



Dynamical properties of atmospheric and sea level extremes in the North-East Atlantic

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Abstract

Understanding the dynamics of extremes during storms is critical for improving their forecasts. In this work, we focus on both the atmospheric and ocean extremes, by considering the atmospheric pressure and the storm surges. Their properties are analysed using a dynamical systems approach. To this end, a novel metric is introduced: the normalised phase space velocity v . This allows us to track the acceleration of the dynamics near extreme events. We discuss its advantages compared to other indicators and compare their performances. We study the correlations between different storm intensity metrics and local dimensions d and v , as well as the evolution of these metrics near extreme events, uncovering the physical processes at play behind these observations. These findings provide new insights into the temporal structure of storm surges.

Keywords Storm surges · Dynamical systems · Phase space velocity · Local dimensions

1 Introduction

Since the beginning of the industrial era, the Earth has been undergoing profound and rapid changes (Masson-Delmotte 2021), many of which are unprecedented in the Holocene. Among these changes, extreme sea levels are rising and pose an increasing threat to coastal areas. Changes in extreme sea levels result from a combination of factors: mean sea level rise (Marcos and Woodworth 2017; Wahl and Chambers 2015), changes in tides (Haigh et al. 2019; Pineau-Guillou et al. 2021) and storm surges (Talke et al. 2014; Cheynel et al. 2025). Better understanding and characterizing these extremes is crucial, as they can lead to significant flooding (e.g., the Xynthia storm, Bertin et al. (2014)). This raises an important scientific question: how can we improve our understanding of the dynamics of extreme storm surges?

To address this, several approaches have been developed. Classical methods include extreme value analysis (Marcos and Woodworth 2017; Cheynel et al. 2025), numerical ocean

modeling (Fernández-Montblanc et al. 2020), and statistical approaches based on multilinear regression or neural networks (Tadesse et al. 2022; Harter et al. 2024). These methods are useful for estimating extreme levels and identifying contributing factors. However, some of these methods often rely on strong assumptions (e.g., stationarity, independence, linearity) and may not fully capture the dynamic complexity of the ocean-atmosphere system.

In contrast, tools from dynamical systems theory offer a complementary perspective. These methods reduce the system's state to two parameters: (1) a local dimension, which informs on the number of active degrees of freedom at a given time, and (2) a persistence indicator, which quantifies the tendency of the system to remain in or escape from a given state. While promising, these methods have mainly been applied to atmospheric circulation (Faranda et al. 2017, 2019; Hochman et al. 2021), and to a lesser extent to oceanic dynamics (Alberti 2023; Liu et al. 2021). To our knowledge, they have never been applied to storm surges at a regional scale. This study aims to explore that direction, by applying dynamical systems theory tools to the analysis of extreme storm surges and improving our understanding of their underlying dynamics.

Our objective is to characterise the sea level surge (i.e., the chaotic component of the sea level dynamics) using methods originating from dynamical systems theory

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(Lucarini et al. 2016; Faranda et al. 2017; Caby et al. 2019). We estimate the dynamical properties of both the atmosphere and ocean, focusing on extreme events. We consider the North-East Atlantic, including the North Sea, which is known to have important storm surge events (Menéndez and Woodworth 2010; Choi et al. 2018; Pineau-Guillou 2020).

After presenting the atmospheric and sea level data, we discuss the methods used in the literature and their limitations, in particular when it comes to the computation of the persistence index. This motivates the introduction of a novel indicator v that we call: *normalised phase space velocity*. This provides information on the stability of a state. We then study the relations between the values of these indicators and diverse storm intensity metrics, showing that extremes in these metrics correspond to extremes in surge intensity. In the discussion, we describe the evolution of the dynamics near such extreme events.

2 Data

In this study, we are interested in both the atmospheric dynamics and the sea level surge dynamics in the North-East Atlantic.

2.1 Sea-level atmospheric pressure (SLP) and wind data

We use the 20th Century Reanalysis version 3 (20CR, (Compo 2011; Slivinski 2019)). This dataset consists of sea level pressure and wind speed every 3 hours on a 1° grid from 1836 to 2015. We focus on the North Atlantic, delimited by the longitudes 40°W and 10°E and the latitudes 30°N and 70°N (see extension Fig. 3a). For computational efficiency, we restrain the studied period to 1960–2010. For this 50-year period, the 20CR reanalysis is known to reproduce more faithfully measurements (Slivinski 2019). We also restrain the temporal resolution to 6 hours (instead of the native 3-hour resolution). Finally, this leads to considering $N = 74512$ time steps and $n_x \times n_y = 51 \times 51 = 2601$ grid-points. We chose 20CR over the ERA5 reanalysis for consistency with the surge data that we now present.

2.2 Sea level surge data

We use surge data from ClimEx 1900–2015 sea level hind-cast in the North Atlantic (Cheynel et al. 1900–2015, 1900–2015). The dataset consists of hourly sea levels and surges over 1900–2015 in the North Atlantic, on a 0.1° grid. This sea level dataset comes from numerical simulations using a global ocean model TUGO (Lyard et al. 2021), forced by tidal potential and 20CR atmospheric reanalysis. Surge

values were obtained removing the tide and the mean sea level from the hourly sea levels.

As the surges produce smaller-scale patterns compared to those of the atmosphere, we further restrain the zone to the eastern part of the North-East Atlantic: 20°W – 10°E and 40°N – 60°N (see the extension on Fig. 3b). This zone is known for its intense surge events (Cheynel et al. 2025). In addition, still for computational efficiency, we restrict the period to 1960–2010, and subsample the temporal resolution to 6 hours (instead of 1 hour), and the spatial resolution to 0.4° (instead of 0.1°). This leads to consider $N = 74512$ time steps and $n_x \times n_y = 51 \times 76 = 3876$ grid points.

3 Methods

3.1 Data as a dynamical system

We model the evolution of both SLP and surge as dynamical systems. At each time step i from 1 to n , the dynamical system is in a given state. Denoting by X_i the vector of size $N = n_x \times n_y$ containing the values of the observable of interest (SLP or surge) at the different grid points at time t_i , the successive outputs $X_1, X_2, \dots, X_n$ can be thought of as a trajectory of a dynamical system evolving in the configuration space $M = \mathbb{R}^N$. Such high-dimensional nonlinear physical systems usually do not visit all their configuration space, but settle on a lower-dimensional set of configurations A called the attractor, which often exhibits a complex (multi)-fractal structure. Its dynamical properties can be described by two parameters: the local dimension $d(z)$ and a persistence index, usually denoted $\theta(z)$. The local dimension, that describes the local fractal geometry of the attractor at z , also quantifies the effective number of degrees of freedom of the system near the state z . It relates to notions of complexity and local predictability (Dong et al. 2024), as a lower local dimension corresponds to a lower number of directions in which the system can evolve. In different studies, $d(z)$ is computed together with a persistence index $\theta(z)$ and these metrics are used to characterise the dynamical properties of the system at different states z . θ takes values between 0 and 1, and the higher it is, the less persistent is the state z .

3.2 Dynamical metrics d , θ , and v

To define these metrics formally, we assume that the system admits a physical measure μ , which has the property that for all subset E of M (where M is the set of all possible states), $\mu(E)$ corresponds to the probability that the system is in a state of E .

The first metric, the local dimension $d(z)$, is defined as:

$$d(z) = \lim_{r \rightarrow 0} \frac{\log \mu(B_r(z))}{\log r}, \quad (1)$$

where $B_r(z)$ denotes a ball in the phase space of radius r centered at z (z is the state of the system). An estimate of this quantity, or rather a pre-asymptotic version of it Caby et al. (2019), can be obtained by analysing the recurrences of the system near z , using the framework of Extreme Value Theory. In particular, by fitting the empirical distribution of the exceedances of the process

$$Y_z(i) := -\log \|X_i - z\|$$

over a high threshold $u_d(z)$ Faranda et al. (2017), where $\|\cdot\|$ denotes the Euclidean norm in $\mathbb{R}^N$. In this work, we take $u_d(z)$ equal to the quantile 0.95 of the Y_z distribution. Intuitively, the local dimension measures the uncertainty of the future of a given state by measuring the number of “paths” the system can follow. A dimension of 1 suggests that the system has only one possible path and is fully determined without uncertainty. A dimension larger than 1 suggests that there is more than one possible path and so there is uncertainty in the future state of the system. Following this logic, large values of the local dimension characterise a state with “many paths” and so large uncertainty in the future state of the system.

The second metric, the persistence indicator $\theta(z)$, relates to the Extremal Index of the process Y_z , a parameter originating from Extreme Value Theory which quantifies the tendency of this process to form clusters of exceedances. Under some assumptions usually satisfied by chaotic systems, it is formally defined as Lucarini et al. (2016)

$$\theta(z) = \lim_{u \rightarrow +\infty} \frac{\mu(Y_z(i) > u, Y_z(i+1) > u)}{\mu(Y_z(i) > u)}. \quad (2)$$

Different estimators of θ exist in the literature, which all rely on counting the number of successive exceedances of the process $Y_z(i)$ over a high threshold u_θ Caby et al. (2020), which corresponds to visits of the system in a ball $B_r(z)$ of small radius $r = e^{-u_\theta}$ centered at z . To ensure that enough data are available to estimate it, this radius usually corresponds to a given quantile q of the empirical distribution of $Y_z(i)$ Faranda et al. (2017), yielding an estimator that we refer to as θ_v . As this distribution depends on the state z , this fixed quantile q assigns different radii of balls for different states z , and depends strongly on $d(z)$, as this quantity modulates the distribution of the distances between the state z and its nearest neighbours Platzer et al. (2021). As we will see, this creates strong correlations between the values of $d(z)$ and $\theta_v(z)$, which can be observed in different publications (Faranda et al. 2017; Liu et al. 2021; Alberti 2023; Faranda

2024; Hochman et al. 2021). Such a variable radius renders the interpretation of θ_v as a persistence indicator challenging since it assigns different scales to different states (making comparison of θ_v for different z virtually impossible). These issues can be resolved by fixing a threshold u_θ common to all the states z . This is how we proceed in this work, fine-tuning u_θ in such a way that the values of our inverse persistence indicator, which we refer to simply as θ , have a uniform spread. However, proceeding that way can be challenging in practice since the distributions of $Y_z(i)$ can vary widely depending on the state z .

To solve the issues related to the computation of the inverse persistence metric (i.e., with a variable or fixed radius), we propose a new metric that does not depend on any radius but still accounts for the stability of a state. This new metric, $v(z)$, is called the *normalised phase space velocity*, and is defined for a state $z = X_i$ as:

$$v(z) = \frac{\|X_i - X_{i+1}\|}{k}, \quad (3)$$

where $k = \max_{i=1, \dots, N-1} \|X_i - X_{i+1}\|$ is a normalising constant chosen so that v is comprised between 0 and 1, in such a way that $v = 0$ at the most persistent situation, that is when $X_i = X_{i+1}$, and $v = 1$ at the least persistent observed state. It is a dimensionless quantity that is easily computed, does not require fixing a radius or a quantile of the distribution, and provides an intuitive notion of the stability of a state; the higher is v , the faster the system tends to escape a given state (i.e., largest change between previous state, X_i , and following state, X_{i+1}). We provide the Python code for its computation in the appendix Sect. 1. It allows a fair comparison of the stability of states within a system, but also in different systems. By construction, it yields the same results whether we consider data or anomalies, and it is practically insensitive to the choice of climatology and seasonality. Although different in nature, we will see that v and θ are strongly correlated, provided θ is computed with a fixed threshold. Note that all the different metrics (d, θ_v, θ, v) are synthesized in Table 1.

4 Results

Following these definitions, we can compute the dynamical properties of atmospheric and surge data. This also allows for the computation the joint distributions of the local dimension (d) and the different metrics (θ, v or θ_v) for both the SLP and surge data (Fig. 1).

Table 1 Metrics characterising a dynamical system at the state z

Name	Interpretation	Computation	References
$d(z)$	Local dim.	Degrees of freedom	Eq. 1
$\theta_v(z)$	persistence index (classical)	Unclear	Eq. 2, var. radius r
$\theta(z)$	Persistence index	Persistence	Eq. 2, fixed r
$v(z)$	Norm. velocity	Instability	Eq. 3, no radius
			Faranda et al. (2017); Faranda (2024)
			Faranda et al. (2017); Faranda (2024); Cabby et al. (2020)
			New method (this paper)
			New metric (this paper)

4.1 The choice of stability metric

The d - θ distributions of the atmospheric pressure and oceanic surges are computed using the inverse persistence index θ with a 'fixed' radius for the persistence (Fig. 1a and d). The distribution is very different when using a 'variable' radius (see θ_v/d diagram, Fig. 1c and f), and similar to the distribution obtained in previous studies (see for example Fig. 2 in Faranda et al. (2017) for SLP data on a similar geographic region). Notably, fixing the radius (Fig. 1a, as proposed in the present paper) changes the distribution and removes its correlations with d (Pearson coefficient $r = -0.08$ for a fixed radius, against $r = 0.30$ for a variable radius). Indeed, a variable radius (due to a fixed quantile) assigns a lower

threshold u_θ to rarer events, which corresponds to a ball of larger radius $r = e^{-u_\theta}$ in the phase space compared to more typical states. The persistence of these states in that ball is therefore increased compared to when computed at a fixed radius, leading to a decrease in the estimates of θ_v . This favours assigning a higher persistence to rarer states. As the estimated value of d also depends strongly on the radius r (Lucarini et al. 2016; Platzer et al. 2021; Pons et al. 2023), this also creates important correlations between d and θ_v , which can be observed in different publications and in our analysis (Fig. 1c and f). A variable radius also prevents a uniform comparison of the various states. In addition, our method of computation of θ with a fixed radius assigns low persistence to stormy and unstable states and high persistence to calm states (see Fig. 1), whereas the contrary is obtained with a variable radius. Indeed, this means that there is more change during storm-states than during calm-states. However, since different states may have very different recurrence patterns, fixing a common radius for all the states may select many extreme events for some states and very few for other states. It is therefore difficult in practice to choose a good radius without yielding estimators that may not be at convergence for some states and select too few extreme events for other states. Furthermore, it requires a fair amount of fine-tuning of r to get non-degenerate distributions of θ . To go further and avoid the use of any radius, we propose a novel metric v (instead of θ), which provides an intuitive, easy-to-compute metric of stability of a state.

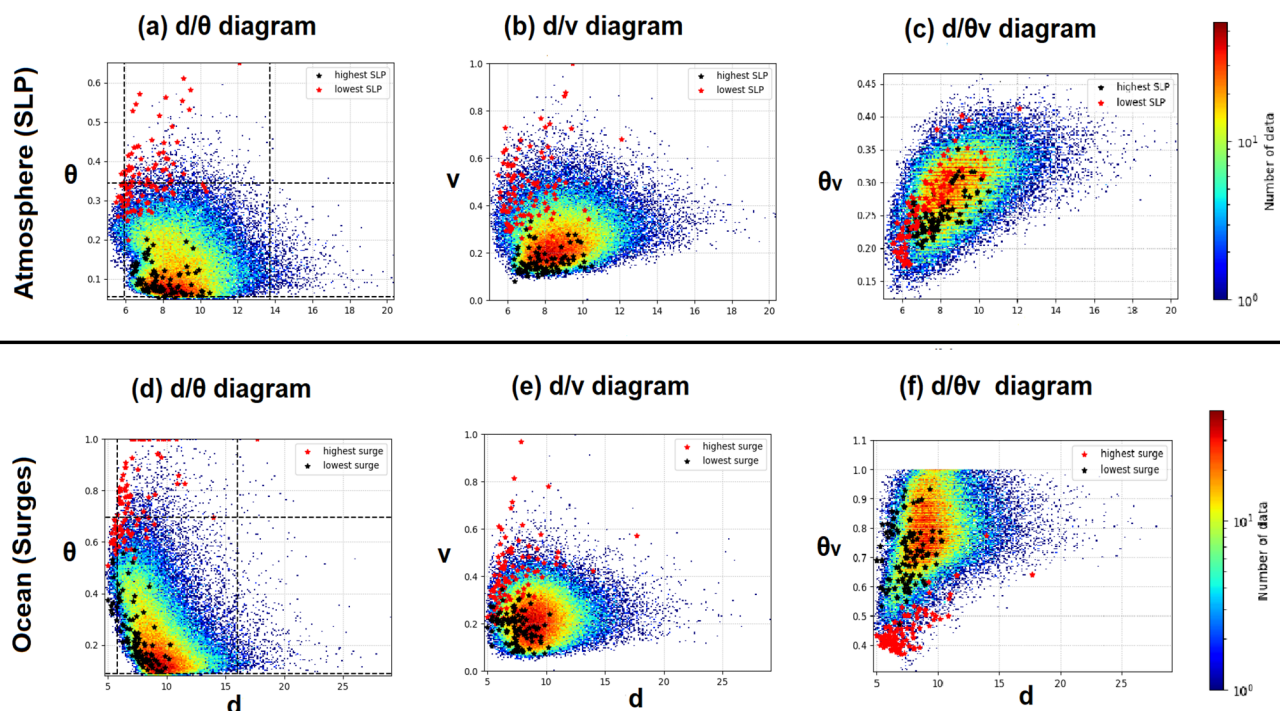


Fig. 1 Distribution of the local dimension and stability of atmospheric pressure (upper panels) and ocean surge (lower panels), for the different indicators described in the text

This metric estimates the relative change between two successive states. In the rest of this work, we will use this metric ν to investigate the stability of a state (Fig. 1b and e).

4.2 General observations

For both SLP and surge data (Fig. 1), the mean dimension ($\bar{d}_{\text{atm}} = 8.9$ and $\bar{d}_{\text{ocean}} = 9.5$) of the attractor is much smaller than the phase space dimension $n_x \times n_y$ (2601 for the atmosphere and 3876 for the ocean), indicating strong correlations between the dynamics of the different grid points, which is not surprising. The dimensions of the surge dynamics are on average higher than their atmospheric counterpart (9.5 against 8.9), although they correspond to a smaller geographic region, indicating that the surge dynamics is not a direct response to the atmospheric pressure. Indeed, the wind also plays a role. Moreover, once the surge wave is generated, its propagation is influenced by the bathymetry and coastal geometry.

4.3 Extreme events

The North Atlantic is crossed by mid-latitude extratropical winter storms, occurring mainly in winter (December-January). Extreme meteorological events are characterized by low atmospheric pressure (typically lower than 983 hPa) and strong surface winds (typically more than 25 m s^{-1} Pineau-Guillou et al. (2018)). The most extreme events of the dataset (Fig. 2) occur on December 25th, 1986 and December 15th, 2010 for low and high atmospheric pressure, respectively, and on January 26th, 1990 and March 3rd, 1987 for positive and negative oceanic surges, respectively. For the atmospheric pressure, it corresponds to large-scale patterns.

The low-pressure structure is characterised by an anomaly centered southwest of Iceland, and the high-pressure is centered west of the British Islands and the Bay of Biscay. The oceanic storm surges are more local and the strongest surges are located east of the North Sea along the coasts of Denmark and the Netherlands.

The storm surge is the sea level response to extreme meteorological events, which are caused by low SLP and strong winds. Low pressures modify the sea level through the inverse barometer effect, a fall of 1 hPa leading to a surge of 1 cm Pugh and Woodworth (2014). In addition, strong winds also play a role through the Ekman transport, pushing the water masses at 90 degrees from the wind direction. For this reason, in the Gulf of Biscay, as soon as westerlies are coming from the south-west rather than the west, the storm surges are enhanced (e.g., Xynthia storm Bertin et al. 2014; Pineau-Guillou et al. 2012). Storm surges typically reach a few tens of centimeters to 2 or 3 meters in hot spot regions such as the North Sea, where the wind stress effect is enhanced by the very shallow waters (see Fig. 2b).

In addition to stormy events, extreme events also include events with abnormally high atmospheric pressure (i.e., anticyclones) or abnormally high negative surge. These events are much less studied than storms, as they are not as destructive.

To investigate extreme events, we selected independently the 100 most extreme states for the atmospheric pressure and the surges. The criteria for selecting these events are the lowest sea level pressure (over the whole domain) for the atmosphere and the highest surge (still over the whole domain) for the surges. Extreme states, either stormy (black stars, low SLP or high surges, see Fig. 1) or calm (red stars, high SLP or negative surges), are characterized by low local

Fig. 2 Most extreme atmospheric (left) and surge (right) events. The arrows show the surface wind direction and intensity

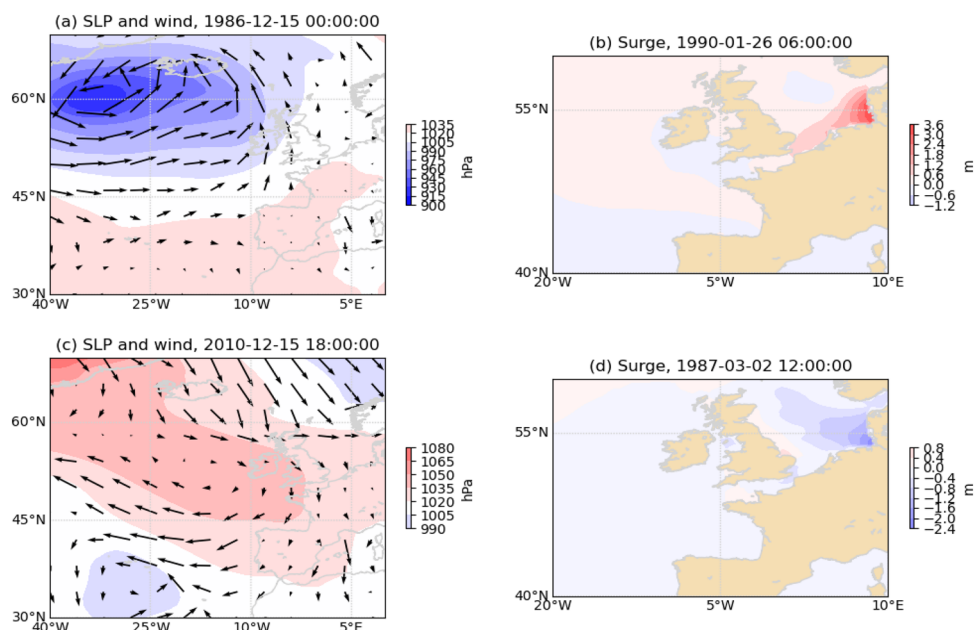
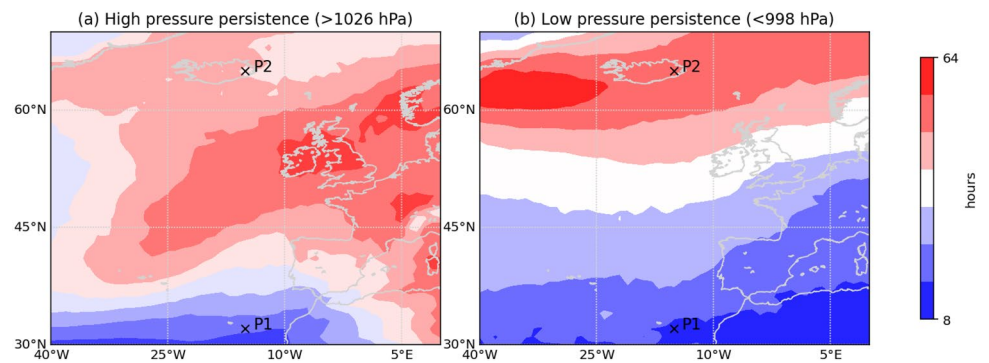
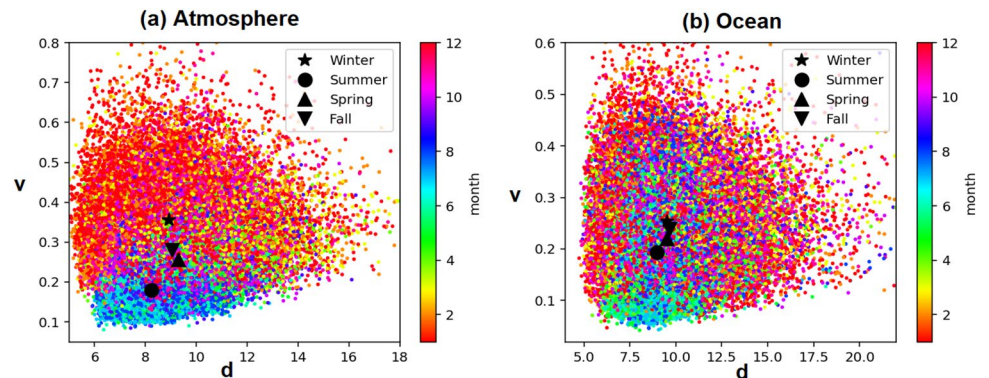


Fig. 3 Average duration of **a** low and **b** high pressures states**Fig. 4** **a** and **b** Distributions of the local dimension (d) and velocity (v) coloured as a function of the time of year (in month) for atmospheric pressure and oceanic surges, respectively. The black symbols represent the average seasonal situations

dimensions, indicating that they only form from a low set of predictors (e.g., westerlies). They however differ significantly in their phase space velocity, stormy states being much more unstable (high v) than calm ones (low v). These results are independent of the choice of metric v or θ , and we observe strong correlations between the two (Pearson coefficients $r = 0.48$ for the ocean and $r = 0.69$ for the atmosphere). To give consistency to these statements, we compute the average duration of both anticyclones and low-pressure states at different locations in the selected zone (Fig. 3). At each point of the grid, an anticyclone duration is computed as the time elapsed between the instant when the SLP becomes larger than the quantile 0.9 of the global SLP distribution over the whole zone, that is 1026 hPa, and the instant when the SLP becomes smaller than this value. We then average this duration over all observed events. The low-pressure duration is obtained in a similar fashion, considering SLP values inferior to the 0.1 quantile of the distribution at this point (998 hPa). We retained the quantiles 0.1 and 0.9 as indicative thresholds, acknowledging that they are not standard definitions of anticyclonic or stormy states but are necessary to define thresholds for our analysis. Overall, high pressures last longer on average than low pressures structures, which is consistent with the low values of the velocity v for these states.

Checking the metric dependency on the time of year (Fig. 4), we find a strong correlation with the month of the year. The summer states tend to be more persistent compared

to the winter ones. We test if this is still valid for seasonal averages by averaging over three months following: Winter (Dec-Feb), Spring (Mar-May), Summer (Jun-Aug), and Fall (Sep-Nov). We find that this dependency is also true for the seasonal mean (black triangles in Fig. 4). Summer is usually associated with stable anticyclonic states and steady surge dynamics.

4.4 Relations with storm characteristics

Storms are characterized by different patterns, such as the minimum atmospheric pressure, the maximum wind speed, the storm spatial extension, and the maximum surge, for example. Here, we relate the local dimension d and the phase space velocity v with these variables, which are commonly used to quantify storm intensities (Fig. 5). Note that the spatial extension of the storm is defined as the area delimited by the grid points whose atmospheric pressure is lower than 998 hPa (0.1 quantile of the total SLP distribution). To infer a possible relation, we have used the Pearson correlation coefficients applied to the 100 most extreme storms, which are the states with the lowest SLP and highest surge, respectively.

We first focus on the atmosphere dynamics (Fig. 5a-c). We find that the more intense stormy indicators (lowest atmospheric pressure, strongest winds, largest storms) are all located in the upper left corner of the d - v diagram (i.e., minimum dimension and largest velocity). This is not

Fig. 5 **a-c** and **d-f** Distributions of storm intensity metrics as a function of the local dimension and of the phase space velocity (v) for atmosphere pressure and oceanic surges, respectively. The storm intensity metrics are **(a)** the minimum of atmospheric pressure, **(d)** the maximum of oceanic surges, **(b)** and **(e)** the wind maximum, and **(c)** and **(f)** the spatial extension of the depression

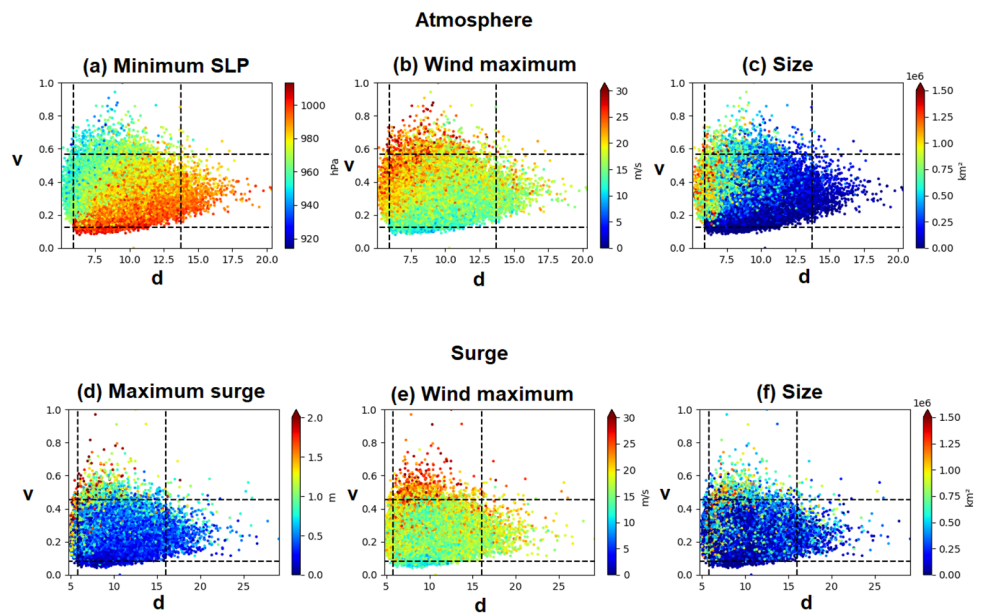


Table 2 Pearson correlations between dynamical metrics and different physical observables

	Wind max	Storm extension	Minimum SLP	Maximum surge
d atmosphere	-0.44	-0.73	0.13	-0.50
d ocean	0.52	0.22	-0.10	0.24
v atmosphere	0.28	-0.02	0.13	-0.21
v ocean	0.31	0.01	0.13	0.27

Absolute values larger than or equal to 0.5 are in bold

surprising, as we showed previously that the storms were associated with a low dimension and a high instability (i.e., high v , see the red stars in Fig. 1). On the contrary, the calmest states (high pressure and no wind) are almost all located in the lower part of the d - v diagram (small v), which is again not surprising, as high-pressure systems are associated with high persistence (see the black stars in Fig. 1). The dimension of the atmospheric system d is strongly negatively correlated with the spatial extension of the low-pressure structure ($r = -0.73$, see Tab. 2 and Fig. 5c). Indeed, the bigger the storm, the more points of the grid have a common evolution, and the dynamics is constrained to lower dimensional sets of configurations. The largest storm structures reach up to $2 \times 10^6 \text{ km}^2$. In addition, we observe a quite strong negative correlation between the dimension of the atmospheric system d and the maximum wind speed ($r = -0.44$, Fig. 5b), the latter being correlated with the amplitude and the gradient of the depression structure. Finally, the correlation is weaker with the atmospheric pressure ($r = 0.13$), even if the states with the lowest atmospheric pressure clearly occur for small dimensions. Here, the relation of the minimum atmospheric pressure seems related to a combination of the two dynamical metrics

(Fig. 5). This explains the rather low correlation found when only one of the two dynamical metrics is used (Tab. 2).

Similarly, for the surges, storm events are located in the upper part of the d - v distribution (Fig. 1d), indicating unstable dynamics (i.e., uncertainty in the future state of the system). In this case, however, the dimensions are more positively correlated with the maximum wind speed than the storm spatial extension (Tab. 2). This may be mostly due to the local influence of the wind, which decorrelates the surge dynamics at different gridpoints and overall increases the dimensions. For both atmospheric pressure and storm surges, we notice, from the d - v distribution, that intense events (low minimum pressure, high maximum surge, maximum wind, and large storm spatial extension) correspond to low local dimension and low persistence. This indicates that stormy states are associated with more constrained configurations and accelerated dynamics.

5 Evolution of the dynamical metrics around extreme events

We showed in the previous sections that extremes in SLP and storm surges are associated with low dimensions and that stormy and calm states are differentiated by large and small v respectively. This is consistent with previous studies that showed that important linear changes are related to the occurrence of extremes (regime shift) in idealized chaotic models (Evans 2004; Sévellec and Fedorov 2014). We now discuss the evolution of these metrics near such extreme events, that is, 5 days before and after the storm. To this purpose, we study the evolution of a mean storm situation (average of the 100 largest states with lower SLP or higher

Fig. 6 Evolution of d and v for the atmospheric dynamics for extreme events. In the left panel (a), the large black triangles represent the average seasonal situations and the curves represent typical excursions for a storm event (upper curve) and for a high-pressure event (lower curve). These typical evolutions are built using 100 lower and higher SLP, respectively. The small red and black stars represent the 100 states with the lowest and highest SLP respectively. The right panels represent the evolution of individual metrics d and v for stormy events (b and c) and for calm events (d and e)

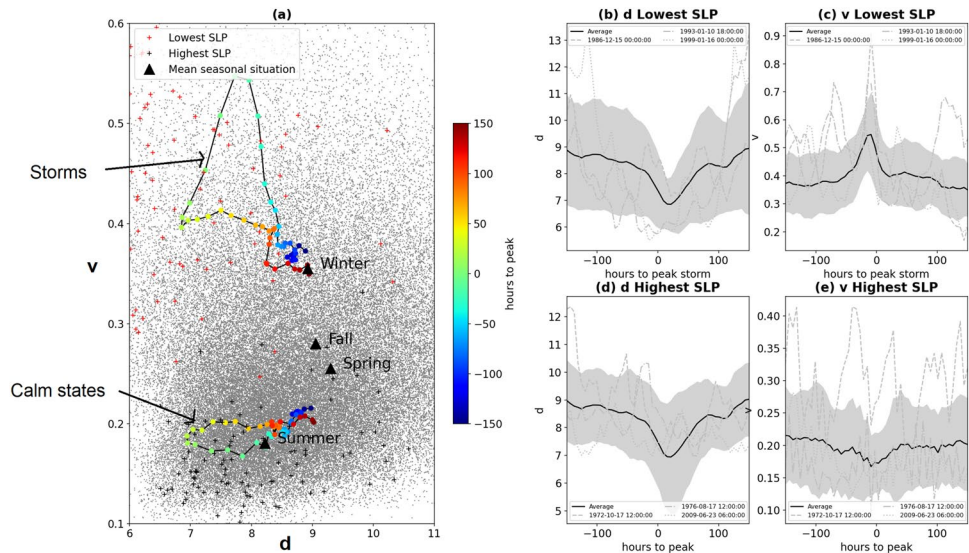
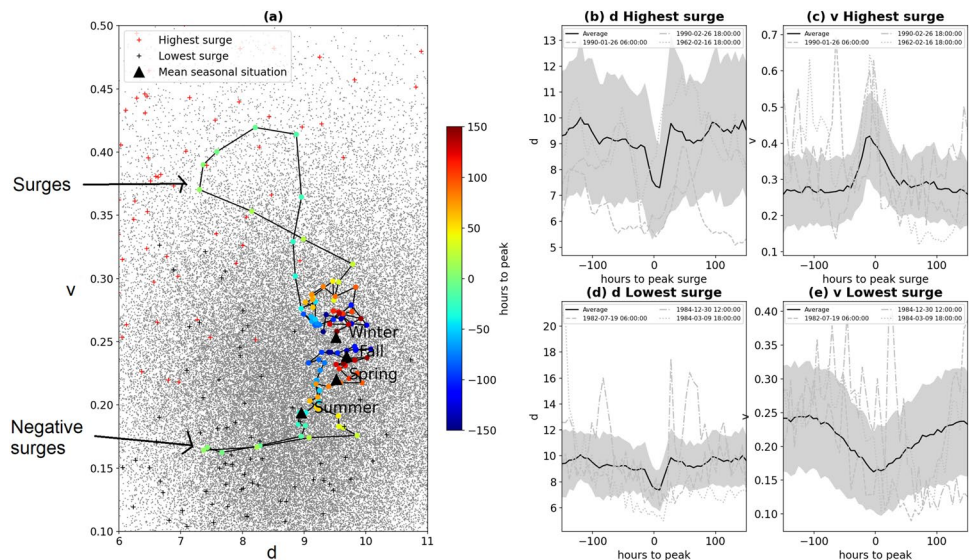


Fig. 7 Evolution of d and v for the sea level surge dynamics for extreme events. In the left panel, the black triangles represent the average seasonal situations and the curves represent typical excursions for a large surge event (upper curve) and for a large negative surge event (lower curve). These typical evolutions are built using 100 higher and lower surges, respectively. The small red and black stars represent the 100 states with the highest and lowest surges respectively. The right panels represent the evolution of individual metrics d and v at large surge events (b and c) and for negative surge events (d and e)



surges) in the dynamical metric d - v space within a time window of 5 days (120 hours) centered at the peak of the event (upper curve on Fig. 6a for SLP and 7a for storm surge). These storms typically start from a standard winter situation (black triangle named “winter” in Fig. 6a), then undergo a sharp increase in v , indicating a strong acceleration of the dynamics (Fig. 6a). This phase is followed by a decrease of both the local dimensions and v around the peak of the storm. After the peak storm, the energy has been released, and the situation slowly goes back to a more normal state, towards an average value of d . (Fig. 6a).

On the other hand, the evolution of the high SLP states and negative surges (calm situations) affects mostly the dimension (and not v), which decreases until the peak of the event and then returns to a more typical summer situation. v remains rather constant, although we observe a small

decrease in phase-space velocity, more noticeably for the surge system (Fig. 6a vs 7a).

To further investigate the dynamical metric change we diagnose their evolution for the average as well as for the 3 most extreme events (Figs. 6b-e and 7b-e). The averaged values show a sharp decrease of the local dimensions a few hours before the peak of the event. For highest SLP and lowest surge events, v decreases over a longer period, up to 4 days before and after the peak, while for low pressures and large surges, the acceleration of the dynamics is more intense and occurs over a shorter period, only 50 hours before the peak of the event. This can be because calm situations are usually more persistent and less turbulent (i.e., laminar flow are less inductive to fast changes), as discussed in the previous section. Individual events (in dotted line) show strong variations in both d and v , as the large values of the standard deviation also suggest, rendering the use of

these metrics as predictors for large storms challenging. We note a similar evolution for the local dimension at the peak of different weather regimes (see Fig. 4 in Hochman et al. (2021)) and at the transition between these regimes (Platzer et al. 2023).

6 Conclusion and perspectives

We have characterised the dynamics of aspects of the atmospheric and oceanic systems with a focus on extreme events. We applied tools taking their roots in the dynamical system theory to the atmospheric pressure and the storm surges in the North-East Atlantic, over the period 1960–2010 using 6-hour data on a 1° -grid for the atmosphere, and 0.2° -grid for the ocean.

Concerning the method, we showed that computing the inverse persistence index θ with a fixed radius rather than with a variable radius θ_v , as done in other publications (Faranda et al. 2017; Hochman et al. 2021), assigns low persistence to stormy and unstable states and high persistence to calm states. In addition, our novel indicator v , which avoids the use of any radius, provides an intuitive, easy-to-compute metric of stability of a state.

In terms of physics, we found that the dimensions d of the attractors of both atmospheric pressure and oceanic surge dynamics are much smaller than that of their ambient phase space (on average, d is around 10, while $n = 2601$ and 3676 respectively) and get even smaller at extreme events. In addition, extremes are found to be associated with lower dimensions d , and storms are found to be more unstable (higher v) than calm states (lower v). We looked at the evolution of these metrics around extreme events and found significant changes in the dynamical properties of both atmospheric and oceanic systems leading up to these events.

Although the method to compute d is known to underestimate the local dimensions (Pons et al. 2023), the small

values obtained here (around 10) indicate strong correlations between the dynamics at different grid points, which can be caused by the displacement of large-scale structures like storms and surge waves but could also emerge from geographic zones with coherent dynamics.

In perspective, such homogeneous regions can be determined with clustering algorithms (e.g. k-means Wilks 2011). In practice, choosing a number of clusters in these methods can be challenging. The average value of the local dimensions, which describes the number of variables needed to describe the dynamics, constitutes a good candidate. They could also inform on the number of modes needed to decompose the dynamics with different methods (e.g., EOFs, harmonic analysis, dynamic mode decomposition Mezic 2013).

As a lower dimension is interpreted as fewer directions in which a system can evolve to and from a particular state, local dimensions are often associated with predictability. Different publications explore the link between local dimensions and various notions of short-term predictability of the system (Dong et al. 2024; Faranda et al. 2017; Hochman et al. 2022). Such relations are important, in particular in the context of prediction by analogues and require more understanding.

The evolution of these metrics (local dimension d and persistence index) with climate change was investigated in Faranda et al. (2019) for the case of atmospheric circulation and in the context of a warming ocean. A similar analysis could be performed for extreme surge events to assess their predictability and temporal scale.

Codes for v and θ

We provide a Python code for the normalised phase space velocity $v(z)$ for a data array X for which the column i is the data vector at time $i = 0, \dots, n$:

```

import numpy as np

def v(X):
    n=np.shape(X)[1]
    vv=np.zeros(n)
    for i in range(n-1):
        vv[i]=np.linalg.norm(X[i+1,:]-X[i,:])
    mm=np.max(vv)
    vv=vv/mm
    return vv

```

The following code is for the computation of θ for a fixed threshold u for the same data array X at time $t = 0, \dots, n$:

```

import numpy as np
from fastdist import fastdist

def extremal_index(X,t,u):
    dist= fastdist.vector_to_matrix_distance(X[t,:],X,
    fastdist.euclidean,'euclidean')
    dist=np.delete(dist,t)
    dist=np.log(dist)
    N = np.sum(dist > u)
    nn = 0

    for i in range(len(dist) - 1):
        if dist[i] > u and dist[i + 1] > u:
            nn += 1
    theta = 1- nn/N
    return theta

```

Example of a program computing v and θ for prmsl data downloaded from 20CR:

```

import numpy as np
import xarray as xr
from fastdist import fastdist

#Data preparation

cr = xr.open_mfdataset('/Data_20CR_ATLN/prmsl/*.nc')
nt=cr.sizes['time']
crstacked=cr.to_stacked_array('gridpoint',sample_dims=['time'])
crstacked=cr.stack(gridpoint=('lat','lon'))
Xcr=crstacked.prmsl.values

#Computation of phase-space velocity

v=v(Xcr)

#Computation of theta with fixed quantile u=0.1

u=-10
theta=np.zeros(nt)
for t in range(nt):
    theta[t]=extremal_index(Xcr,t,u)

```

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Data availability The 20CR (Twentieth Century Reanalysis) Version 3 atmospheric data (Slivinski et al., 2019) are available on the 20CR website https://psl.noaa.gov/data/20thC_Rean/. Support for the Twentieth Century Reanalysis Project version 3 data set is provided by the U.S. Department of Energy, Office of Science Biological and Environmental Research (BER), by the National Oceanic and Atmospheric Administration Climate Program Office, and by the NOAA Physical Sciences Laboratory. The ClimEx 1900–2015 sea level hindcast is available on SEANOE (Sea Scientific Open Data Publication) website <https://doi.org/10.12770/da6a6b5c-586f-4a70-be43-863182fb10ad>.

Declarations

Conflict of interest We declare no competing interests.

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