

## Bifurcation Structure of Thermohaline Millennial Oscillations

A. COLIN DE VERDIÈRE, M. BEN JELLOUL, AND F. SÉVELLEC

*Laboratoire de Physique des Océans, Brest, France*

(Manuscript received 5 December 2005, in final form 30 March 2006)

### ABSTRACT

The question of the generation of millennial oscillations by internal ocean dynamics is studied through deliberate use of the simplest geometry and surface forcing, namely a hemispheric ocean with time-independent mixed boundary conditions (autonomous system). The lowest-order model that supports free oscillations has three horizontal and two vertical boxes. The essential ingredients permitting the existence of the oscillations are turbulent mixing and freshwater forcing. The finite amplitude oscillations share the advective–convective–diffusive characteristics of neighboring stable thermal and haline steady states. There are limits to the quantity of precipitation in polar regions for the existence of oscillatory states. When the freshwater forcing amplitude is increased, the system evolves from a stable thermal state through a global bifurcation to a finite amplitude limit cycle. The period of the limit cycle remains constant when freshwater is increased until at a second global bifurcation it becomes infinite with a logarithmic behavior characteristic of a homoclinic bifurcation. For still higher values of freshwater, the system locks into the stable haline steady state. These results are confirmed through the use of a two-dimensional latitude–depth model. A sensitivity study carried out with the latter shows that the period (away from the logarithmic singularity) varies as (vertical mixing)<sup>−1/3</sup>. The implications of these results for the Dansgaard–Oeschger oscillations of the last glacial period are threefold: First, internal ocean dynamics in a salt-conserving ocean basin and with time-independent boundary conditions are sufficient to allow free transitions between a strong thermal and a weak haline circulation regime provided that the precipitation in polar oceans does not exceed a certain threshold. It is noteworthy that the snow accumulation rates of the last glacial period were about a fourth of Holocene values. Second, the period of the oscillatory state is determined internally, a possible alternative to studies that require external periodic forcing. The range of the periods when estimated with present determinations of oceanic mixing easily accommodates the observations. Third, if the abrupt warming that signals the beginning of a Dansgaard–Oeschger event is interpreted through the present modeling results, its cause is linked to the efficiency of mixing to accumulate heat for a considerable amount of time in the deep ocean when the thermohaline circulation is weak.

### 1. Introduction

There is a considerable body of observations from Greenland ice cores and deep sea sediments that shows that the Greenland climate has undergone quasiperiodic oscillations of 1.5 kyr period with  $O(10^{\circ}\text{C})$  air temperature variations during the last glacial period [see the reviews by Alley et al. (1999), National Research Council Committee on Abrupt Climate Change (2002), Dansgaard et al. (1993), Grootes et al. (1993), and Severinghaus and Brook (1999)]. Active discussion persists

today to find both the geographical extent and the causes of the reorganization of the atmosphere–ocean–ice system. In the words of the Alley et al. review, “the Dansgaard–Oeschger (D/O) oscillations is an oceanic process, often triggered by meltwater changes but possibly oscillating freely in response to some as-yet-unidentified process(es).” In their review Clark et al. (2002) indicate that *the concept of multiple equilibria of the thermohaline circulation and the transitions between these states is now commonly invoked as a mechanism to explain the abrupt climate changes that were characteristic of the last glaciation*. This was first proposed by Broecker et al. (1990) as follows: The North Atlantic is a net basin of evaporation, which requires at steady state an export of salt by the circulation (North Atlantic Deep Water leaves the basin). When this circulation is weakened, the salinity will increase in response to the

---

*Corresponding author address:* Alain Colin de Verdière, Laboratoire de Physique des Océans, Université de Bretagne Occidentale, 6 Ave. Le Gorgeu, C. S. 93837, 29238 Brest CEDEX 3, France.  
E-mail: acolindv@univ-brest.fr

net evaporation. At the same time ice sheets will grow due to the reduced warming of the polar regions caused by reduced meridional heat transports. Because the salinity and, therefore, the density of the basin is increasing, polar convection will resume at some point, and the circulation will shift into its active mode, causing the salinity to increase in the polar regions because of the renewed salt transport. At the same time the heat released through advection and convection causes ice to melt and the meltwater input brings the opposite shift of the circulation into its weakened state. At odds with this conceptual description, Gregory et al. (2003) used a coupled ocean–ice–atmosphere model to point out that the Atlantic is fresher (and not saltier) when the circulation is weak. Sakai and Peltier in a series of papers (Sakai and Peltier 1995, 1997, 1999) showed that millennial oscillations could be generated under internal ocean dynamics and brought attention to the similarities of these oscillations with the Dansgaard–Oeschger oscillation. Their oscillations were forced by a net excess of freshwater input at the surface, supposedly caused by the occurrence of high-latitude ice runoff in glacial times. They showed further that the period was strongly dependent of that imposed forcing value. More recently, an intermediate-complexity climate model was used by Ganopolski and Rahmstorf (2001) to show that during glacial times the oceanic circulation oscillated between a cold mode (with deep-water formation south of Iceland) and a warm mode (with deep-water formation similar to modern) when forced by periodic variations of the freshwater forcing in the latitude belt  $50^{\circ}$ – $80^{\circ}$ N. The origin of the periodicity, which was left unspecified, had to be searched in other components of the climate system. Heinrich events, which follow the release of massive iceberg discharges and punctuate the 100 kyr glacial record, were used by Timmermann et al. (2003) to precondition the ocean for the subsequent oscillations forced by stochastic freshwater forcing. A coherent resonance (or autonomous stochastic resonance) mechanism was invoked to describe the oscillations used as prototypes for the D/O. The application of this mechanism in the present context was discussed previously in several studies (Alley et al. 2001; Velez-Belchi et al. 2001; Ganopolski and Rahmstorf 2002): the weak and strong states of the thermohaline circulation are stable and separated by a hill. Under certain conditions, a combination of periodic forcing and noise is sufficient to climb the hill and allow quasiperiodic visits of the two stable states.

Focusing on the internal oscillations of the thermohaline circulation (THC) per se, the millennial band was found in 2D and 3D oceanic GCMs as an oscilla-

tion between strong and weak circulation regimes associated with polar convection being on or off respectively (Marotzke 1989; Weaver and Sarachik 1991; Weaver et al. 1993; Huang 1994). Such oscillations were named “deep decoupling oscillations” and were studied in much more detail by Winton and Sarachik (1993), who explored their sensitivities to forcing distributions and vertical mixing. Although there is little doubt that THC dynamics are involved somehow in the D/O of glacial times, there appears to be a considerable diversity of forcings, extra components (ice, atmosphere), and number of degrees of freedom that seems critical for different authors to generate millennial oscillations, and simple physical models do not abound. The first was apparently that of Winton (1993), who succeeded in generating such oscillations in a three-box model (two surface boxes above a single deep one). Winton’s oscillations are special in the sense that they depend entirely upon the nonlinearity of the equation of state, whereas Winton and Sarachik (1993) had mentioned that this was not essential for the existence of deep decoupling oscillations. Although it is not impossible that the nonlinearity of the equation of state plays an important role in geophysical applications, it is disturbing to have it play the leading role in conceptual models. Should not the major nonlinear effects be concerned instead with the transport of heat and salt by the circulation? Following their numerical explorations, Sakai and Peltier (1999) proposed a three-box model (one surface, one deep subtropical box, and one full-depth polar box) whose salinities were the only active variables. They were able to find oscillatory regimes similar to those found earlier in their 2D model. However, their dynamical system lacks an important ingredient, namely the diffusive warming of deep water as a precursor to the flush when the circulation shifts from the weak state to the strong state (Marotzke 1989; Weaver and Sarachik 1991). Many authors have tried to find THC oscillations with a small number of degrees of freedom [see the reviews of Weaver and Hughes (1992) and Whitehead (1995) and the recent one by Dijkstra and Ghil (2005)]. Ruddick and Zhang (1996) showed that no oscillations were possible in the original (or slightly modified) Stommel (1961) two-box model. Gargett and Ferron (1996), testing the impact of double diffusive processes, were able to obtain oscillations in a  $2 \times 2$  box model and stressed the sensitivity to differential mixing of temperature and salinity. Self-sustained oscillations over periods of centuries to millennia were found by Zhang et al. (2002) in a  $2 \times 1$  box model. The instability of the haline state in that model was linked to the use of a sophisticated latitude-

dependent, convecting scheme parameterizing the effect of local convection. Using a  $3 \times 2$  box model, Huang et al. (1992) described the multiple steady states that appear when more latitudinal structure is allowed but did not find oscillations. A very different model, which neglects entirely advective transports and allows periodic convection, is that of Welander (1982). Indeed, oscillations between a diffusive state and a convective state occur in the case of warm, salty waters overlying a cold, fresh reservoir. This model was analyzed in much more details by Cessi (1996), who constructed the bifurcation diagram as a function of freshwater flux and found that the period of these finite amplitude oscillations varied logarithmically with the control parameter. These findings were discussed in the light of dynamical system theory by Abshagen and Timmermann (2004). Cessi further suggested that the behavior of the Welander convective model would be important for the oscillations of oceanic circulation models and, indeed, the present study will show that her intuition was far reaching for the millennial oscillations.

The present paper originated from this desire to generate and study millennial oscillations in the simplest physical context in order to find out the conditions of their existence. It was thought important to deal with a *salt conserving* oceanic basin, excluding thereby the type of freshwater forcing that does not average to zero and does not allow salt conservation. If the focus is on intrinsic oceanic oscillations, it is important to consider an autonomous system and exclude the stochastic forcings used by many authors. To concentrate on the effects of the nonlinear temperature and salinity advection terms, a *linear* equation of state will be used, given that many of the past studies used a nonlinear equation of state, which may by itself generate oscillations according to the process put forward by Winton (1993). In contrast with Zhang et al. (2002), a standard convection scheme will be used in which temperature and salinity are mixed vertically whenever the density contrast is unstable. The geometry is restricted to that of a *single hemisphere* even though a double-hemisphere geometry may be quite important for geophysical applications (Rahmstorf 1996; Blunier and Brook 2001). The simplest model that demonstrated millennial oscillations with these requirements is a 3 (horizontal)  $\times$  2 (vertical) box model quite similar to Huang et al. (1992) but with an important addition through the inclusion of *oceanic mixing*. One may react to this on the grounds that oceanic mixing stands among the least understood oceanic processes and that it is not satisfying to have a simplified model to depend crucially on mixing. The obvious answer is that albeit not well understood, oceanic turbulence is known to mix water properties both

along and across isopycnal surfaces with mixing efficiency whose order of magnitude are known globally (Munk and Wunsch 1998). Recall also how important the vertical mixing is for THC theories at *steady state* (Bryan 1987; Colin de Verdière 1988) to admit that it is not surprising to find that vertical mixing plays a central role for the millennial transients. Alternative proposals by Huang (1999) and Nilsson and Walin (2001) consider that the energy available for mixing should be imposed rather than the mixing coefficients. Since observations do not yet allow us to support either choice, we have kept the traditional approach of using constant mixing coefficients, essentially because the alternative introduces an additional source of nonlinearity in the (vertical) diffusive terms, thereby departing from our objective to concentrate on the nonlinear effects due to temperature and salinity advection.

The exploration of this  $3 \times 2$  model in section 2 has two objectives: first find out how the existence of oscillatory states depend on the geographical distribution of freshwater forcing and second identify the structure of the bifurcations that border the oscillatory regimes. Section 3 shows that several of the previous results extend to the case of a 2D (latitude–depth) model ocean. Geophysical implications are discussed in section 4.

## 2. The $3 \times 2$ box model

The existence of a saddle node bifurcation in Stommel's (1961) two-box model is well known (see, e.g., Thual and McWilliams 1992). When the salinity forcing is low, two thermal equilibrium states are possible, only one of which (the one with the largest circulation) is stable. When the salinity forcing increases, these two equilibrium states approach each other to finally collide at a saddle node bifurcation. When the salinity forcing increases further, the thermal state disappears and the solution tends to a stable haline state with circulation in the opposite sense. The analogy with Stommel's model has been widely used to explain the multiple steady states of the THC in oceanic (Bryan 1986) or coupled GCMs (Manabe and Stouffer 1988). On the other hand, no oscillatory states have been found in the original or modified Stommel's model (Ruddick and Zhang 1996).

A very different model, which exhibits oscillatory states, is that of Welander (1982). A single oceanic box can communicate with an upper reservoir mimicking the atmosphere, relaxing its temperature and salinity to values of the upper reservoir with different time scales. In addition, the temperature and salinity variables may also adjust instantaneously to the values of a lower reservoir if the density of the box becomes larger than that

of the lower reservoir parameterizing the intense mixing associated with polar convection. Otherwise weak mixing with the lower reservoir happens as pertains to a stably stratified system. Under certain values of external parameters, the solution flip-flops between the convecting and conductive states. However these oscillations are found in a parameter regime, which translates to a case of evaporation making geophysical comparisons difficult since convection occurs in polar oceans where precipitation occurs. In spite of this difficulty, the analysis of the oscillations in the Welander model made by Cessi (1996) will be shown to bear considerably on the present results.

In searching for a box model sustaining oscillations, the underlying idea was to find one that would allow the multiple states and saddle node bifurcation of the Stommel model and the *flip-flop* oscillations found by Welander with external parameters (such as the values of the variables in the reservoirs of Welander's model) kept to a minimum. The simplest addition that comes to mind was to add two deep boxes to the Stommel model. Such a  $2 \times 2$  model shows the collapse phase of the THC when the salinity forcing is strong enough but remains in that collapsed "haline mode" under constant boundary conditions. Whatever the initial and forcing conditions, no oscillatory states were ever found. The difficulty lies with the strong stability of the haline state. Although the temperature of the deep boxes increases diffusively tending to the interruption of polar convection, the stabilizing salinity difference of the polar boxes (halocline) always dominates over the destabilizing temperature differences, and the system does not ever flip back, as Welander's model, to one of active convection. However, if a net evaporation is imposed (in which case the overall salt content increases), the haline state is also unstable and oscillations are possible. Given the desire to work with a conservative system, degrees of freedom were first added in the vertical (two additional deep boxes) to study a  $2 \times 3$  model, hoping that the system could oscillate between states of shallow or deep convection. This expectation was deceived and the next trial was to add two other boxes in the horizontal to make a  $3 \times 2$  model, which is the one studied in what follows.

#### a. Description of the $3 \times 2$ box model

The chosen geometry is shown in Fig. 1. The horizontal and vertical areas between the boxes can be chosen to conform to spherical sectors of given latitude and longitude. On the other hand, the surface (or deep) boxes have the same common depth. Temperature and salinity evolve according to surface forcings, advection by meridional flow, and diffusion. The meridional flow

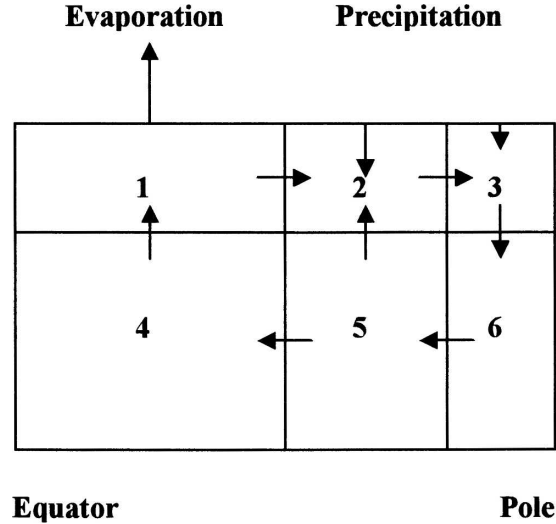


FIG. 1. The geometry of the  $3 \times 2$  box model. Mass transports are positive when the flow is in the direction of the arrows.

results from a simple balance between a Rayleigh type frictional dissipation and the meridional pressure gradient. The pressure is computed hydrostatically from the density assuming no net barotropic pressure forces between boxes. The temperature equations for the generic box  $i$  in interaction with the  $j$  neighboring boxes can be written as

$$m_i C_p \dot{T}_i = Q_i A_i + \rho_o C_p \psi_{ij} T_j + \rho_o C_p D_{ij} T_j + C_o(T_i), \quad (1)$$

where  $\rho_o$  is a reference density ( $=10^3 \text{ kg m}^{-3}$ ),  $C_p$  the heat capacity ( $= 4000 \text{ J kg}^{-1} \text{ }^\circ\text{C}^{-1}$ ), and  $A_i$  the horizontal area of box  $i$ . The mass  $m_i$  is kept constant (the small changes caused by freshwater exchange at the air-sea interface or continental runoff are neglected). The heat fluxes at the surface,  $Q_i$ , are written as

$$Q_i = \lambda(T_{ai} - T_i),$$

where the  $T_{ai}$  are constant atmospheric temperatures and  $\lambda$  an exchange coefficient. The advection operator is represented by the  $6 \times 6$  matrix of  $\psi_{ij}$ , where  $\psi_{ij}$  are volume transports between box  $i$  and  $j$ . Upstream differencing is used to compute the advection terms and necessitates a formulation that depends on the sense of the circulation, which is given in the appendix. Similarly  $D_{ij}$  is a  $6 \times 6$  diffusivity matrix that includes both lateral and vertical mixing between adjacent boxes. Vertical mixing between box  $i$  and  $j$  is given by

$$F_{ij}^V = \rho_o C_p \lambda_v A_i (T_j - T_i),$$

where  $j$  is the index of the box immediately below (or above) box  $i$ . The vertical exchange coefficient  $\lambda_v$  is

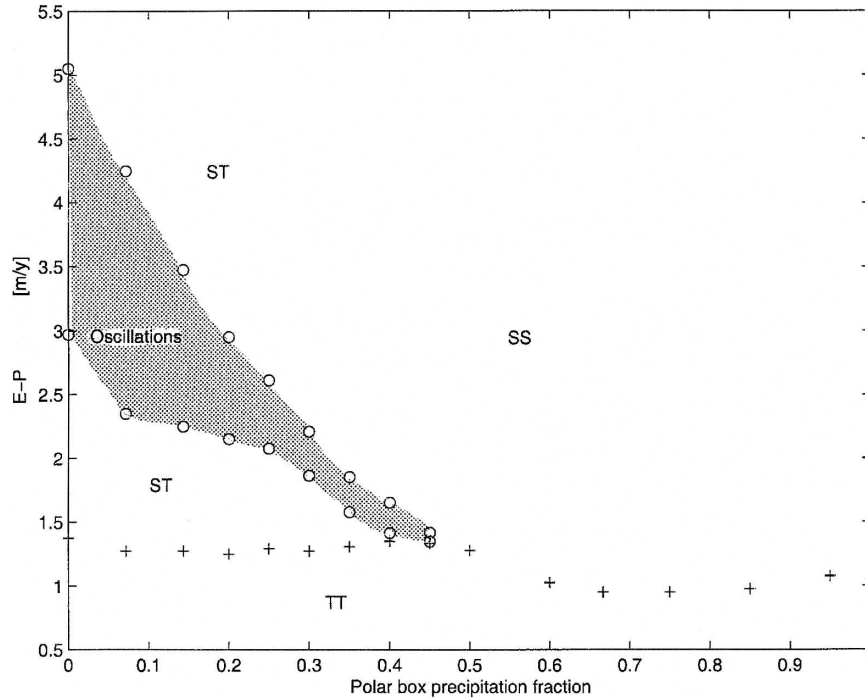


FIG. 2. Domains of existence of steady states (TT, ST, and SS) and oscillations against the precipitation fraction in the polar box and the  $E - P$  net forcing amplitude.

constant and  $A_i$  is the horizontal area of box  $i$  (or  $j$ ). Similarly lateral mixing between box  $i$  and box  $j$  (with the same common depth) is written as

$$F_{ij}^H = \rho_o C_p \lambda_H A_{ij} (T_j - T_i),$$

where  $\lambda_H$  is a horizontal constant exchange coefficient and  $A_{ij}$  the vertical area between the two boxes. Using these definitions, the divergence of the fluxes allows the construction of the matrix  $\mathbf{D}$  given in the appendix. Note in particular that the added degrees of freedom, compared to the Stommel (1961), model, allow distinguishing between advective and diffusive terms. The last term  $C_o(T_i)$  is the convection scheme that mixes instantaneously upper and lower boxes in case the vertical density difference leads to static instability. The convective mixing is carried out in one time step and preserves the thermal and salt content of the two boxes. Similar equations are used for the salinity variables:

$$m_i \dot{S}_i = S_o F_i A_i + \psi_{ij} S_j + D_{ij} S_j + C_o(S_i), \quad (2)$$

where  $S_o$  is a reference salinity ( $= 35$  psu) and  $F_i$  the freshwater forcing that is the net evaporation minus precipitation flux ( $\text{kg s}^{-1} \text{m}^{-2}$ ) received by box  $i$ . A linear equation of state  $\rho/\rho_o = -\alpha T + \beta S$  is used to compute density ( $\alpha = 2 \times 10^{-4} \text{ } ^\circ\text{C}^{-1}$ ,  $\beta = 0.8 \times 10^{-3} \text{ psu}^{-1}$ ). Using hydrostatics and the fact that the pres-

sure gradient forces only baroclinic flow allows one to relate the pressure in an upper box  $i$  to the density in that box and that immediately below (of index  $i + 3$ ) as

$$P_i = -\frac{1}{2} h_{i+3} / h g (h_i \rho_i + h_{i+3} \rho_{i+3}) \quad \text{and} \\ P_{i+3} = -h_i / h_{i+3} P_i. \quad (3)$$

Here  $h_i$  is the thickness of box  $i$ ,  $g$  is gravity, and  $h$  the full depth. Expressions (3) guarantee that the net pressure force (averaged vertically) vanishes. A simple balance between the horizontal pressure gradient and a Rayleigh friction term allows one to write:

$$\psi_1 = C A_{11} (P_1 - P_2) \quad (4a)$$

$$\psi_2 = C A_{21} (P_2 - P_3), \quad (4b)$$

where  $\psi_1$  and  $\psi_2$  are the volume transports between box 1 and 2, 2 and 3, respectively. If  $\psi_1$  and  $\psi_2$  are positive, the circulation has the sense as shown in Fig. 1. Because the volume of each box is conserved, the transports between box 4 and 5, 5 and 6 are  $-\psi_1$ , and  $-\psi_2$ , respectively. Continuity applied to box 2 (or 5) allows one to determine the upwelling transport  $\psi_w$  between box 2 and 5:

$$\psi_w = \psi_2 - \psi_1.$$

TABLE 1. Parameters of the  $3 \times 2$  box model.

|                                                                                                            |
|------------------------------------------------------------------------------------------------------------|
| Longitudinal widths: $60^\circ$                                                                            |
| Latitudinal widths: $0^\circ\text{--}45^\circ$ , $45^\circ\text{--}60^\circ$ , $60^\circ\text{--}90^\circ$ |
| Upper (lower) layer depth (m): 1000 (3500)                                                                 |
| Restoring surface temperatures: $T_a = 25^\circ$ , $12.5^\circ$ , $0^\circ\text{C}$                        |
| Pivot values of mixing exchange coefficients ( $\text{m s}^{-1}$ ):                                        |
| $\lambda = 7.5 \times 10^{-6}$ , $\lambda_v = 1.25 \times 10^{-7}$ , $\lambda_h = 1.75 \times 10^{-3}$     |

In such a crude model of the THC the strength of the advective transports can be simply controlled by varying parameter  $C$  in (4), which is the inverse of a friction coefficient [all multiplicative constants in (3) are lumped into parameter  $C$ ]. In practice  $C$  was chosen to give mass transports appropriate to the present North Atlantic climate.

A systematic search for oscillations was carried out using standard present values of mixing parameters (Table 1). The sizes of the boxes, which represent a single hemisphere, were kept fixed. The oscillations were not easy to find, and, once found, forcing and mixing parameters were systematically varied to delineate their domain of existence.

#### b. The influence of the freshwater fluxes

Foremost in the existence of millennial oscillations is the geographical distribution of the evaporation/precipitation,  $E - P$ , forcing at the surface. Given a climatological external temperature field to relax to, it was found that only rather special distributions allowed oscillations. To allow the exploration of the amplitude and the geographical distribution, the forcing was written as

$$S_0 F_i A_i = p q_i,$$

where  $q_i$  is the fraction between  $-1$  and  $+1$  of the net freshwater forcing in box  $i$  and  $p$  is the amplitude of the forcing, itself related to the imposed freshwater flux by

$$p = S_0(E - P)A_i.$$

The conservation of salt requires furthermore that

$$\sum q_i = 0.$$

With a large tropical/subtropical box extending from the equator to  $45^\circ$ , a net evaporation was chosen in that box ( $q_1 = 1$ ), the equivalent precipitation being shared in various proportions by the subpolar box 2 and polar box 3. With this choice ( $q_2 + q_3 = -1$ ), the exploration in the solutions was reduced to a two-dimensional forcing space ( $p$ ,  $q_3$ ). The model given by Eqs. (1) to (4) was then integrated forward for 10 kiloyears for given values of  $p$  and  $q_3$  with results summarized in Fig. 2.

When  $E - P$  is low, steady states are found. Both  $\psi_1$  and  $\psi_2$  are positive and, since the circulation has the thermal sense of Fig. 1, such states are noted TT. Convection occurs in the polar boxes 3 and 6 and cold deep water is formed. When the precipitation in box 3 is weak enough ( $|q_3| \leq 0.45$ ), an increase of  $E - P$  shows first that a new steady state emerges for which the circulation between boxes 1 and 2 has reversed ( $\psi_1 < 0$ ), indicating that the pressure gradient is now dominated by the salinity gradient (a state therefore called ST). When  $E - P$  increases further, millennial oscillations appear spontaneously. For  $|q_3| > 0.45$  the transition from the TT state leads to a stable haline state with reverse circulations ( $\psi_1$  and  $\psi_2 < 0$ ) in both subtropical and polar boxes (hence the acronym SS), and no oscillations have been found in this area of parameter space. Coming back to the region supporting oscillations, the bifurcation diagram is now described in more detail in Fig. 3 for the particular distribution of the freshwater flux  $q_i$  factors  $[1, -0.7, -0.3]$ . As the freshwater forcing increases, the transport in the thermal mode decreases much as in the Stommel box model, then the first bifurcation to the ST mode appears in which  $\psi_1$  is now directed equatorward: then the oscillatory regime in which  $\psi_1$  oscillates between large positive values overshooting those of the nearby TT mode and negative values typical of the ST mode appears. If the forcing increases further, the system reaches the stable steady state (the SS mode). Note that, in Fig. 3, there is coexistence of the ST regime with the oscillations. This steady branch was constructed by continuity with stable ST states that exist outside the oscillatory window. In practice, the model was initialized from a previous ST state with a weakly modified freshwater flux amplitude. Consequently this ST regime appears stable (unstable) to small (large) amplitude perturbations of the initial conditions. In the oscillatory regime one can see that the amplitude of the oscillation increases roughly linearly with the strength of the forcing. The maximum overshoots the value of nearby thermal steady states, while the minimum is close to the values of the haline SS steady state, a behavior noted by Cessi (1996) in her analysis of the Welander model. What about the period of the oscillations? Figure 4 shows that the period varies weakly near the first bifurcation (lower  $E - P$  value) but increases very rapidly near the second (higher  $E - P$  value). The curve giving the period as  $\log(p - p_c)$ , where  $p_c$  is the  $E - P$  value at the second bifurcation, provides a very suggestive fit to the data, even far from the second bifurcation. This behavior is very different from the supercritical Hopf bifurcations found, for instance, in 3D models at interdecadal peri-

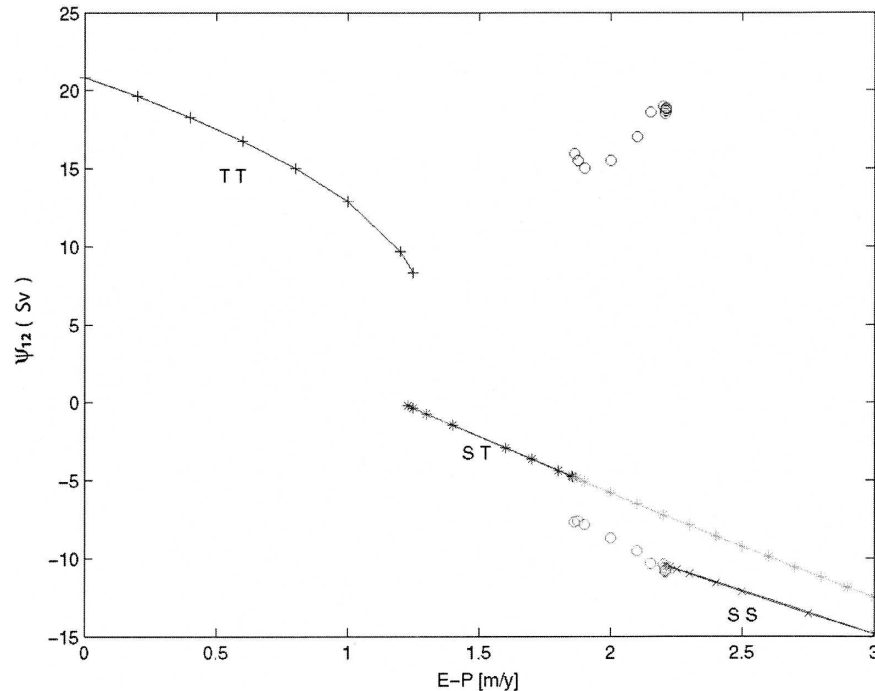


FIG. 3. The bifurcation diagram of  $\psi_1$  as a function of the  $E - P$  forcing amplitude for a given geographical distribution of that forcing  $q_i = [1, -0.7, -0.3]$ . The open circles represent the extrema reached by  $\psi_1$  during the oscillation. Note the presence of the stable steady state ST (\*) within the oscillation window.

ods: here the amplitudes of the oscillations cannot be linearized near a fixed unstable point and do not vanish when the supercriticality goes to zero. Strogatz (1994) describes three types of such *global* bifurcation of cycles in systems with one or two degrees of freedom: at the *saddle node bifurcation of cycles* a half stable cycle

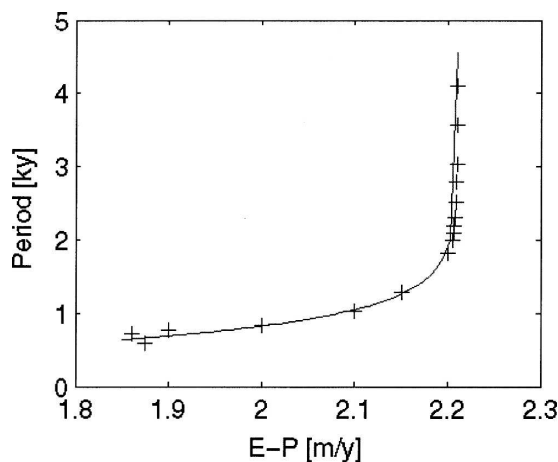


FIG. 4. The period of oscillation as a function of  $E - P$  forcing amplitude for the  $3 \times 2$  box model. The solid line is the log scaling characteristic of a global, homoclinic bifurcation.

is “born out of the blue sky” and as the control parameter increases, it splits into a pair of stable and unstable cycles. In this case the period and amplitude of oscillations are  $O(1)$ , independent of the control parameter. This might be the situation appropriate for the first bifurcation. Then Strogatz describes two kinds of *infinite period bifurcations*. The first is described nicely by the one degree of freedom dynamical system on the circle  $\dot{\theta} = \mu - \sin\theta$ , which represents the dynamics of an overdamped pendulum (with  $\theta$  the angle of the rod,  $\mu$  the dissipation, and  $\sin\theta$  proportional to the torque of the weight). If  $\mu < 1$ , there are two fixed points  $\theta_1$  and  $\theta_2$  equal to  $\sin^{-1}(\mu)$  and only the one with  $0 < \theta < \pi/2$  is linearly stable. When  $\mu_c = 1$ , the two fixed points collide to make a half-stable fixed point at  $\pi/2$  through a saddle node bifurcation. When  $\mu$  is larger than  $\mu_c$ , however, there are no more fixed points and the motion is periodic. If  $\mu$  approaches  $\mu_c$  from above, the period becomes infinite and near this limit the period behaves as  $(\mu - \mu_c)^{-1/2}$ . For this infinite period bifurcation the amplitude of the oscillation is  $O(1)$ . The second infinite period bifurcation described by Strogatz is the *saddle-loop or homoclinic bifurcation*. Before the bifurcation a stable limit cycle passes close to a saddle point: during part of the cycle the solution moves toward the saddle

point and then is repelled near the unstable branch of the saddle. As the control parameter increases, the limit cycle reaches the saddle point, creating a so-called homoclinic orbit, and the oscillation is destroyed. The amplitude of the oscillation is again  $O(1)$ , but now the period scales as the logarithm of the control parameter. This choice is clearly what is preferred for the second bifurcation in Figs. 3 and 4. As the system escapes the limit cycle, it jumps on the stable haline fixed point (SS state). If the forcing is decreased when the system is at the SS state, the same oscillations appear and no hysteresis is found. The similarity of this second bifurcation with that of the Welander model analyzed by Cessi (1996) and Abshagen and Timmermann (2004) is rather compelling. Much as in the Welander model, the period near the second bifurcation is controlled by the duration of the weak diffusive stage, the convective stage occupying a small fraction of the cycle. This description was obtained for values of dissipation consistent with the present amplitude of the THC. For lower dissipation (higher  $C$ ) it is possible to generate centennial oscillations whose advective character is clearly different (Sévellec et al. 2006, hereafter SHB). The mapping of the bifurcations in this parameter regime has yet to be carried out.

### c. The role of the mixing

The fact that mixing was not explicitly considered is likely to explain why such oscillations were not reported by Huang et al. (1992). First of all, eddy mixing (both lateral and vertical) is ubiquitous in the ocean. Although the mechanisms of mixing are not well known, the strength of mixing is roughly estimated through lateral and vertical turbulent diffusivity coefficients  $K_h$  and  $K_v$  of order  $10^3$  and  $10^{-4}$   $\text{m}^2 \text{s}^{-1}$ , respectively. It is natural to include it for the following reason: although dominant at the scales that carry the mixing, the influence of mixing at the scale of the planetary overturning can be estimated by comparing advective time scales to diffusive time scales. Peclet numbers,  $UL/K_h$  and  $Wh/K_v$ , serve this purpose. With  $U = 10^{-2}$   $\text{m s}^{-1}$ ,  $W = 10^{-7}$   $\text{m s}^{-1}$ ,  $L = 1000$  km, and  $h = 1$  km taken as scales for the THC,  $UL/K_h \sim 10$ , and  $Wh/K_v \sim 1$ . Because of the values of these Peclet numbers, it is well known that mixing must be included in the construction of steady states (Bryan 1987; Colin de Verdière 1988). The following results will show that mixing plays a similar role for the millennial oscillations, which should be no surprise given that the oscillations are really transitions between two unstable states reminiscent of the neighboring steady states. Given the estimate of the Peclet numbers, equivalent values in the

box model are found when  $\lambda_h (=K_h/L)$  and  $\lambda_v (=K_v/h)$  are  $10^{-3}$  and  $10^{-7}$   $\text{m s}^{-1}$ , respectively.

The existence of the oscillatory states of the model was explored by varying the mixing coefficients between zero and the above values. As  $\lambda_v$  decreases, the value of the freshwater forcing at the first bifurcation decreases, so oscillations happen more easily. If the  $E - P$  thresholds for the occurrence of oscillations in Fig. 2 are judged too high in comparison to the climatology, they can be lowered easily using this sensitivity to  $\lambda_v$ . On the other hand, the parameter has little impact on the amplitudes of the oscillation, which decrease somewhat. However the period at that bifurcation increases from 700 yr when  $\lambda_v = 10^{-7}$   $\text{m s}^{-1}$  to more than 2500 yr when  $\lambda_v = 0$ . The vertical mixing does not appear crucial to the existence of the oscillations in the box model but is certainly an important parameter in the determination of the periods. That vertical mixing, not essential here, must be taken with caution since upstream differencing, necessary for such box models, is associated with an implicit diffusion that plays a stronger role when explicit diffusion is low. Only the consideration of more finely resolved models can help here, and the 2D models of section 4 were included to do just that.

If  $\lambda_h$  is now decreased from values of  $5 \times 10^{-3}$   $\text{m s}^{-1}$ , the freshwater forcing at the bifurcation to oscillatory states decreases similarly, while the amplitude of the oscillations increases somewhat. The period also varies, but no systematic trend can be extracted. One of the differences with vertical mixing however is that the oscillations “require” some amount of lateral mixing, and they disappear entirely if  $\lambda_h$  is less than  $5 \times 10^{-4}$   $\text{m s}^{-1}$ . By describing the processes at work during the different phases of the oscillation, it is possible to find out where and when the lateral mixing becomes important.

### d. The description of the oscillation

The oscillation is really one between two regimes, one with strong meridional (mass and heat) transport, air-sea heat fluxes, and polar convection (*on* state) and the other with reduced meridional transports, air-sea heat fluxes, and no convection (*off* state). In the *on* state, the advection is carried out by a single direct thermal cell with negligible upwelling in the middle subtropical box. On the other hand, the advective cell of the *off* state has a dipole structure: a direct cell in the subpolar and polar boxes and a weak reversed (haline) cell between the subtropical and subpolar now with a strong upwelling in the middle box. Surface (deep) temperatures are higher (lower) in the *on* state. The salinity anomalies are more intense in the *off* state because with

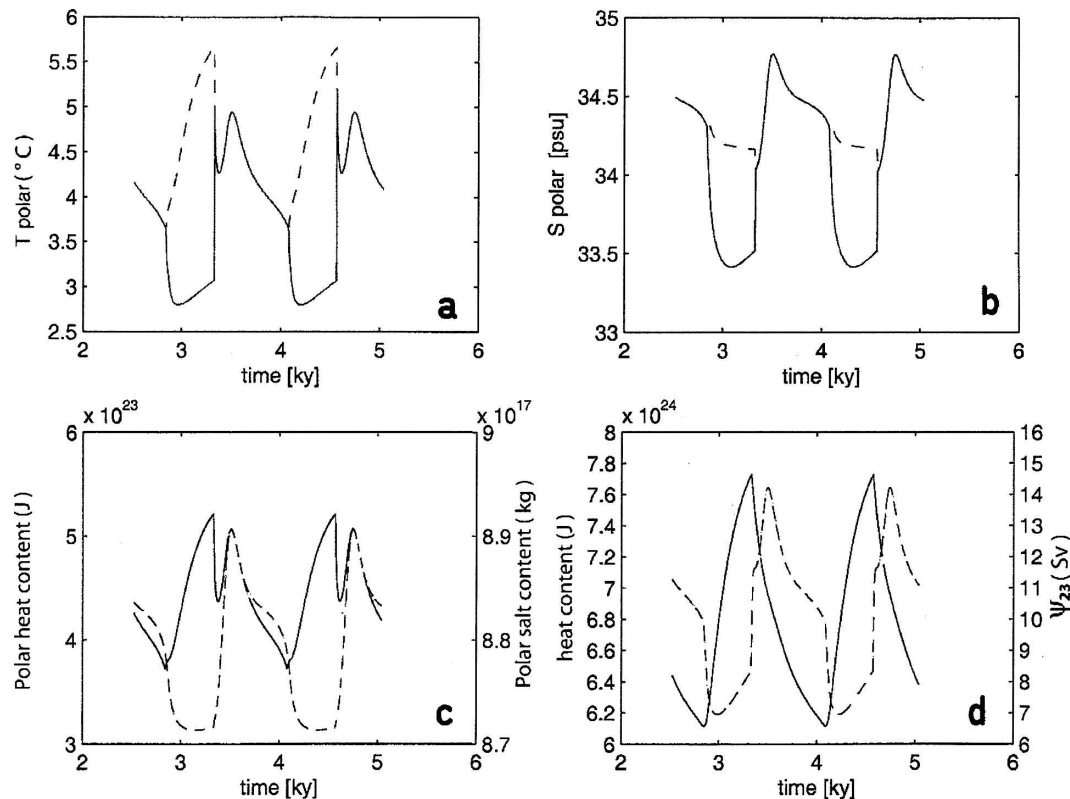


FIG. 5. Note the deep diffusive warming and the buildup of the halocline in the *off* state in (a) temperature and (b) salinity of the upper (solid) and lower (dashed) polar boxes. (c) The heat and salt content of the polar boxes as a function of time: in the *off* state, the heat content builds up diffusively. In the *on* state the heat content decreases through increased surface heat losses. The salt content builds up explosively as convection resumes but then decreases as the mass transport weakens. (d) The total heat content and the mass transport between the subpolar and polar region as a function of time: the two variables are in quadrature with the mass transport lagging the heat content.

less advection the constant freshwater forcing builds up stronger anomalies. An oscillation occurs when these two states are unstable, and a time series of the important variables is shown in Fig. 5. By carefully following each term in the model equations, a description of the different phases of the oscillation can be offered. Suppose that the system is in the *off* state. The deep subtropical box 4 warms due to vertical mixing and downwelling, while lateral mixing cools it. In deep boxes 5 and 6, the warming by lateral mixing becomes the dominant term, while advection is associated with a weaker cooling. As a result, the deep temperatures rise slowly in response to the nearly constant surface temperatures of the subtropical box. This rise causes an increase of the poleward pressure gradient through Eq. (3), and the transports  $\psi_1$  and  $\psi_2$  increase as well. During this *diffusive phase* the salinities are rather constant except for the salinity of the polar box, which increases in response to the stronger advective salt transport (the flow has the direction of a direct cell). With both the deep

temperature and surface salinity slowly rising, a *convection event* is bound to occur in the polar box. The heat stored in the deep layer is lost quickly to the atmosphere by suddenly active air–sea fluxes. Because the deep salinities have higher values, the strong convective mixing increases the surface salinity anomaly. Both higher surface salinity and lower deep temperature in the polar boxes now cause the pressure gradient (hence the transport) to rise instantaneously to high values. With such high transport, the salinities keep increasing in the polar box and in turn force the mass transport to increase even further through the well-known positive feedback between the amplitude of the THC and salt transport (Broecker et al. 1990; Rooth 1982; Walin 1985). This is the *salt advection* phase. However, at some point the mass transport reaches a maximum and then decreases. The cause of this decrease is due to the temperature gradient between the tropical and subtropical boxes, which has started to *decrease* both at the surface and at depth as soon as the convection has been

reinitiated. During this phase the advection of heat in the deep subtropical box is rather small because heat is imported and exported at similar temperatures. The deep box loses more heat by lateral mixing than can be gained by vertical mixing from the upper box. In the upper box, the strong export of heat by advection is balanced by the increased heat gain at the surface, but again a net heat loss due to lateral mixing is observed. Because of this thermally induced weakening of the mass transport, the salt advection feedback works in reverse to amplify an initial decrease of the mass transport: as it decreases, the salinity anomalies build up, increasing the equatorward salinity gradient and reducing further the transport. With the freshwater forcing becoming dominant, it is no surprise to see the convection being interrupted at some point. As soon as it disappears, the surface salinities of middle and polar boxes decrease very rapidly and the advective transports reach low values because the salinity contributions to the transports are now maximum. The *off* state is reached during which deep temperatures will rise again slowly.

Through this account, the three processes—eddy diffusion of heat, salt advection, and polar convection—emerge. The first is crucial at definite times in the oscillation: to trigger the slow decrease from the *on* state and to allow the resumption of convection at the end of the *off* state. The second is the well-known salinity–circulation positive feedback of Rooth (1982) and Walin (1985), while the third allows a catastrophic break away from the grip of the haline *off* state.

### 3. The 2D model

The next step in this study was to confirm some of the previous conclusions with higher resolution models yet economical enough to integrate for many thousands of years. The two-dimensional latitude–depth model has been widely used by many investigators (Marotzke et al. 1988; Wright and Stocker 1991; Thual and McWilliams 1992; Winton and Sarachik 1993; Sakai and Peltier 1995; Saravanan and McWilliams 1995; Rossby 1998; Paillard and Cortijo 1999) to investigate thermally forced circulation and the multiple steady states of the THC under varying freshwater fluxes with dissipative dynamics. The geometry does not allow one to include the rotation of the earth [although some simplified effects have been discussed by Wright and Stocker (1992)], but there is a general perception that the crude dynamics that relate the meridional pressure gradient to dissipation in such models is appropriate for the zonally averaged dynamics found in 3D models. There is no guarantee however since, after zonal aver-

aging of the meridional momentum equation, the additional Coriolis acceleration due to the zonal flow is still present. The oscillations found in the interdecadal frequency band, for instance, are sensitive to the choice, 2D or 3D, of the geometry because zonally propagating baroclinic Rossby waves play such an important role in these motions. Because the size of the oceanic basin induces a lower bound to the Rossby wave frequencies, the ultralow frequency forcing, of interest herein, is unlikely to excite them and the 2D representation might be an acceptable choice. Alternatively the use of 2D models may be seen as just one more step on the road of increasing realism and complexity. As a matter of fact, the nonlinear processes active in the THC *require* use of such an hierarchy of models to test the physics of a proposed mechanism.

#### a. The 2D model

The spherical model has a uniform grid in  $\sin(\text{lat})$  and depth with 25 grid points over a single hemisphere and 30 in the vertical. Use is made of a staggered grid to implement conservation of tracer, tracer variance, and energy. Constant horizontal ( $10^3 \text{ m}^2 \text{ s}^{-1}$ ) and vertical ( $10^{-4} \text{ m}^2 \text{ s}^{-1}$ ) diffusive coefficients are used for the tracers, identical for temperature and salinity. Rayleigh dissipation was preferred to the more traditional Laplacian form because the upper limb of the overturning circulation simulated with the former is closer to the surface and agrees better with that calculated in 3D geometry. The overturning streamfunction  $\psi$  is linked to meridional and vertical velocities as

$$v = (1 - y^2)^{-1/2} \psi_z$$

$$w = -\psi_y,$$

where  $y = \sin(\text{lat})$  is the meridional coordinate and  $z$  the vertical one. With the additional assumption of hydrostatic pressure, the relation between  $\psi$  and the density gradient reduces to

$$\varepsilon \partial^2 \psi / \partial z^2 = g / \rho_o (1 - y^2) \partial \rho / \partial y \quad (5a)$$

where  $\varepsilon$  is the Rayleigh friction coefficient [with values in the range from 2 to  $4 \times (10^{-3} \text{ s}^{-1})$ ] and  $\rho_o$  a constant reference density that appears when the Boussinesq approximation is made.

$$\frac{\partial T}{\partial t} + \frac{\partial \psi}{\partial z} \frac{\partial T}{\partial y} - \frac{\partial \psi}{\partial y} \frac{\partial T}{\partial z} = K_v \frac{\partial^2 T}{\partial z^2} + K_h \frac{\partial}{\partial y} (1 - y^2) \frac{\partial T}{\partial y} + C_o(T). \quad (5b)$$

Equation (5b) is the temperature evolution equation (with a similar one for salinity). Temperature and salinity are advanced in time with a standard leapfrog

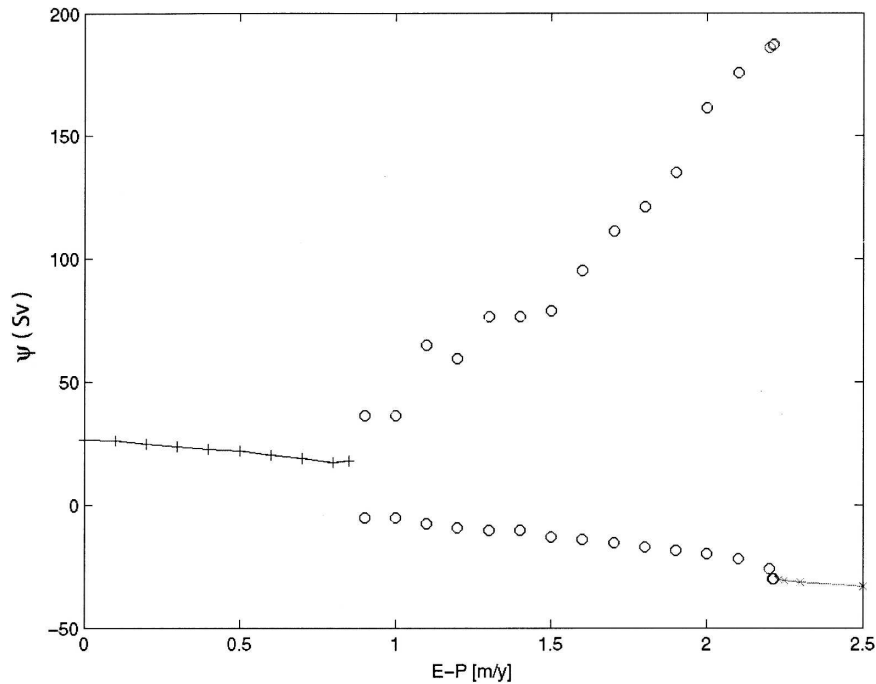


FIG. 6. The bifurcation diagram as a function of  $E - P$  forcing amplitude for the 2D model. Open circles are oscillation extrema of  $\psi$ . Thermal (haline) steady states are labeled with + ( $\times$ ) symbols.

scheme (with an Euler forward increment every 20 steps to suppress mode splitting). Density is computed with the same linear equation of state as previously used. Finally  $\psi$  is obtained from (5a) with zero boundary conditions at top and bottom. In the simulations with a single hemisphere described here, the meridional gradient of the tracer fields vanishes at the equator ( $y = 0$ ). The surface temperature was restored to the climatological SST distribution used by Saravanan and Mc Williams (1995):

$$\text{SST}(y) = 19[1 - P_2(y)]$$

with  $P_2(y) = \frac{1}{2}(3y^2 - 1)$  a second-order Legendre polynomial. The freshwater flux distribution was chosen to be

$$E - P = p \cos(3\theta)$$

with  $\theta$  the latitude  $= \sin^{-1}y$ . This function has maximum evaporation (precipitation) at the equator ( $60^\circ$ ). The “reduced” precipitation near the pole was chosen to conform to the forcing distributions susceptible to make the ocean oscillate carried out previously in the box model. It is similar to the distribution used by Winton and Sarachik (1993) to generate millennial oscillations in their 2D model.

#### b. The bifurcations of the 2D model

All the results presented in the following have been obtained with this choice of forcing, the amplitude  $p$  being the only parameter that was varied. Figure 6 shows the value of the maximum of the overturning streamfunction obtained after the model solutions have been integrated for at least 10 kiloyears for each value of  $p$ . For  $p$  less than  $\sim 0.9 \text{ m yr}^{-1}$ , the THC is in a steady thermal state. For  $p$  in the range from 0.9 to  $2.25 \text{ m yr}^{-1}$ , the model solution oscillates between two states: a strong thermal state and a weak haline state. For still larger  $p$ , the THC resumes to a steady haline state. Some trials to isolate steady regimes within the oscillation regime (as the steady state ST in the  $3 \times 2$  box model: Fig. 3) were not successful. This does not mean that they do not exist but simply that a few direct time integrations with various initial conditions failed to locate them. Apart from this, the bifurcation diagrams of the box model or the 2D model compare well in the neighborhood of the oscillation window (Figs. 3 and 6): similar overshoots of the oscillation maxima and minima close to the nearby haline steady state. When the period of the oscillations is plotted against the amplitude of the freshwater forcing, it exhibits the transition to infinite period found earlier for the second bifurcation (Fig. 7). The scaling as  $\log(p - p_c)$ , which is

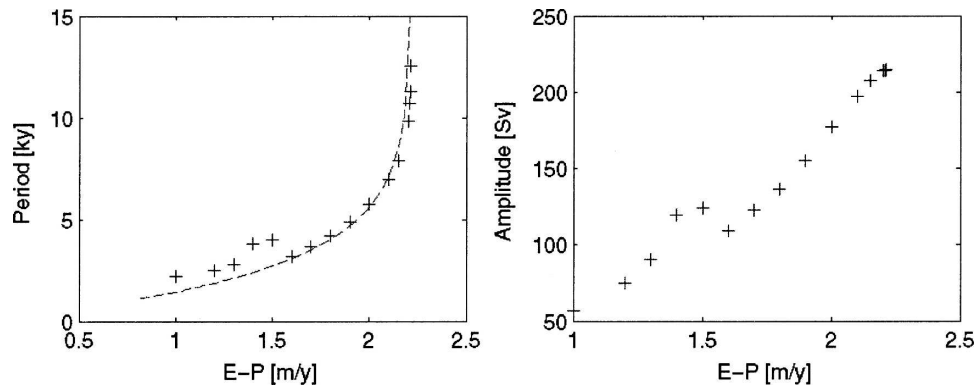


FIG. 7. The (a) period and (b) amplitude of the oscillations as a function of  $E - P$  forcing amplitude. The dashed line in (a) is the log scaling characteristic of a global, homoclinic bifurcation.

also indicated, is shown to be an excellent fit. The amplitude of the oscillations grows roughly linearly with  $p$  with, again, the minimum at values close to the stable haline fixed point.

#### c. The millennial oscillations in the 2D model

The streamfunctions of the two states of the oscillations, the strong thermal and the weak haline ones, are shown in Fig. 8. Their shape is similar to those found in the box model. Note in particular the small positive cell that occurs near the pole when the THC is in the weak state. As in the box model, the *strong decrease, near the pole*, of the *freshwater profile*, and not of the relaxing temperature, permits the *inversion* of the *density gradient* and creates this *small positive cell*. The difficulty to find these oscillations is linked to the fact that the haline state is very stable. The halocline formed at the pole is difficult to break unless the precipitation is reduced there. The conditions for the onset of convection are met when the deep water temperatures become sufficiently high through a combination of diffusion and advection, and the presence of the small polar positive cell is crucial to the resumption of convection. Conversely the reduction of the precipitation imposed in the north is crucial for the breaking of the halocline and for the existence of the small positive transport cell. The time evolutions of some variables are shown in Fig. 9, and the striking similarities with Fig. 5 bear witness to the fact that the *same oscillations* are being observed in the low-order model and the 2D model.

#### d. Sensitivity to vertical mixing

It is of course more costly to investigate parameter sensitivities in the 2D model and only that respective to vertical mixing is being reported here, given that only the global values of mixing are well constrained (Munk and Wunsch 1998). Indeed the coefficient  $K_v$  has been

varied between  $0.25 \times 10^{-4}$  and  $5 \times 10^{-4} \text{ m}^2 \text{ s}^{-1}$ , keeping constant the momentum dissipation  $\varepsilon = 1.85 \times 10^{-3} \text{ s}^{-1}$ . For each value of  $K_v$ , the first bifurcation has been searched empirically by varying  $E - P$  and, once found, the period has been measured. In a manner similar to what happens in the box model, Fig. 10 shows that the value of  $E - P$  at this first bifurcation increases as  $K_v$  increases. It varies by a factor of 2 over the range of plausible values for vertical mixing. Since  $\psi$  increases with  $K_v$ , an increase of vertical mixing leads to a more advective regime with the result that the haline feedback is less able to induce the first bifurcation (SHB). Consistent with the importance of diffusive warming of the deep layers in the *off* state, the figure shows that the period increases with the decrease of the vertical mixing. With pure diffusive control, the period would be inversely proportional to the mixing coefficient, but this is not what the figure shows and another scaling law has to be found. Given the advective-diffusive nature of the oscillation, assume that the period  $T$  scales as the inverse of a horizontal velocity scale  $U$ :

$$T = L/U,$$

where  $L$  is some appropriate basin scale. The circulation  $U$  is influenced by the vertical mixing, as in steady-state theories. Then following Wright and Stocker (1992), a scaling for  $U$  is derived assuming a single hemispheric basin and a constant vertical mixing coefficient. Introduce  $\delta$  as a vertical scale (the depth of the pycnocline) and assume that the horizontal density gradient is imposed externally (which is only true in the strong thermal phase of the oscillation): First,  $U$  is related to the streamfunction scale  $\psi$  as

$$U\delta = \psi.$$

But, (5a) can be used to relate scales  $\psi$  and  $\Delta\rho$  as

$$\psi = g\Delta\rho/\rho_o(L\varepsilon)^{-1}\delta^2.$$

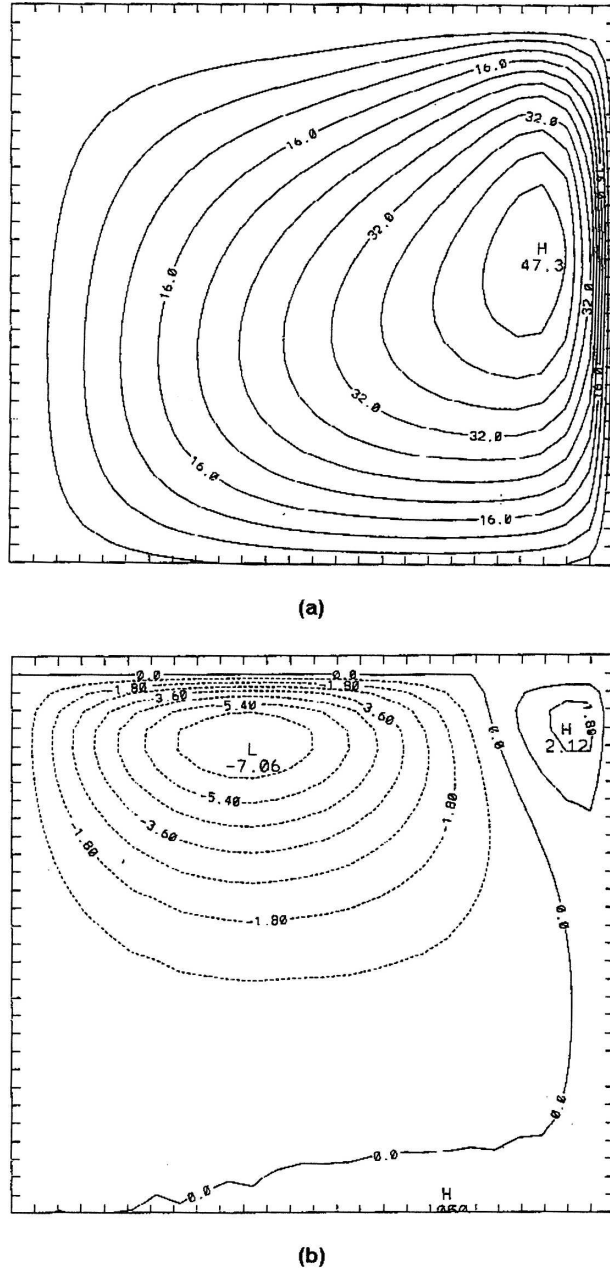


FIG. 8. The 2D overturning streamfunction in (a) the *on* state and (b) the *off* state. The volume transport values that are indicated have been obtained for a width equal to the circumference of the earth.

Finally, when vertical mixing equilibrates advection in the tracer equation,

$$\psi = K_v L / \delta.$$

From this, the following scaling is obtained for  $\delta$  and  $U$ :

$$\delta = [K_v L^2 \varepsilon / g \Delta \rho / \rho_o]^{1/3}$$

$$U = [K_v L^{-1} (g \Delta \rho / \rho_o \varepsilon)^2]^{1/3}.$$

The end result is the same as Welander's (1971) scaling valid for the 3D steady-state THC [see the reviews by Bryan (1991), Gnanadesikan (1999), and Saenko and Weaver (2003)]. But now, the period  $T$ , which is inversely proportional to velocity  $U$ , scales as  $[K_v]^{-1/3}$  and the amplitude of the oscillation in terms of  $\psi$  as  $[K_v]^{2/3}$ . Figure 10 shows that this scaling is meaningful for  $K_v$  larger than  $0.5 \times 10^{-4} \text{ m}^2 \text{ s}^{-1}$ , reinforcing the contention that the oscillation has a truly mixed advective–diffusive origin. For the lowest values of the vertical mixing, the scaling law for the period becomes poor, and lateral mixing would need to be included in the buoyancy balance scaling.

#### 4. Discussion

Given that intrinsic millennial oscillations of the THC are probably at the heart of Dansgaard–Oeschger oscillations, the present study has followed the lead of the early works on the subject (Winton–Sarachik 1993) through the deliberate choice of simple physical and geometrical conditions, putting aside the many additions that comparisons with the climate system of the last glacial period may require. Indeed, the conditions of existence and the characteristics of millennial oscillations have been explored herein with a *salt-conserving* system, the sole focus on *internal ocean* dynamics, a *linear* equation of state, a standard convection scheme, and with *time independent* boundary conditions.<sup>1</sup> If the boundaries between steady states and oscillatory states are to be delineated, the latter is essential. These choices allow concentrating on the essential ingredients controlling the *free oscillation*, so to speak. In contrast with many geophysical fluid oscillations, this one cannot be linearized as there is no way to reduce the amplitude to zero with a control parameter (as in a supercritical Hopf bifurcation, for instance). The first conclusion, of course, is that such elementary conditions are sufficient to generate finite amplitude, millennial oscillations that do not require the presence of periodic or net nonzero freshwater forcing, freshwater noise, a nonlinear equation of state, or interactions with ice or atmosphere. All these additions might turn out to be important for comparison with the paleoclimatic data but are considered here as simply shaping the intrinsic oceanic *free oscillation*. In spite of the simplified context of the study, the millennial oscillation is a complex,

<sup>1</sup> Note that although the boundary condition on temperature is fixed, the surface heat fluxes are time dependent and it is these fluxes that provide the coupling with the atmosphere.

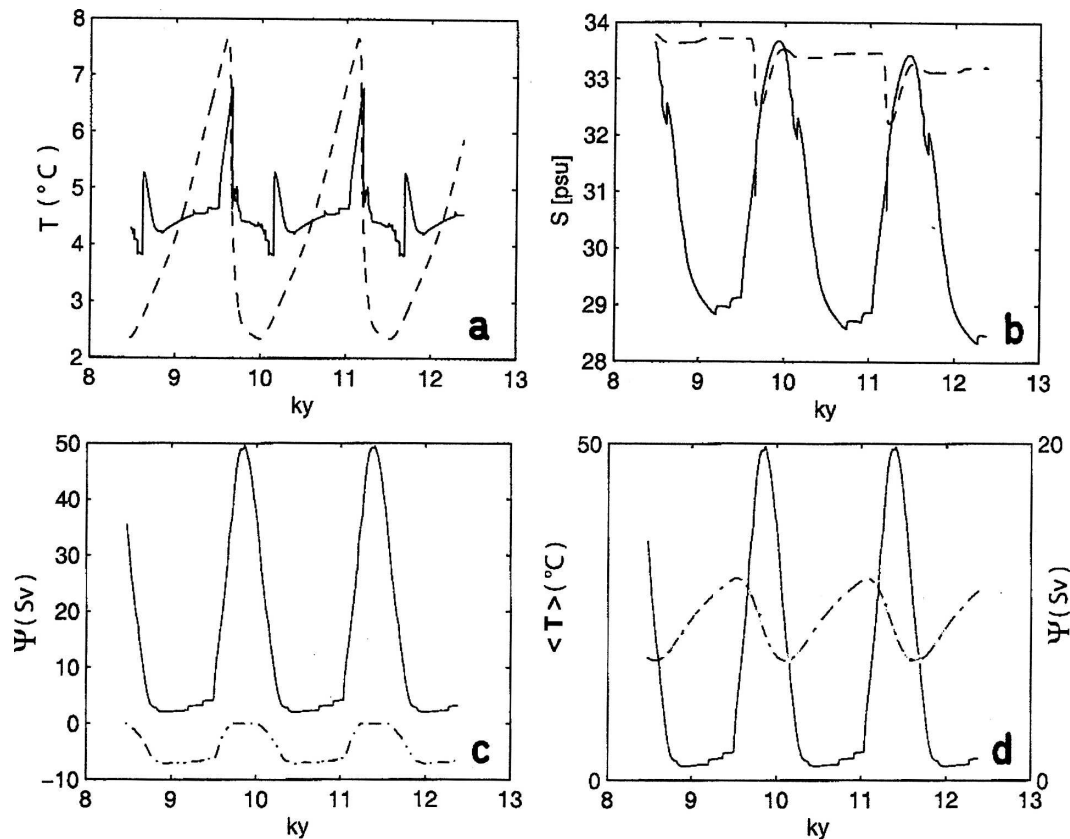


FIG. 9. Results of the 2D model to be compared with Fig. 5. (a) Temperature and (b) salinity of the upper (solid), lower (dashed) polar regions. (c) Maxima and minima of the overturning streamfunction as a function of time. (d) The total heat content and the maxima of the overturning in the 2D model.

nonlinear phenomenon that relies on the presence of turbulent mixing (as do steady states when they exist). The mechanism at work is different from the stochastic resonance recently proposed by Alley et al. (2001) and modeled by Velez-Belchi et al. (2001) and Ganopolski and Rahmstorf (2002). Alley et al. (2001) show that the distribution of waiting times of the abrupt warm D/O events in the Greenland Ice Core Project (GRIP)  $\delta\text{O}_{18}$  record is consistent with the stochastic resonance hypothesis by exhibiting occurrences at period  $T$  ( $\sim 1.5$  kyr),  $2T$ , and  $3T$ . The stochastic resonance mechanism requires a bistable system, a weak periodic forcing, and stochastic noise. They propose that the origin of the freshwater noise is due to the outburst floods from ice-dammed lakes but consider that the cause of the periodic forcing is more uncertain as may arise from deep-ocean processes (Broecker et al. 1990), solar forcing (Mayewski et al. 1997), tidal forcing (Keeling and Whorf 2000), and the El Niño–Southern Oscillation (Cane and Clement 1999). The present study supports the idea that external periodic forcing is unnecessary since internal oscillations are permitted in the relevant

parameter range through ocean processes only. The remark that the millennial frequency range being five octaves above the Milankovitch forcing range and five octaves below the tidal range made by Munk et al. (2002) does not favor the external origin, and they suggest discriminating between the two on the basis of signal bandwidth. It may come as a surprise that the exactly periodic nature of our internal oscillations is not consistent with the broad nature of the geophysical records, but clearly the contradiction comes from the drastic spatial filtering of our models, which leave aside nearly all classical fluid instabilities and all interactions with other climate subsystems: the addition of an ice-induced, stochastic freshwater noise forcing as proposed by Alley et al. (2001) is one possibility to reproduce the irregular character of the distribution of waiting times between warm events.

When further implications of the present work for the climate system are considered below, the qualitative nature of the conclusions must be emphasized because such a study provides maps of bifurcation diagrams whose precise boundaries depend in reality on

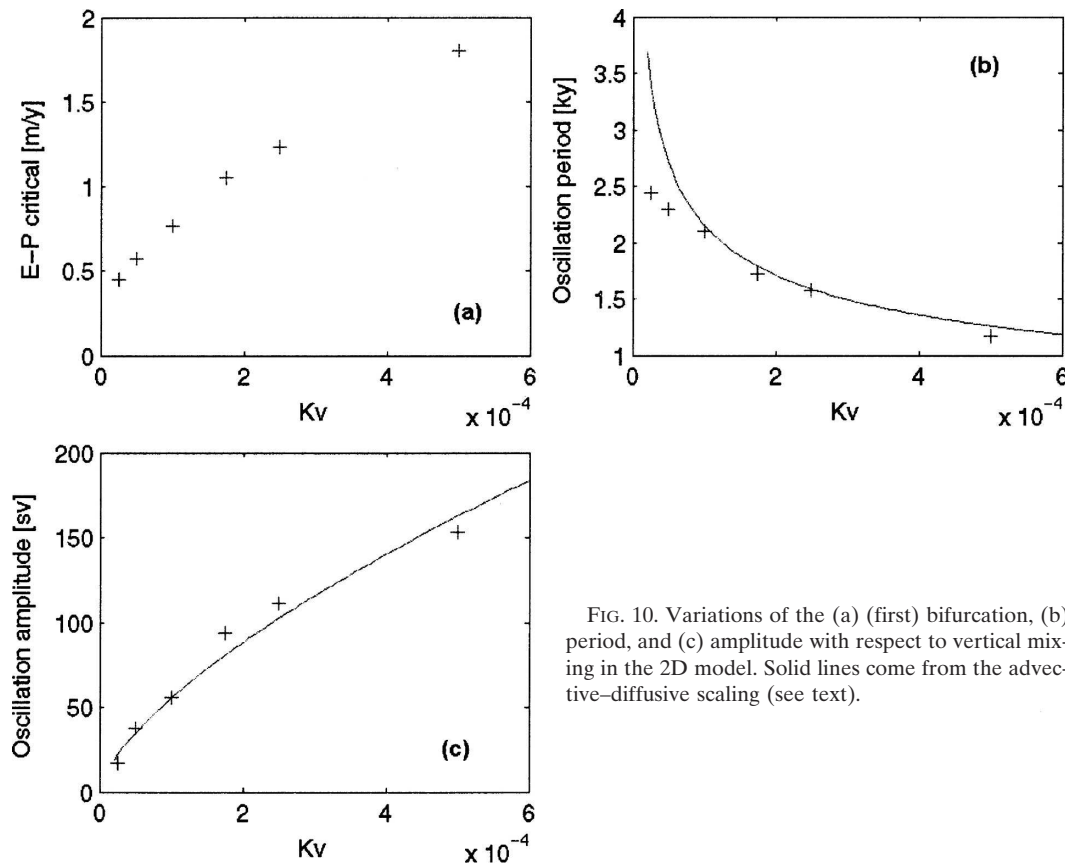


FIG. 10. Variations of the (a) (first) bifurcation, (b) period, and (c) amplitude with respect to vertical mixing in the 2D model. Solid lines come from the advective-diffusive scaling (see text).

the many interactions and processes that have been neglected *purposefully*.

#### a. The forcing distribution

The ease of exploration of the box model bifurcations has shown that two conditions must be met for oscillations to appear. First, the freshwater forcing must be strong enough to bring the THC close to its saddle node (in a subpolar region) and, second, the percentage of precipitation that occurs in a polar region must not exceed a certain value. This second condition is particularly interesting since it is known that precipitation, as measured by the rate of snow accumulation on ice sheets, was about one-fourth at glacial times of what it is today (Alley et al. 1993; Cuffey and Clow 1997; Jouzel et al. 1997). Could this be one of the reasons why the Holocene has not experienced some of the strong D/O signals? The importance of the geographic distribution of forcing may well be model dependent since the early 3D simulations of Winton and Sarachik (1993) and Huang (1994) showed that millennial oscillations could be produced without a reduction in precipitation over polar regions. However, these studies used a nonlinear

equation of state, so the Winton's (1993) mechanism cannot be excluded. What needs to be tested with 3D models is whether the bifurcation to oscillatory states "with a reduction of freshwater input in polar regions" occurs earlier with respect to the overall amplitude of the freshwater forcing than the models without such a polar reduction.

#### b. The period of the millennial oscillations

All of the solutions explored here show that the periods are well defined at and well beyond the first bifurcation (the low critical value in  $E - P$ ). However, at a second bifurcation (higher critical value in  $E - P$ ), the period becomes infinite. This is of course one of the important results of the Sakai and Peltier's (1995) study. However their proposed exponential fit of the period with their *nonzero net* freshwater forcing (their Fig. 9) is not appropriate. In a manner similar to what has been found by Cessi (1996) and Abshagen and Timmermann (2004) for the Welander model, the period appears to vary as the logarithm of distance to the bifurcation, a signature of a homoclinic global bifurcation. Although it remains to give a geophysical value to

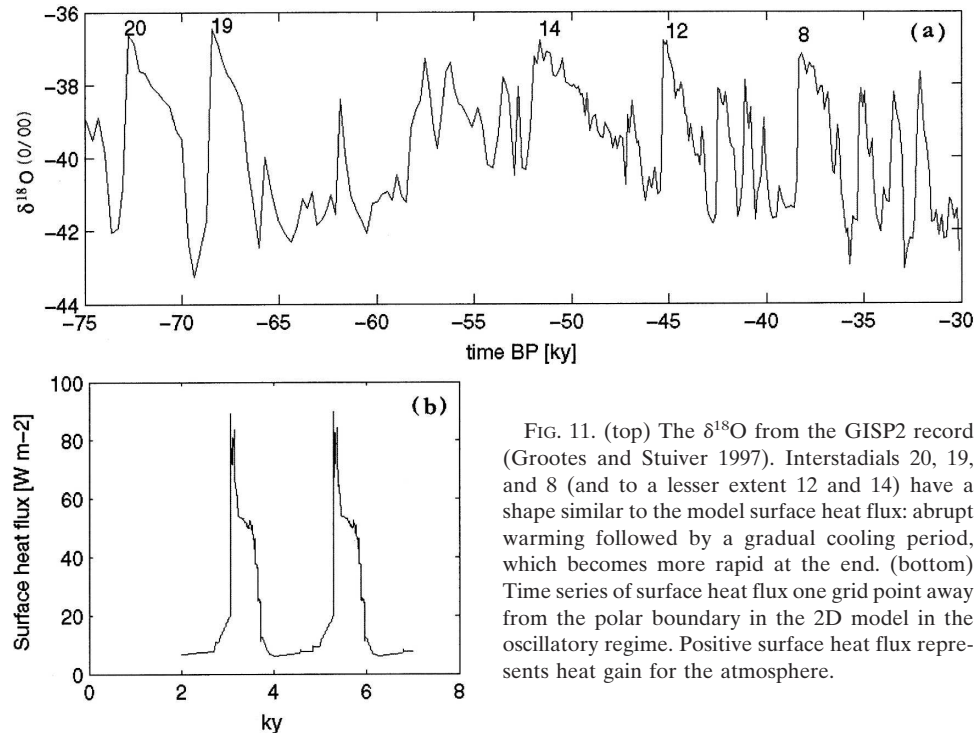


FIG. 11. (top) The  $\delta^{18}\text{O}$  from the GISP2 record (Grootes and Stuiver 1997). Interstadials 20, 19, and 8 (and to a lesser extent 12 and 14) have a shape similar to the model surface heat flux: abrupt warming followed by a gradual cooling period, which becomes more rapid at the end. (bottom) Time series of surface heat flux one grid point away from the polar boundary in the 2D model in the oscillatory regime. Positive surface heat flux represents heat gain for the atmosphere.

this infinite period singularity, it may be useful to identify this kind of signature in climate models.

The value of the period near the first global bifurcation in  $E - P$  is in the millennial range, in agreement with the paleo-observations, but the value in a given experiment depends on both overturning strength and many parameterized mixing processes. However, with what is known today of mixing processes and of the intensity of the THC (which was used to calibrate momentum dissipation) and assuming similar values for the past, this type of study predicts the right order of magnitude for the paleoperiods of the D/O. The scaling of the period as  $(\text{vertical mixing})^{-1/3}$ , shown in Fig. 10, is suspected to extend to a 3D situation, given the similarities of the 2D argument with Welander's 3D scaling. Because of this power law, uncertainties in vertical mixing are expected to influence the period only weakly.

#### c. The bifurcation thresholds

Since it can be shown that the bifurcation thresholds depend on four adimensional parameters in addition to the spatial structure of the forcings, it is clear that only a subset of that control space has been explored. Nevertheless, if we hypothesize that the strong sensitivity of the bifurcation thresholds to vertical mixing in 2D (Fig. 10) extends to 3D, the diversity of the response of climate models on these long time scales is likely to be

linked to variations in mixing in the models. As much as weather is dependent on poorly known initial states, the occurrence of THC oscillations in climate models will be dependent on poorly known mixing rates. There remains the further possibility of additional stable steady states in the oscillation window, such as state ST in the  $3 \times 2$  box model, which remains to be found in 2D (or 3D). In such a case the ocean would also express sensitivity to initial conditions in the oscillation window (as well as outside).

#### d. The abrupt warming

The surface heat flux near the polar boundary in the oscillatory regime of the 2D model shown in Fig. 11 has a shape that is strikingly similar to some of the D/O events seen in ice cores by Dansgaard et al. (1993). Figure 11 shows an abrupt warming followed by a plateau and a faster decline, features which are shared by certain events of the  $\delta^{18}\text{O}$  records. This is specially true of the interstadials 20, 19, 14, 12, and 8. Since Greenland atmospheric temperatures are expected to follow the curve of surface heat flux with a quadrature, the similarity with the  $\delta^{18}\text{O}$  curve, proxy for atmospheric temperature, is anything but fortuitous. The abrupt warming that signals the beginning of a D/O event is linked in the present models to the efficiency of vertical mixing to store heat for a considerable amount of time in

the deep ocean when the thermohaline circulation is weak and to the efficiency of mesoscale turbulence to make that heat available in polar regions (as shown in the  $3 \times 2$  model). At the end of an *off* state these regions are now heated from below and may convect if the halocline can be broken, which explains why the precipitation in such regions cannot be too large. When the convection occurs, two things happen: first the heat stored in the deep ocean for about a millennium will be made available to the polar atmosphere and, second, the thermohaline circulation shifts to a thermal mode as the meridional pressure gradient becomes temperature dominated, not so much because the temperature gradient increases but because the salinity gradient decreases. In this sense it can be said that the resumption of polar convection at the end of the *off* state drives the circulation in contrast with discussions of the present-day *on* state of the THC (Marotzke and Scott 1999). This abrupt release at high latitudes of the heat accumulated at low latitudes when the circulation is *off* has already been proposed by Paillard and Labeyrie (1994), who analyzed the solutions of an oceanic box model forced by an oscillating ice sheet simulating the Heinrich events. The combination of long-term heat storage in the deep ocean and convection for rapid climate change was also proposed albeit in a different context by Adkins et al. (2005): the presence of cold salty water of southern origin in the Atlantic Ocean at glacial times makes a stable stratification and allows geothermal flux to slowly heat the deep waters until convection of thermobaric origin (the dependence of seawater density on temperature and pressure) sets in. Whether these processes (with ice sheets or geothermal flux, respectively) or the one outlined in the present study is the most relevant remains to be studied.

Internal ocean dynamics with *autonomous*, *salt-conserving*, *mixed* boundary conditions and a *linear* equation of state are self-sufficient for the existence of millennial oscillations. The first bifurcation to oscillatory states is of a global nature (finite amplitude), while the second, also global but of infinite period, shares the property of the period scaling as the logarithm of the

distance to critical freshwater flux of the Welander model found by Cessi (1996). Obviously these bifurcation properties remain to be tested with respect to other subgrid-scale processes (since only a subset of them have been considered here) and in conjunction with other components of the climate system. In particular, the survival of these modes of oscillation when the stronger advective dynamics of 3D models are taken into account remains to be tested. The central question, however, remains whether the parameter space in which glacial oceans operated allowed the emergence of these self-sustained internal oscillations.

**Acknowledgments.** A. Colin de Verdière wishes to thank the directors of the 2003 Aosta School on “Paleo Climates: Dynamics and Observations,” and A. Provenzale, D. Schrag, and E. Tziperman for providing so many interesting discussions on the present subject. Many comments on a first draft of this paper by O. Marchal have been most helpful. Discussions with T. Huck and L. Te Raa are gratefully acknowledged.

## APPENDIX

### Formulation of the Advective Matrix and Diffusion Operator

With an upstream differencing scheme, the advection terms have different forms according to the sign of the circulation. Following Thual and McWilliams (1992),  $\psi_i^+$  and  $\psi_i^-$  are defined as

$$\psi_i^+ = \frac{1}{2}(\psi_i + |\psi_i|)$$

$$\psi_i^- = \frac{1}{2}(\psi_i - |\psi_i|)$$

so that, when the transport  $\psi_i$  is positive, the circulation has the sense of that chosen in Fig. 3,  $\psi_i^+ = \psi_i$  and  $\psi_i^- = 0$  (and conversely when  $\psi_i$  is negative,  $\psi_i^+ = 0$  and  $\psi_i^- = \psi_i$ ).

The elements  $\psi_{ij}$  of the  $6 \times 6$  advective matrix are the following:

$$\begin{bmatrix} -(\psi_1^+ + \psi_1^-) & \psi_1^- & 0 & \psi_1^+ & 0 & 0 \\ \psi_1^+ & -(\psi_2^+ + \psi_1^- + \psi_w^-) & \psi_2^- & 0 & \psi_w^+ & 0 \\ 0 & \psi_2^+ & -(\psi_2^+ + \psi_2^-) & 0 & 0 & \psi_2^- \\ \psi_1^- & 0 & 0 & -(\psi_1^+ + \psi_1^-) & \psi_1^+ & 0 \\ 0 & \psi_w^- & 0 & \psi_1^- & -(\psi_1^+ + \psi_w^+ + \psi_2^-) & \psi_2^+ \\ 0 & 0 & \psi_2^+ & 0 & \psi_2^- & -(\psi_2^+ + \psi_2^-) \end{bmatrix}.$$

Similarly, the diffusive matrix  $\mathbf{D}$  that allows one to compute the diffusive terms is

$$\mathbf{D} = \begin{bmatrix} -(\lambda_1 + \mu_{11}) & \mu_{11} & 0 & \lambda_1 & 0 & 0 \\ \mu_{11} & -(\lambda_2 + \mu_{11} + \mu_{21}) & \mu_{21} & 0 & \lambda_2 & 0 \\ 0 & \mu_{21} & -(\lambda_3 + \mu_{21}) & 0 & 0 & \lambda_3 \\ \lambda_1 & 0 & 0 & -(\lambda_1 + \mu_{11}) & \mu_{11} & 0 \\ 0 & \lambda_2 & 0 & \mu_{11} & -(\mu_{11} + \mu_{21} + \lambda_2) & \mu_{21} \\ 0 & 0 & \lambda_3 & 0 & \mu_{21} & -(\lambda_3 + \mu_{21}) \end{bmatrix},$$

where  $\lambda_i = \lambda_v A_i$ ,  $\mu_{11} = \lambda_h A_{11}$ , and  $\mu_{21} = \lambda_h A_{21}$ .

#### REFERENCES

- Abshagen, J., and A. Timmermann, 2004: An organizing center for thermohaline excitability. *J. Phys. Oceanogr.*, **34**, 2756–2760.
- Adkins, J. F., A. P. Ingersoll, and C. Pasquero, 2005: Rapid climate change and conditional instability of the glacial deep ocean from the thermobaric effect and geothermal heating. *Quat. Sci. Rev.*, **24**, 581–594.
- Alley, R. B., and Coauthors, 1993: Abrupt increase in Greenland snow accumulation at the end of the Younger Dryas event. *Nature*, **362**, 527–529.
- , P. U. Clark, L. D. Keigwin, and R. S. Webb, 1999: Making sense of millennial-scale climate change. *Mechanisms of Global Climate Change at Millennial Time Scales*, *Geophys. Monogr.*, Vol. 112, Amer. Geophys. Union, 385–394.
- , S. Anandakrishnan, and P. Jung, 2001: Stochastic resonance in the North Atlantic. *Paleoceanography*, **16**, 190–198.
- Blunier, T., and E. J. Brook, 2001: Timing of millennial-scale climate change in Antarctica and Greenland during the last glaciation. *Science*, **291**, 109–112.
- Broecker, W. S., G. Bond, and M. Klas, 1990: A salt oscillator in the glacial Atlantic? 1. The concept. *Paleoceanography*, **5**, 469–477.
- Bryan, F., 1986: High latitude salinity effects and interhemispheric thermohaline circulation. *Nature*, **323**, 301–304.
- , 1987: Parameter sensitivity of primitive equation ocean general circulation models. *J. Phys. Oceanogr.*, **17**, 970–985.
- , 1991: Poleward heat transport in the ocean: A review of a hierarchy of models of increasing resolution. *Tellus*, **43A**, 104–115.
- Cane, M., and A. Clement, 1999: A role for the tropical Pacific coupled ocean-atmosphere system on Milankovitch and millennial time scales. Part II: Global impacts. *Mechanisms of Global Climate Change at Millennial Time Scales*, *Geophys. Monogr.*, Vol. 112, Amer. Geophys. Union, 373–383.
- Cessi, P., 1996: Convective adjustment and thermohaline excitability. *J. Phys. Oceanogr.*, **26**, 481–491.
- Clark, P. U., N. G. Piasis, T. F. Stocker, and A. J. Weaver, 2002: The role of the thermohaline circulation in abrupt climate change. *Nature*, **415**, 863–869.
- Colin de Verdière, A., 1988: Buoyancy driven planetary flows. *J. Mar. Res.*, **46**, 215–265.
- Cuffey, K. M., and G. D. Clow, 1997: Temperature, accumulation and ice sheet elevation in central Greenland through the last deglacial transition. *J. Geophys. Res.*, **102**, 26 383–26 396.
- Dansgaard, W., and Coauthors, 1993: Evidence for general instability of past climate from a 250 ky ice-core record. *Nature*, **364**, 218–220.
- Dijkstra, H. A., and M. Ghil, 2005: Low-frequency variability of the large-scale ocean circulation: A dynamical systems approach. *Rev. Geophys.*, **43**, RG3002, doi:10.1029/2002RG000122.
- Ganopolski, A., and S. Rahmstorf, 2001: Rapid changes of glacial climate simulated in a coupled climate model. *Nature*, **409**, 153–158.
- , and —, 2002: Abrupt glacial climate changes due to stochastic resonance. *Phys. Rev. Lett.*, **88**, 038 501–1–038 501–4.
- Gargett, A. E., and B. Ferron, 1996: The effects of differential vertical diffusion of T and S in a box model of thermohaline circulation. *J. Mar. Res.*, **54**, 827–866.
- Gnanadesikan, A., 1999: A simple predictive model for the structure of the oceanic pycnocline. *Science*, **283**, 2077–2079.
- Gregory, J. M., O. A. Saenko, and A. J. Weaver, 2003: The role of the Atlantic freshwater balance in the hysteresis of the meridional overturning circulation. *Climate Dyn.*, **21**, 707–717.
- Grootes, P. M., and M. Stuiver, 1997: Oxygen 18/16 variability in Greenland snow and ice with  $10^3$  to  $10^5$  year time resolution. *J. Geophys. Res.*, **102**, 26 455–26 470.
- , —, J. W. C. White, S. Johnsen, and J. Jouzel, 1993: Comparison of oxygen isotope records from GISP2 and GRIP Greenland ice cores. *Nature*, **336**, 552–554.
- Huang, R. X., 1994: Thermohaline circulation: Energetics and variability in a single-hemisphere basin model. *J. Geophys. Res.*, **99**, 12 471–12 485.
- , 1999: Mixing and energetics of the oceanic thermohaline circulation. *J. Phys. Oceanogr.*, **29**, 727–746.
- , J. R. Luyten, and H. M. Stommel, 1992: Multiple equilibrium states in combined thermal and saline circulation. *J. Phys. Oceanogr.*, **22**, 231–246.
- Jouzel, J. R., and Coauthors, 1997: Validity of the temperature reconstruction from water isotopes in ice cores. *J. Geophys. Res.*, **102**, 26 471–26 487.
- Keeling, C. D., and T. P. Whorf, 2000: The 1800-year oceanic tidal cycle: A possible cause of rapid climate change. *Proc. Natl. Acad. Sci. USA*, **97**, 3814–3819.
- Manabe, S., and R. Stouffer, 1988: Two stable equilibria of a coupled ocean-atmosphere model. *J. Climate*, **1**, 841–866.
- Marotzke, J., 1989: Instabilities and multiple steady states of the thermohaline circulation. *Oceanic Circulation Models: Combining Data and Dynamics*, D. L. T. Anderson and J. Willebrand, Eds., NATO ASI Series, Kluwer, 501–511.
- , and J. R. Scott, 1999: Convective mixing and the thermohaline circulation. *J. Phys. Oceanogr.*, **29**, 2962–2970.
- , P. Welander, and J. Willebrand, 1988: Instability and multiple steady states in a meridional plane model of the thermohaline circulation. *Tellus*, **40A**, 162–172.

- Mayewski, P. A., L. D. Meeker, M. S. Twickler, S. Whitlow, Q. Yang, W. B. Lyons, and M. Prentice, 1997: Major features and forcing of high-latitude Northern Hemisphere atmospheric circulation using a 110000-year-long glaciochemical series. *J. Geophys. Res.*, **102**, 26 345–26 366.
- Munk, W., and C. Wunsch, 1998: Abyssal recipes: Energetics of tidal and wind mixing. *Deep-Sea Res. I*, **45**, 1977–2010.
- , M. Dzieciuch, and S. Jayne, 2002: Millennial climate variability: Is there a tidal connection? *J. Climate*, **15**, 370–385.
- National Research Council Committee on Abrupt Climate Change, 2002: *Abrupt Climate Change: Inevitable Surprises*. National Academy Press, 230 pp.
- Nilsson, J., and G. Walin, 2001: Freshwater forcing as a booster of thermohaline circulation. *Tellus*, **53A**, 629–641.
- Paillard, D., and L. Labeyrie, 1994: Role of the thermohaline circulation in the abrupt warming after Heinrich events. *Nature*, **372**, 162–164.
- , and E. Cortijo, 1999: A simulation of the Atlantic meridional circulation during Heinrich event 4 using reconstructed sea surface temperatures and salinities. *Paleoceanography*, **14**, 716–724.
- Rahmstorf, S., 1996: On the freshwater forcing and transport of the Atlantic thermohaline circulation. *Climate Dyn.*, **12**, 799–811.
- Rooth, C., 1982: Hydrology and ocean circulation. *Progress in Oceanography*, Vol. 11, Pergamon, 131–149.
- Rosby, T., 1998: Numerical experiments with a fluid heated non-uniformly from below. *Tellus*, **50A**, 242–257.
- Ruddick, B., and L. Zhang, 1996: Qualitative behavior and non-oscillation of Stommel's thermohaline box model. *J. Climate*, **9**, 2768–2777.
- Saenko, O. A., and A. J. Weaver, 2003: Southern Ocean upwelling and eddies: Sensitivity of the global overturning to the surface density range. *Tellus*, **55A**, 106–111.
- Sakai, K., and W. R. Peltier, 1995: A simple model of the Atlantic thermohaline circulation: Internal and forced variability with paleoclimatological implications. *J. Geophys. Res.*, **100** (C7), 13 455–13 479.
- , and —, 1997: Dansgaard–Oeschger oscillations in a coupled atmosphere–ocean climate model. *J. Climate*, **10**, 949–970.
- , and —, 1999: A dynamical system model of the Dansgaard–Oeschger oscillations and the origin of the Bond cycle. *J. Climate*, **12**, 2238–2255.
- Saravanan, R., and J. C. McWilliams, 1995: Multiple equilibria, natural variability, and climate transitions in an idealized ocean–atmosphere model. *J. Climate*, **8**, 2296–2323.
- Sévellec, F., T. Huck, and M. Benjelloul, 2006: On the mechanism of centennial thermohaline oscillations. *J. Mar. Res.*, in press.
- Severinghaus, J., and E. Brook, 1999: Abrupt climate change at the end of the last glacial period inferred from trapped air in polar ice. *Science*, **286**, 930–934.
- Stommel, M. H., 1961: Thermohaline convection with two stable regimes of flow. *Tellus*, **13**, 224–230.
- Strogatz, S. H., 1994: *Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry and Engineering*. Perseus Books, 498 pp.
- Thual, O., and J. C. McWilliams, 1992: The catastrophe structure of thermohaline convection in a two-dimensional fluid model and a comparison with low-order box models. *Geophys. Astrophys. Fluid Dyn.*, **64**, 67–95.
- Timmermann, A., H. Gildor, M. Schultz, and E. Tziperman, 2003: Coherent resonant millennial-scale climate oscillations triggered by massive meltwater pulses. *J. Climate*, **16**, 2569–2585.
- Velez-Belchi, P., A. Alvarez, P. Colet, and J. Tintore, 2001: Stochastic resonance in the thermohaline circulation. *Geophys. Res. Lett.*, **28**, 2053–2056.
- Walsh, G., 1985: The thermohaline circulation and the control of ice ages. *Palaeogeogr. Palaeoclimatol. Palaeoecol.*, **50**, 323–332.
- Weaver, A. J., and E. S. Sarachik, 1991: The role of mixed boundary conditions in numerical models of the ocean's climate. *J. Phys. Oceanogr.*, **21**, 1470–1493.
- , and T. M. C. Hughes, 1992: Stability and variability of the thermohaline circulation and its link to climate. *Trends Phys. Oceanogr.*, **1**, 15–70.
- , J. Marotzke, P. F. Cummins, and E. S. Sarachik, 1993: Stability and variability of the thermohaline circulation. *J. Phys. Oceanogr.*, **23**, 39–60.
- Welander, P., 1971: The thermocline problem. *Philos. Trans. Roy. Soc. London*, **A270**, 415–421.
- , 1982: A simple heat salt oscillator. *Dyn. Atmos. Oceans*, **6**, 233–242.
- Whitehead, J. A., 1995: Thermohaline ocean processes and models. *Annu. Rev. Fluid Mech.*, **27**, 89–113.
- Winton, M., 1993: Deep decoupling oscillations of the oceanic thermohaline circulation. *Ice in the Climate System*, W. R. Peltier, Ed., NATO ASI Series, Vol. 112, Springer-Verlag, 417–432.
- , and E. S. Sarachik, 1993: Thermohaline oscillations induced by strong steady salinity forcing of ocean general circulation models. *J. Phys. Oceanogr.*, **23**, 1389–1410.
- Wright, D. G., and T. F. Stocker, 1991: A zonally averaged ocean model for the thermohaline circulation. Part I: Model development and flow dynamics. *J. Phys. Oceanogr.*, **21**, 1713–1724.
- , and —, 1992: Sensitivities of a zonally averaged global ocean circulation model. *J. Geophys. Res.*, **97** (C8), 12 707–12 730.
- Zhang, R., M. Follows, and J. Marshall, 2002: Mechanisms of thermohaline mode switching with application to warm equable climates. *J. Climate*, **15**, 2056–2072.