1}$. The horizontal dashed lines correspond to the time-mean of the zonal ACC transport ($\bar{\psi}$), with values given in the title.

synchronicity index as:

$$\chi^2 = \frac{\sigma_V^2}{\frac{1}{N} \sum_{i=1}^N \sigma_{V_i}^2}, \quad (11)$$

where χ is the synchronicity index. Here the synchronicity index varies from 0 to 1. If $\chi = 0$, the system is asynchronized, the individual loops do not vary together and the zonal-mean temperature difference amplitude variance tends to 0. If $\chi = 1$, the system is synchronized, individual loop vary together equivalently to their expected value, the variance computed for individual loop or for the zonal mean is equivalent.

To test the impact of the diffusion on the synchronicity of the lattice and its other properties, we made experiments for 16 values of zonal diffusion coefficients: 1, 2, 5, 10, 20, 50, 100, 200, 500, 1000, 2000, 5000, 10000, 20000, 50000, and 100000 $\text{m}^2 \text{ s}^{-1}$. We took advantage of the chaotic nature of the studied system to compute 30 numerical simulations for each values of the diffusivity coefficients. This allows us to compute an expected value together with its uncertainty. We find three regime of synchronicity depending on the amplitude of the diffusion coefficients (Fig. 7). The first regime ($\chi < 0.1$ — free regime) corresponds to individual loop evolving freely without significant impact from their neighbours. The second regime ($0.1 \leq \chi \leq 0.9$ — the intermediate regime) corresponds to a partial synchronicity, where individual loop feels neighbouring loops up to a given spatial-scale (that will be further discussed in the next subsection). The third regime ($\chi > 0.9$ — locked regime) corresponds to a full synchronicity and all the loops evolve together, all feeling each other regardless of their distance.

These three regimes have different properties in term of mean flow and variability. Indeed, the lattice properties change suggesting a bifurcation or a phase transition occurring in the lattice between

the three regimes. The free regime and locked regime have expected similarity: individual loop ($\kappa \rightarrow 0$) and averaged locked-loops ($\kappa \rightarrow +\infty$) are equivalent and led to similar behaviour. They correspond to an unrealistic westward flow of $\sim -40 \text{ Sv}$ and a variability peaking in a range between 20 and 40 yr. When the intermediate regime, corresponding to partial synchronicity, occurs, the lattice behaviour is dramatically different: the flow is now eastward with an amplitude of 121 Sv and a maximum spectral signature of 8.1 yr. Hence in this intermediate regime, the mean flow is reverse, becoming realistic, and the spectral signature is significantly shortened. This overall phase transition is consistent with chimera behaviour [70–72], where a few loops become synchronized (with a burst of negative transport, Fig. 6), whereas the rest remain overall asynchronized.

4.2. Spatial footprint and role of extreme events

To better describe how the local coupling leads to synchronicity and behavioural change of the lattice, we first define the synchrony level at any location of the lattice (j , longitude related to the index of the loop within the lattice) as well as any subnumber ($1 < \phi \leq N$) of oscillators: $\chi(j, \phi)$. This reads:

$$\chi(\phi, j)^2 = \frac{\left\langle \left(\frac{1}{\phi} \sum_{i=1}^{\phi} V_{\phi} \right)_t \right\rangle^2 - \left\langle \frac{1}{\phi} \sum_{i=1}^{\phi} V_{\phi} \right\rangle_t^2}{\frac{1}{\phi} \sum_{i=1}^{\phi} \langle V_{\phi}^2 \rangle_t - \langle V_{\phi} \rangle_t^2}, \quad (12)$$

$$\text{and } V_{\phi}(\phi, j, t) = V_i(j - \frac{1}{2}\phi \leq j < j + \frac{1}{2}\phi, t).$$

Averaging this local synchronicity index over all the location, we find the local synchronicity index (χ_{ϕ}). It reads:

$$\chi_{\phi} = \frac{1}{N} \sum_{j=1}^N \chi(\phi, j). \quad (13)$$

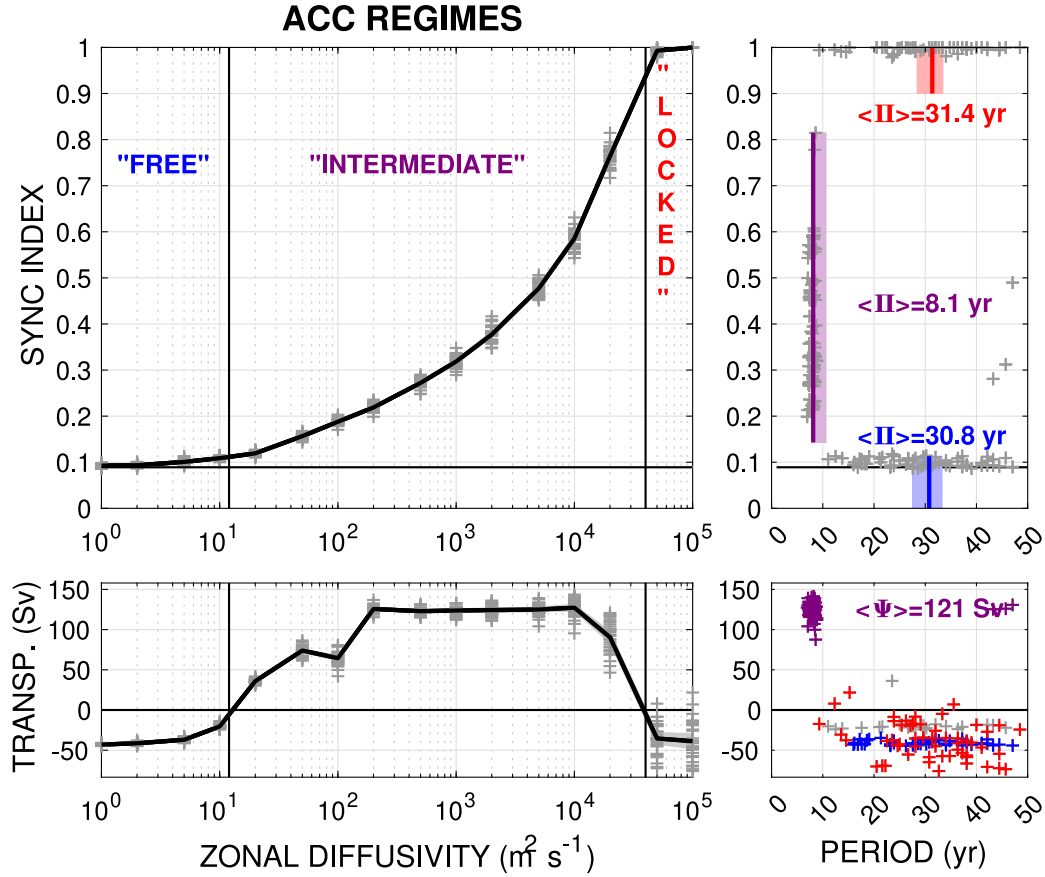


Fig. 7. (top left) Synchrony index (χ) as a function of diffusion coefficient (κ). (bottom left) Transport (Ψ) as a function of diffusion coefficient (κ). (top right) Synchrony index (χ) as a function of the period (Π). (bottom right) Transport (Ψ) as a function of the period (Π). Individual grey crosses correspond to results from individual simulations, black curves correspond to the mean values. The vertical lines separate the three regimes: free ($\chi < 0.1$ — blue), intermediate ($0.1 \leq \chi \leq 0.9$ — purple), and locked ($\chi > 0.9$ — red). The horizontal line denotes the synchronicity achieved by luck given our 200 loops (i.e., for $\kappa = 0$). For each regime, the median period ($\langle \Pi \rangle$) is given by the vertical colour lines (and as an insert) with the 95% uncertainty in shading. Cluster of transport and period are highlighted in colour, median transport value of the intermediate regime is given as an insert ($\langle \Psi \rangle$).

As expected, this decay with distance going from 1 to 0.4 in the case of $\kappa = 2000 \text{ m}^2 \text{ s}^{-1}$ (Fig. 8). From this term, the footprint length scale (l_{fp}) can be computed as the inflexion point of this curve (i.e., the distance over which a loop have lose its maximum influence — the minimum of $d_\phi \chi_\phi$). This footprint length scale is interpreted as the typical distance over which the local-coupling acts [26]. It corresponds to ~ 0.0825 for $\kappa = 2000 \text{ m}^2 \text{ s}^{-1}$ (Fig. 8).

As qualitatively assess in the previous subsection, we find that the footprint length scale increases with the diffusion coefficient. This increase follows a typical square root law. Using a best fit algorithm, we find that a typical time footprint (t_{fp}) of 48.5 yr allows the relation $l_{fp} \simeq \sqrt{\kappa t_{fp}}$. This suggests that the synchronicity is achieved by the propagation of information between loop within the lattice. This scale of propagation is limited to a distance (i.e., footprint length scale) sets by the zonal diffusion and a timescale that appears constant over all experiments. This implies that it is a fundamental constant of the system. This is worth noting that the ratio of the typical time footprint to the median period ($t_{fp}/\langle \Pi \rangle$) is almost an integer (~ 6). However other multiple of $\langle \Pi \rangle$ (e.g., $2 \times \langle \Pi \rangle = 16.2 \text{ yr}$) are not peaking in the spectrum. This suggests that, despite being two fundamental timescales of the system, they might not be directly related.

To test the physical mechanism behind the time scale footprint, we have computed the return period of extreme events. Extreme events correspond to the occurrence of important slow-down (down to negative) transport (Fig. 6). To define it we used the residual circulation (Ω) and set events as value above a threshold ($\Omega_i > \Omega_T$, where

$\Omega_T = -0.1026 \text{ yr}^{-1}$ is the circulation threshold). This corresponds to a circulation that is significantly lower or oppose to the mean circulation (Ω_0). This event occurrence is above the ~ 82 percentile and is independent of the zonal diffusion (Fig. 9). One can compute the return period of these extreme events (t_{xtr}). It is overall quite consistent between experiments (Fig. 9). It varies from 48 yr to 51 yr depending on the diffusivity coefficient. We find that the occurrence of this extreme events, and in particular their return period, set the footprint length scale (Fig. 8), hence the level of synchrony of the system.

Following this logic we can interpret the regime change of the lattice. On the one hand, the absence of synchrony occurs when a cluster is smaller than the zonal extent of a single loop. On the other hand, the full synchrony occurs when there could not have two independent clusters of loops within the lattice. Assuming that a single loop information propagates in the lattice over $2l_{fp}$ (i.e., l_{fp} westward and l_{fp} eastward), the absence of synchrony reads: $2l_{fp} = 2\sqrt{\kappa t_{xtr}} < W/N$. This corresponds to diffusion coefficient threshold of $\kappa = 5 \text{ m}^2 \text{ s}^{-1}$. The full lattice coupling would occurs when there is no room for two independent loops to propagate their information. This corresponds to $4l_{fp} = 4\sqrt{\kappa t_{xtr}} < W$, which leads to $\kappa = 50000 \text{ m}^2 \text{ s}^{-1}$. These two threshold values correspond to the regime change of the lattice as shown by numerical experiments (Fig. 7).

Overall this extreme events allow us to interpret the behaviour of the lattice. Hence the synchronicity comes from a competition between the inherent instability of the wind-feedback, that propagates zonally through the zonal diffusion, and the occurrence of extreme events.

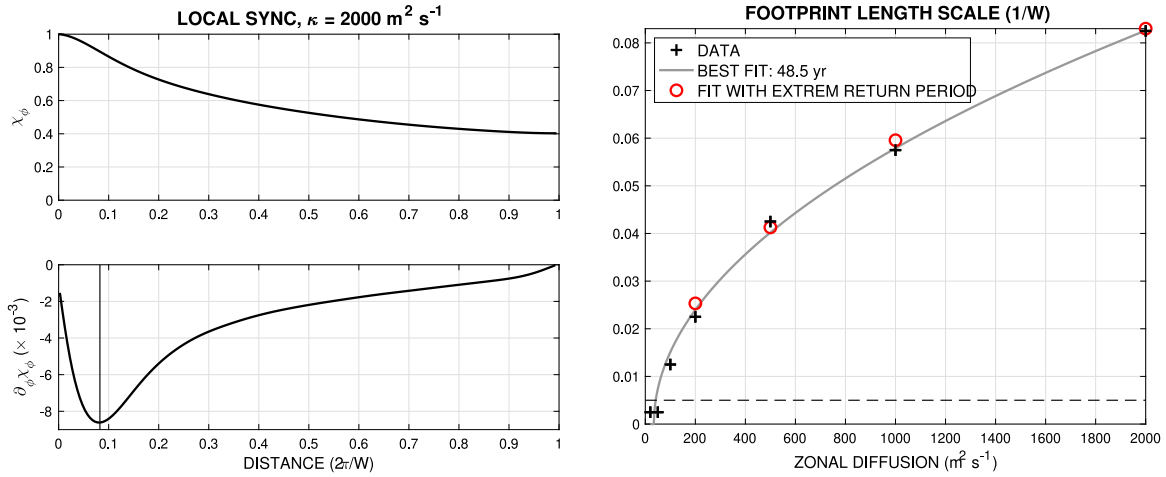


Fig. 8. (top left) Local synchronicity index (χ_ϕ). (bottom left) The distance derivative of the local synchronicity index ($d_\phi \chi_\phi$), its minimum indicating the footprint length scale (l_{fp}), shown by the vertical line. (right) The footprint length scale as a function of zonal diffusion coefficient. (black crosses) Outcomes of the numerical simulations, (grey line) best fit following a square root law ($\sqrt{\kappa t_{fp}}$, where $t_{fp} = 48.5 \text{ yr}$), (red circle) fit using the extreme events return period ($\sqrt{\kappa t_{xtr}}$). The dashed black line corresponds to the minimum distance of the lattice (i.e., $1/N$ — a single loop zonal extent).

The extreme events “flush” the instability and restore the loop to a more normal situation (i.e., eastward transport). This is because of the synchronicity that the normal situation is more easily maintained, where the loops spread the instability and avoid the fast growth of instability and the more frequent occurrence of extreme events. Hence this is through the partial zonal synchronicity that a realistic transport is maintained despite the acknowledgement of the wind feedback. Without this partial synchronicity the circulation is too unstable and the transport is unrealistic. This is the competition between the auto-organization of the lattice and the chaotic nature of individual loops that allows for a realistic transport.

5. Discussion and conclusions

Since the pre-industrial era, the Earth climate is experiencing changes at an unrecorded rate [1]. In particular a vast amount of anthropogenically-induced greenhouse gases has been released into the atmosphere. As a consequence, the atmospheric surface is warming [1]. However, this phenomena is mitigated by the ocean through the absorption of 90% of the heat excess [2]. Hence it is timely to describe and understand the ocean dynamics and its circulation. The ACC is one of the most intense ocean current on Earth. It transports a vast amount of mass zonally [up to 150 Sv, 7] and connects three ocean basins: Atlantic, Indian, and Pacific basins. Through this connection, it redistributes heat, freshwater, and other oceanic properties.

The ACC has been an intense subject of study in physical oceanography. In particular the role of mesoscale eddies, which are ubiquitous in the ocean [17], in setting the ACC is at the core of our current understanding of this climatically-relevant ocean current. Indeed when simple zonal averages are applied, the ACC is characterized by the Deacon cell [a circulation transporting mass meridionally and vertically, 14]. However this cell disappears when acknowledging the role of eddies in transporting ocean properties [16]. This leads to the residual circulation, which acknowledges a mean flow driven by wind, through Ekman transport, and an eddy-induced transport [19,20]. In this framework the ocean isopycnal slope is set so that it induces an eddy flow balancing the wind-driven flow.

In this study we have built on these results to develop a new idealized model, but we have also added two key elements. Firstly, we have incorporated an ocean wind-feedback. This acknowledges the idea that surface ocean exchange heat with the atmosphere, modifying the atmospheric circulation, hence the wind-stress received by the

ocean [21,22]. Secondly, we moved away from a zonally averaged view, to explicitly acknowledge the zonal disparity of the ACC. We show that the former elements drives an instability. This leads to an unrealistic circulation, unless the latter element is introduced allowing for a realistic transport. Hence this idealized model corresponds to a lattice of locally coupled chaotic oscillators.

One of the key parameters of our study is the number of loops. It was set so that loop zonal-scale remains consistent with the use of a mesoscale turbulent closure [18]. This means that we want to keep the width of a single loop larger or equal to $\sim 1^\circ$. On the other hand, we want to have a lattice dynamics that is independent of the number of loops, meaning that the process resolved by the lattice should be as independent as possible from its zonal resolution. In this context, we have tested the use of 100, 200, and 300 loops. We find that for a zonal diffusivity coefficient of $2000 \text{ m}^2 \text{ s}^{-1}$ the three resolutions/configurations lead to the same dynamics (not shown).

We find that increasing the zonal diffusion leads to a bifurcation or a phase transition in the behaviour of the lattice. Three regimes have been determined as a function of the intensity of the local coupling: the free regime — where there is no synchronicity, the intermediate regime — where there is partial synchronicity, and the locked regime — where there is full synchronicity. The free and locked regimes are conducive to unrealistic westward circulations. Overall in these two regimes the lattice behaves quite equivalently (a fully locked lattice being equivalent to a single oscillator). In the intermediate regime, the lattice reaches local phase coherence (partial synchrony). In this regime the transport is realistic (i.e., eastward), with a mean value of 121 Sv, consistent with ACC transport observations [3–7]. An interesting features of the phase transition is the almost plateau-like values attained for the transport and the main spectral signature in the intermediate regime. Investigating if this is an outcome of a chimera type synchronization will be the goal of future investigation [70–73].

There has been extended studies on the phase coherence of lattice [e.g., 27]. In particular, the partial synchrony regime has already been reported in the context of locally coupled Rössler oscillators [28–30]. In a lattice representative of forest fires, [30] found the same triangular shape of intermittency in Hovmöller diagram that the one reported in the present study (Fig. 6). This shape can be rationalized through the loss of synchronicity (laminar behaviour) coming from the edge (turbulent behaviour), as a forest is burning (personal communication with Hugues Chaté). In our case we have rationalized this shape through the

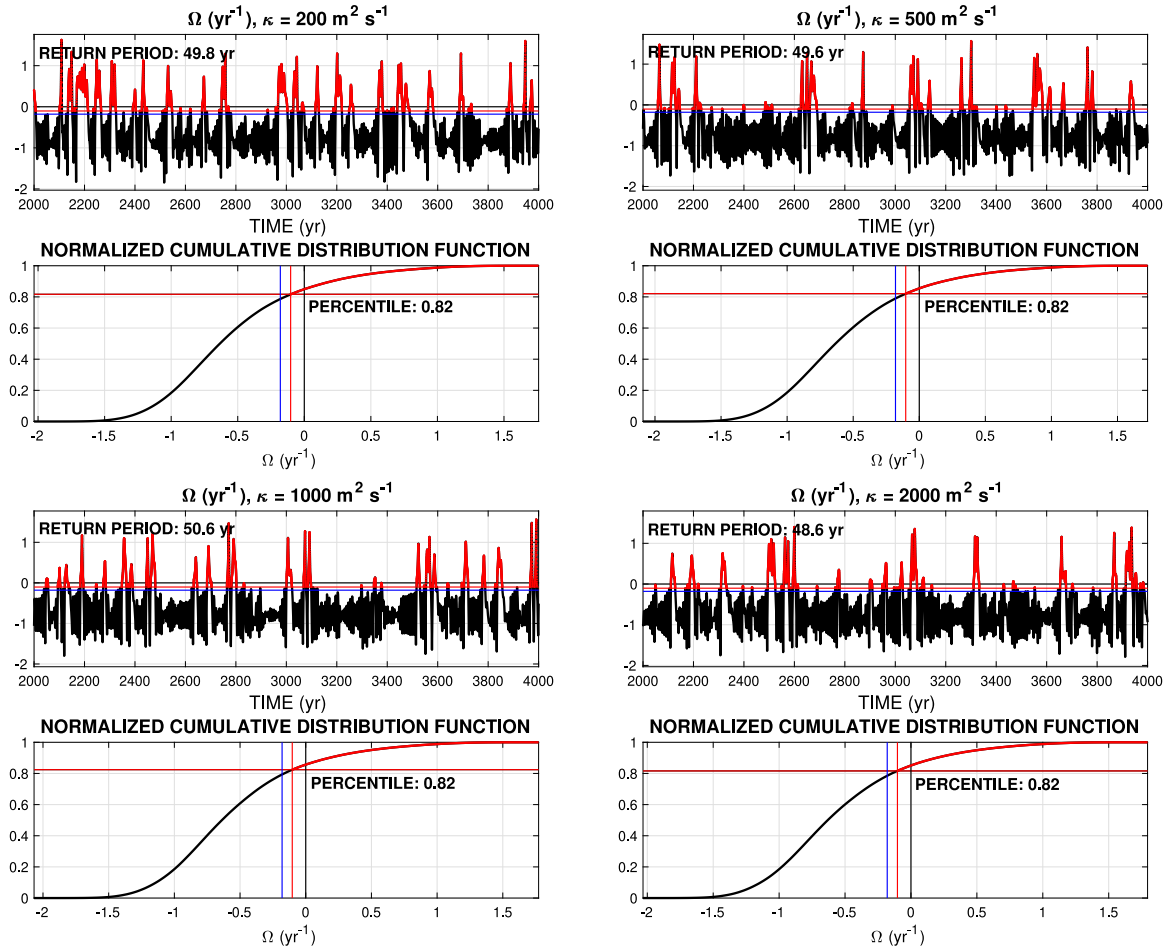


Fig. 9. Time evolution of the residual circulation of one loop of the lattice (Ω_i) and its normalized cumulative distribution as a function of time for four zonal diffusion coefficients (as indicated in the title): (top left) $200 \text{ m}^2 \text{ s}^{-1}$, (top right) $500 \text{ m}^2 \text{ s}^{-1}$, (bottom left) $1000 \text{ m}^2 \text{ s}^{-1}$, and (bottom right) $2000 \text{ m}^2 \text{ s}^{-1}$. The threshold value to define extremes is set to $\Omega > \Omega_T = -0.1026 \text{ yr}^{-1}$ and is indicated by the red lines, whereas extreme values are highlighted in red. The blue lines correspond to $\Omega_0 = -0.18 \text{ yr}^{-1}$. The return period and the percentile of occurrence of extreme events are indicated as inserts in the panels.

footprint length scale: $l_{fp} \simeq \sqrt{\kappa t_{xtr}}$, where $t_{xtr} \simeq 48.5 \text{ yr}$ is the extreme event return period, a universal constant of our studied system.

One key results of our analysis of the idealized model is the appearance of interannual to decadal variability when the wind-feedback is acknowledged (i.e., representing the coupling between the ocean and the atmosphere). In particular, in the realistic intermediate regime (i.e., partial zonal synchrony) a robust period of $\sim 8.1 \text{ yr}$ can be identified. The interannual to decadal variability of the ACC has already been identified in a range of studies using more complex ocean–atmosphere coupled models [e.g., 8–10]. In this context, [8] reported variability with a typical timescale around a decade, consistent with our result.

We have shown that the synchronicity level, measured by the zonal footprint, derives from the competition between the zonal turbulent diffusivity, zonally coupling the dynamics, and the extreme event (>82 percentile) return period ($\sim 48.5 \text{ yr}$), resetting the local meridional-depth dynamics. The realistic ACC transport is achieved for partial zonal synchrony (i.e., the intermediate regime). We conclude that acknowledging the zonal disparity of the ACC is key for realistic dynamics. This is consistent with recent observational studies showing that the zonal disparity is a crucial element of an accurate representation of the ACC [23,24], unlike previously assumed in theoretical descriptions of the ACC [e.g., 19,20].

In future investigation, it will be interesting to test the synchronicity index of the observed ACC. To this purpose, it will be important to

first average-out the turbulent fluxes, by zonally averaging-out over a few degrees of longitudes and over a 1-year time period. Only then the synchronicity index described in (11) can be applied consistently with the proposed analysis. Based on our results (Fig. 7), this index will inform us on the ocean turbulent diffusivity coefficient, which, despite some estimates [e.g., 66,68,69,74], remains an open question in physical oceanography.

The ACC flow has been traditionally viewed as a geostrophic flow driven by a balance between the wind-forced Ekman transport and the eddy-induced transport. This allows for a realistic estimation of the flow and leads to value of $\sim 150 \text{ Sv}$, in broad agreement with observations. Although, there is still a lot of active debates on the role and impact of the eddies for the ACC [eddy compensation vs saturation, 75–80], they remain within this same traditional paradigm. Here we have shown that the addition of ocean–atmosphere wind feedback disrupts this simple view, leading to an un-equilibrated westward mean flow, that is a priori inadequate to reflect observations. Nevertheless when acknowledging the zonal-disparity, a mean realistic eastward flow becomes evident again. This opens the door to an alternative un-equilibrated view of the mean ACC flow as a complex zonal synchrony through eddy-turbulent diffusion. Hence, once again, there are two sides to every story: (i) a traditional one, appealing by its simplicity but failing through the addition of more realistic physics; (ii) an alternative one, more complex but more robust. Our analysis pleads for the further development of an alternative, more complex, paradigm for the ACC.

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: Florian Sevellec reports financial support, administrative support, and equipment, drugs, or supplies were provided by Centre National de la Recherche Scientifique. Florian Sevellec reports a relationship with Centre National de la Recherche Scientifique that includes: funding grants and non-financial support. If there are other authors, they declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgment

This work was supported by ARVOR from LEFE IMAGO and by the OceaniX project funded through the French National Research Agency (ANR) program.

Appendix

In this two-part appendix the derivation of the idealized model is described in two steps. First, in [Appendix A](#), the derivation of a single loop is provided. This mainly follows derivations already provided in [\[32,33\]](#), as well as the long history of derivations of loop models [e.g., [34,35,38,39](#)]. Hence, this derivations is not fully original and is provided for completeness. Then, in [Appendix B](#), the coupling between loops is described. This part is fully original to the present study. In both parts the inherent assumptions of idealized modelling derivations is provided and discussed.

Appendix A. Derivation of a loop

The evolution of temperature within the idealized model can be derived from the conservation of heat and the conservation of mass (i.e. the non-divergence). This reads:

$$\partial_t T + u \partial_x T + v \partial_y T + w \partial_z T = D + F \quad (\text{A.1a})$$

$$\partial_x u + \partial_y v + \partial_z w = 0, \quad (\text{A.1b})$$

where ∂_t , ∂_x , ∂_y , ∂_z are the time, zonal, meridional, and vertical partial derivatives; T is temperature; F is the temperature forcing term; and D is the temperature dissipation term.

To derive the idealized ocean model, we first start with a commonly used description of the ACC as a zonally averaged models [e.g., [46](#)]. This view is at the heart of residual circulation, a key explanation of the ACC dynamics and of the Southern Ocean balance [\[19,20\]](#). The equations reads:

$$\partial_t \bar{T}^\gamma + \bar{v}^\gamma \partial_y \bar{T}^\gamma + \bar{w}^\gamma \partial_z \bar{T}^\gamma = \bar{D}^\gamma + \bar{F}^\gamma \quad (\text{A.2a})$$

$$\partial_y \bar{v}^\gamma + \partial_z \bar{w}^\gamma = 0, \quad (\text{A.2b})$$

where γ represents the zonal average. Here we have assumed that the cross term of the advection can be introduced in the dissipation term ($\bar{D}^\gamma$).

To describe the rotation of the flow mathematically, we can place ourselves in the centre of the ocean basin, inside a simple, closed curve formed by the chosen streamline ([Fig. 1a](#)). We then introduce the angle θ , positive for counterclockwise motion and measured from the ocean bottom, as the new spatial coordinate [\[31\]](#). The flow intensity (overturning rate) will not vary with θ , since the flow is incompressible. We have:

$$\partial_t \tilde{T} + \Omega \partial_\theta \tilde{T} = \tilde{K} \partial_\theta^2 \tilde{T} + \tilde{F}, \quad (\text{A.3})$$

where $\tilde{T}$ is the temperature along a streamline, Ω is the flow along the streamline measured as an overturning rate, $\tilde{K}$ is the diffusivity coefficient, $\tilde{F}$ is a forcing along the streamline.

The periodicity of the loop allows for a Fourier decomposition. This reads:

$$\tilde{T} = \Re \left(\sum_{n \in \mathbb{N}} T_n e^{in\theta} \right), \quad (\text{A.4})$$

where n is the Fourier mode index. We also have:

$$T_n = T_{rn} + iT_{in} = \frac{1}{2\pi} \int_0^{2\pi} \tilde{T} e^{-in\theta} d\theta, \quad (\text{A.5})$$

where r and i indicate the real part and imaginary part, respectively.

Assuming that the surface heat flux (*ildeF*) projects only on the first mode (T_1), we restrict our study to this first Fourier mode. T_1 is associated with the gravest Fourier mode, thus the real and imaginary parts of T_1 are associated with the projections of temperature field onto $\cos(\theta)$ and $\sin(\theta)$. $\theta = 0$ is located at the maximum depth and the angle increases counterclockwise ([Fig. 1a](#)); we have at the bottom $\cos(\theta = 0) = 1$ and $\sin(\theta = 0) = 0$, at the North $\cos(\theta = \pi/2) = 0$ and $\sin(\theta = \pi/2) = 1$, at the top $\cos(\theta = \pi) = -1$ and $\sin(\theta = \pi) = 0$, and at the South $\cos(\theta = 3\pi/2) = 0$ and $\sin(\theta = 3\pi/2) = -1$. Thus, the real and imaginary parts of T_1 represent the ocean bottom-top and North-South temperature gradients. We use the notation $y = T_{i1}$ and $z = T_{r1}$, we rewrite the equations of the incompressible loop model as:

$$\dot{y} = \Omega z - K y, \quad (\text{A.6a})$$

$$\dot{z} = -\Omega y + H - K z, \quad (\text{A.6b})$$

where $\dot{}$ represent the ordinary time derivative. Here, H is Fourier projections of the surface heat flux.

Now that the evolution of temperature in the idealized model has been characterized, there remains the expression of the flow to determine. In the idealized model framework the latitude-depth flow is restricted to flow rotation. This means that it is only represented by a scalar measuring the flow overturning rate: Ω . Following the temporal residual mean framework, we assumed that the total flow is the sum of mean flow and an eddy-induced flow [\[81,82\]](#). Given the zonally-average condition, the mean flow along the latitude depth plain derive from the zonal wind-stress giving rise to a meridional flow (compensated at depth). This is noted Ω_0 and represent the Deacon cell [\[14,15\]](#). The eddy-induced flow is given by meridional density contrast through a linear law: ky , where k is the proportionality coefficient. Here we assumed that the baroclinic instability is mainly dominated by the meridional density contrast. This eddy closure is equivalent to the Gent–McWilliams eddy closure after a linearization around a constant stratification [\[18\]](#). Finally we acknowledge the existence of a wind feedback. This feedback considers that meridional surface ocean temperature contrast (y) warmed and cooled the atmosphere, leading to a meridional atmospheric temperature contrast. By geostrophy this contrast is balanced by a zonal wind, leading to an anomalous Ekman transport and subsequent meridional-depth circulation [\[13\]](#). This term is noted ω . Hence we obtain $\Omega = \Omega_0 + \omega + ky$.

Following [\[21,22\]](#) the wind feedback can be represented by linear term, since this anomalous wind remains small compared to the mean wind. Finally assuming that this wind-feedback is not instantaneous, but takes time to adjust, we get the equation of the anomalous momentum. This reads:

$$\dot{\omega} = -\gamma(\omega + \epsilon y), \quad (\text{A.7})$$

where γ is the inverse of an adjustment timescale and ϵ is the wind feedback coefficient.

Appendix B. Coupling of the loops

Now that the set of equations for a zonal average model has been determined by (A.6) and (A.7), we generalize it for zonal average over a subregion. For consistency with eddy-flow closure, we consider zonal average of at least 1° .

From the previous set of equations the main difference is the need to include the geostrophic advection and the zonal temperature diffusion. However the latter can be simplified by identifying that our loop model is equivalent to a $1\frac{1}{2}$ -layer model. In this context the baroclinic geostrophic advection is perfectly compensated between the zonal and meridional directions. This is a natural consequence of the non-Doppler effect [53–55]. This leaves only the barotropic advection and the vertical advection [52], where the latter induced the non-dispersive propagation of large scale baroclinic Rossby waves. Assuming a zonal mean flow, this two terms can be represented by a zonal velocity ($\bar{u}$) or an equivalent zonal velocity ($\hat{u}$), respectively. This leads to a zonal advection. In the rest of the study, this advection is considered as a Doppler effect. This means that individual loops are transported by $\bar{u}+\hat{u}$, so that their geographic location evolves in time, but this term does not impact the equations. Alternatively, one can assume the validity of the f -plane approximation [48], this would imply $\hat{u} = 0$.

Hence including the zonal diffusion, which coupled the loops, leads to the set of ordinary differential equations representing the loop lattice:

$$\dot{\omega}_i = -\gamma (\omega_i + \epsilon y_i), \quad (\text{B.1a})$$

$$\dot{y}_i = \Omega_i z_i - K y_i + \tilde{\kappa} (-2y_i + y_{i-1} + y_{i+1}), \quad (\text{B.1b})$$

$$\dot{z}_i = -\Omega_i y_i + H - K z_i + \tilde{\kappa} (-2z_i + z_{i-1} + z_{i+1}), \quad (\text{B.1c})$$

where i is the loop index, $\tilde{\kappa}$ is zonal diffusivity coefficient, and $\Omega_i = \Omega_0 + \omega_i + k y_i$.

Data availability

Data will be made available on request.

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