Optimal surface salinity perturbations influencing the ocean circulation

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Alain Colin de Verdière, Thierry Huck et Mahdi Ben Jelloul

NEMOVAR / LEFE - Mars 2007

Climate context (1)

- Climate study → ocean, atmosphere...
- Low frequency Variation → ocean
- Ocean has different variability scale (tide, eddies...)
  low dynamic → ThermoHaline Circulation (THC)

⇒ Variability and stability of the ocean circulation

- Endogenous variability

  → internal process sustains the variability
  (always non-linear)

- Exogenous variability

  → the perturbation sustains the variability
  (could be linear)

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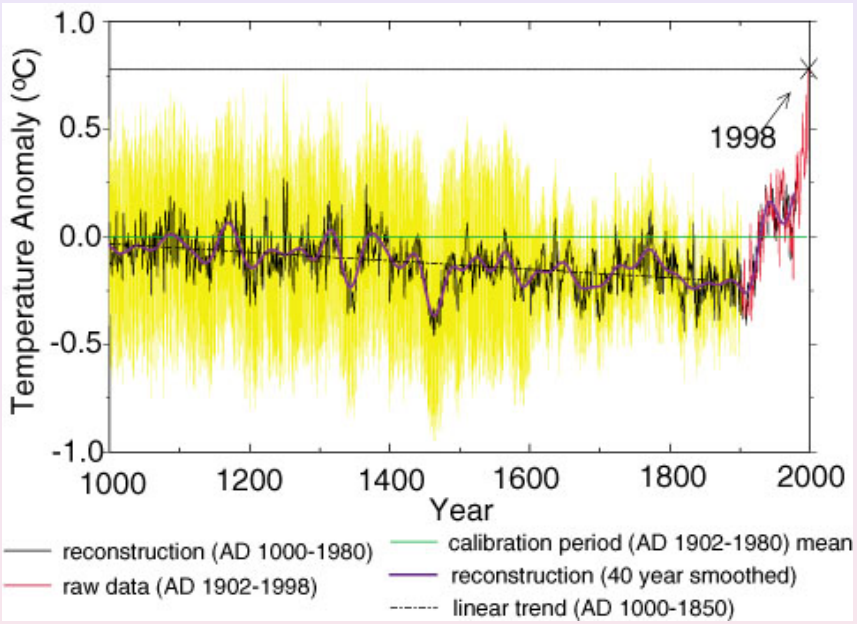

- Strong modification of the global mean temperature during the past century.
- Increase arises after a millennial of a decrease

$\longrightarrow$ Josey and Marsh (2005):

- Increase of the precipitations in the north Atlantic
- SSS modification during the last three decades

$\Rightarrow$ SSS impact on the oceanic circulation

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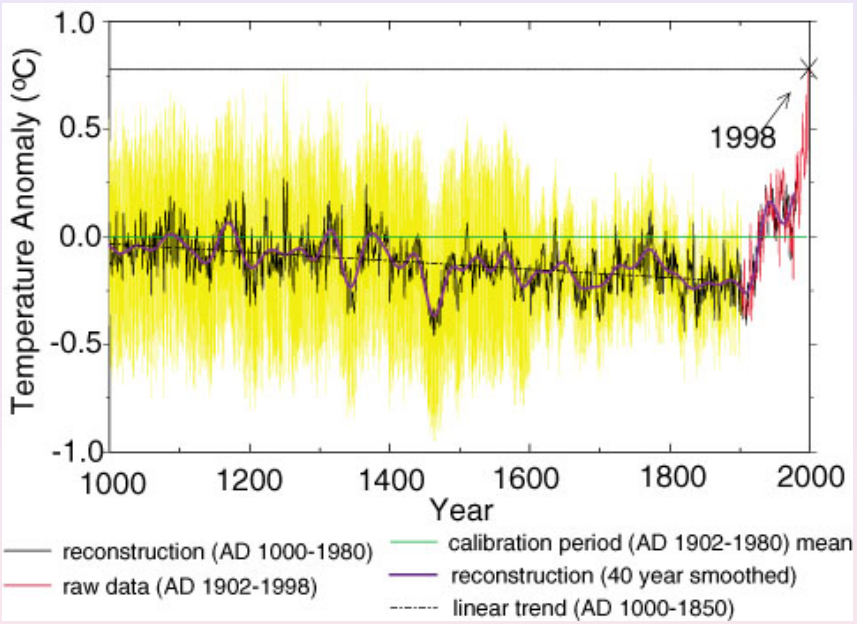

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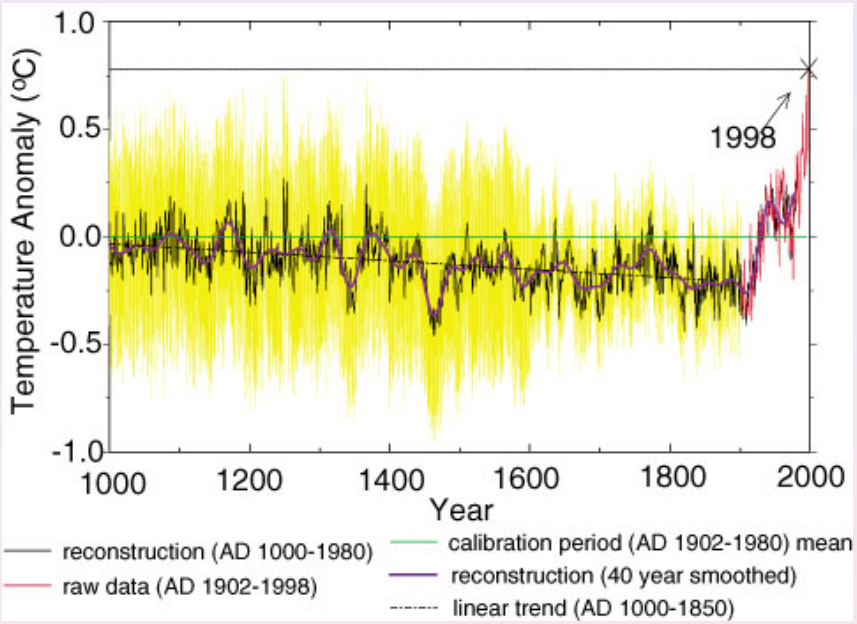

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- Linear approach:
  weak variations (perturbations) of the ocean circulation
- Optimal perturbation:
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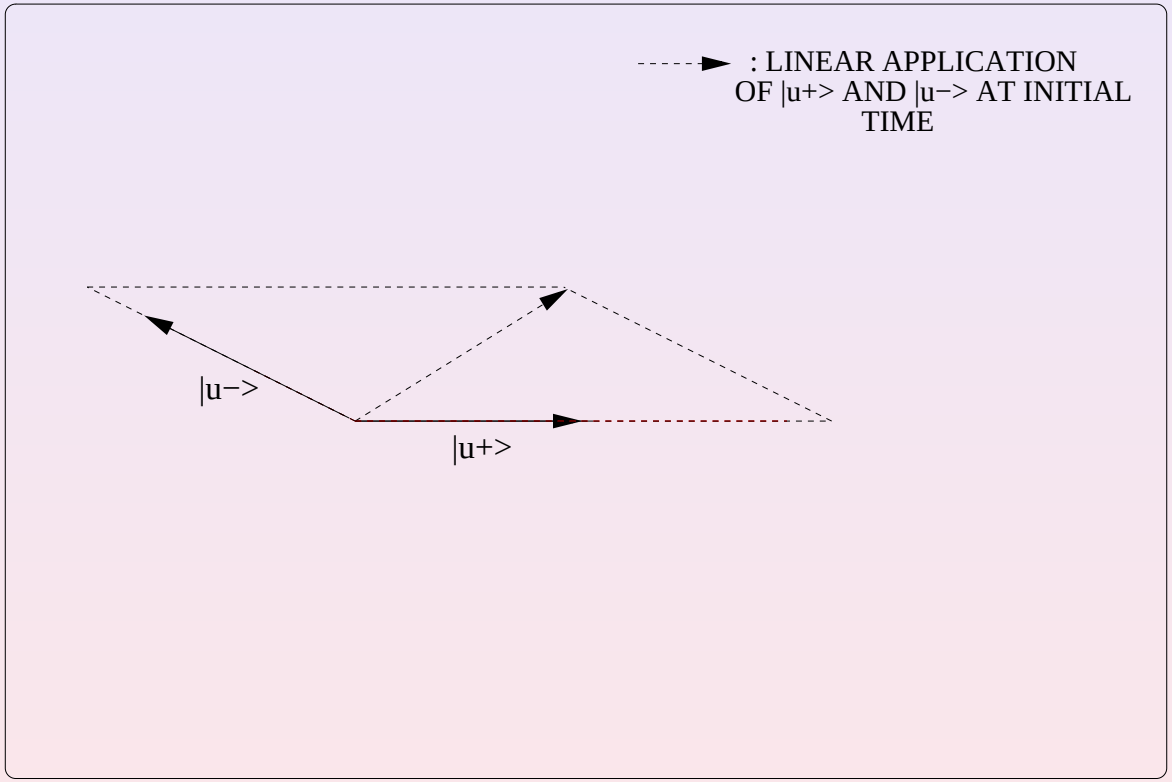
$\Rightarrow$ Finite time growth

Non-orthogonality : finite time growth

Finite time growth due to non-normal eigenvectors in a two dimensional space.



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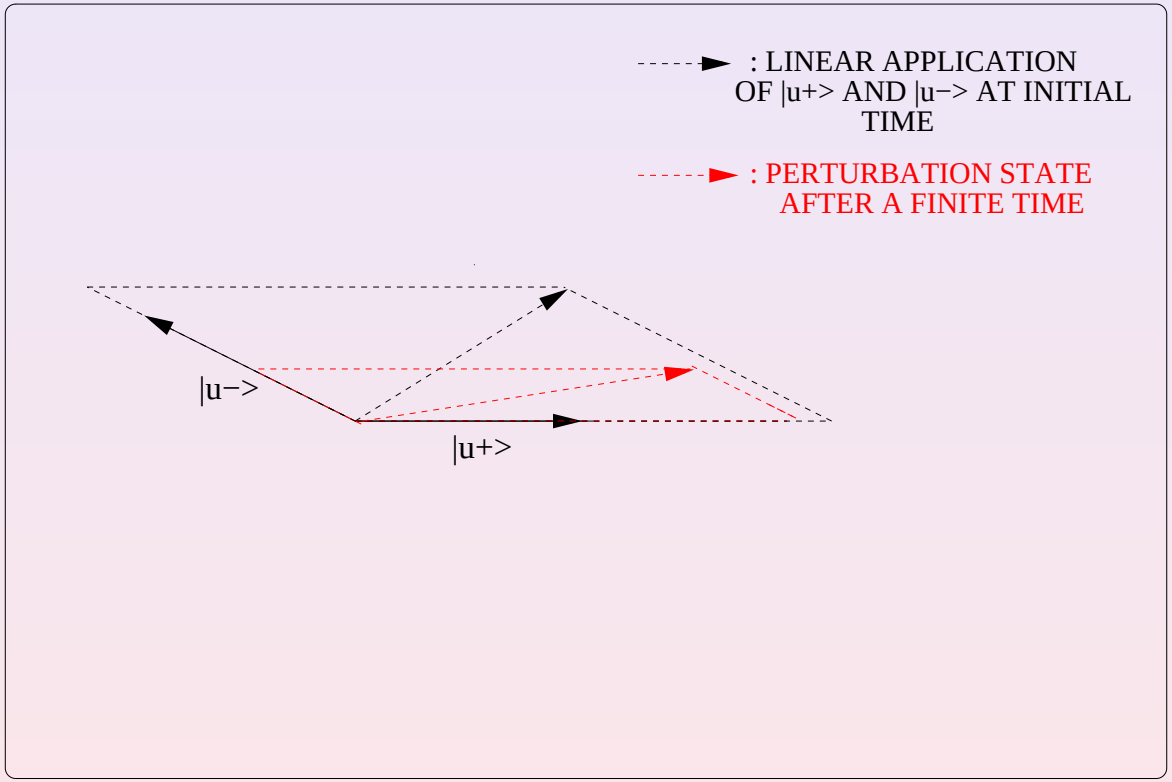

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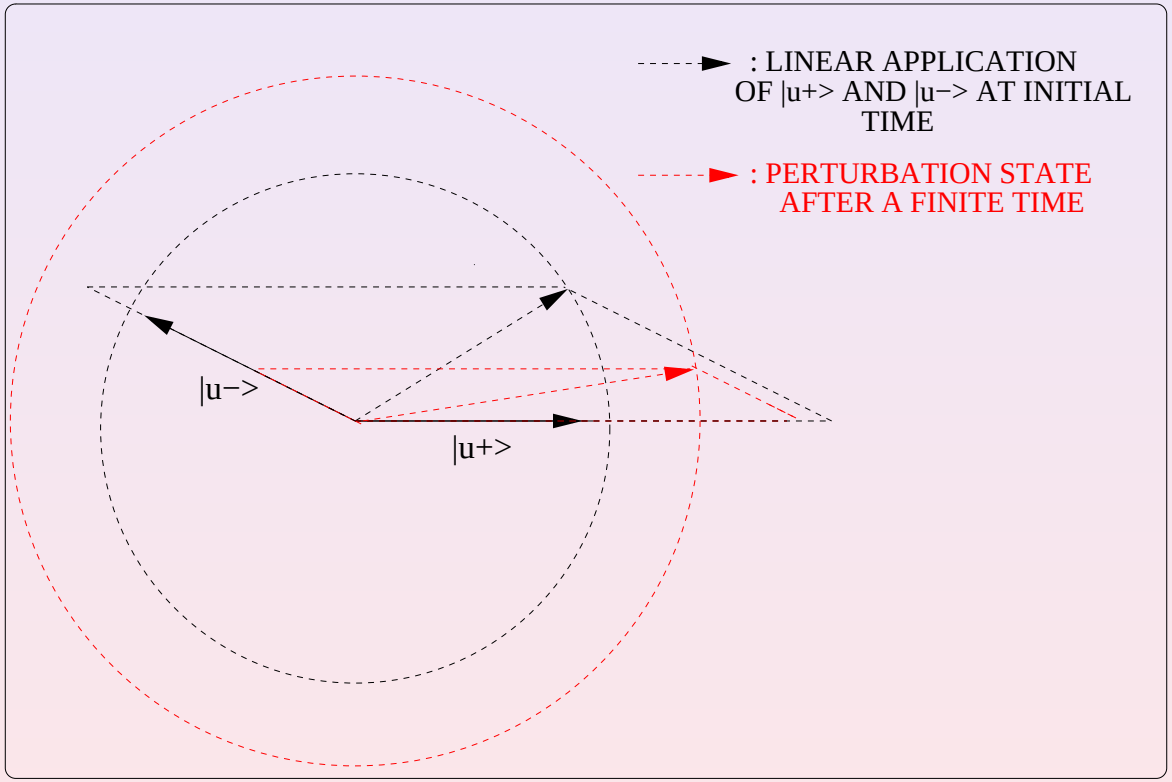

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  - Biorthogonality and maximisation method
  - Model configuration
- Optimal perturbation of the MOC:
  - Optimal SSS
  - Growth mechanism
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Eigenvectors and biorthogonality

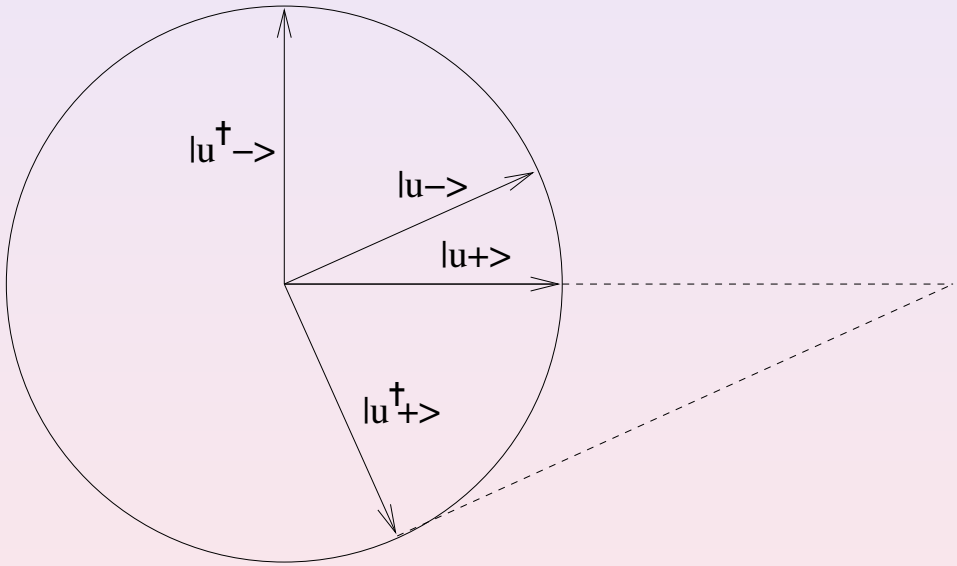
$$\mathbf{A}\,|u_i\rangle = \lambda_i\,|u_i\rangle \text{ and } \mathbf{A}^\dagger\,|u_i^\dagger\rangle = \lambda_i^*\,|u_i^\dagger\rangle$$

Def:

- $|u_i^\dagger\rangle$ is the biorthogonal of $|u_i\rangle$

Properties:

- $|u_i^\dagger\rangle$ biggest contravariant projection on $|u_i\rangle$
- $\langle u_i|u_j^\dagger\rangle = \delta_{ij}$



$$\mathbf{A} = \sum_i |u_i\rangle\,\lambda_i\,\langle u_i^\dagger| \text{ et } \mathbf{A}^\dagger = \sum_i |u_i^\dagger\rangle\,\lambda_i^\dagger\,\langle u_i|$$

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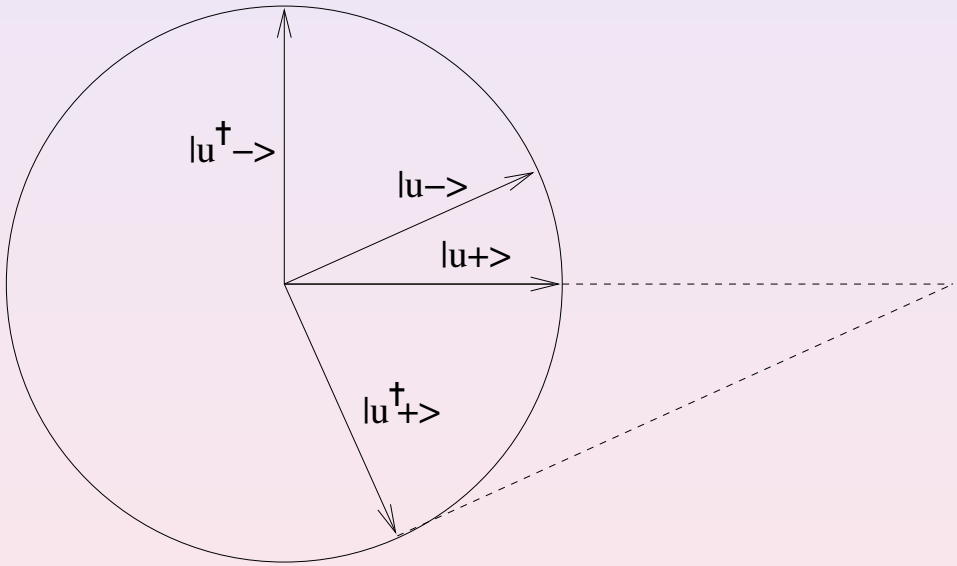

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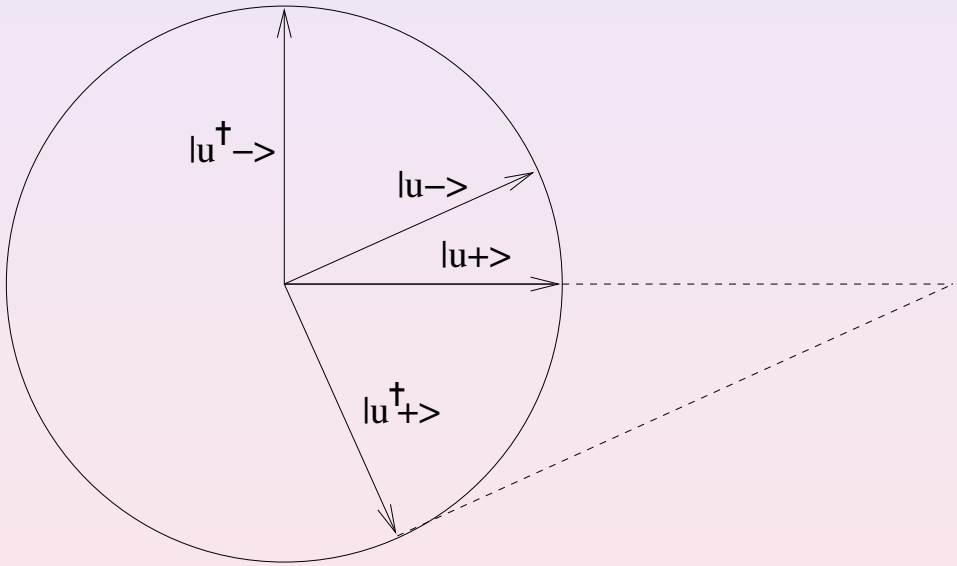

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Maximisation method (Lagrange parameters)

u the perturbation

$\mathbf{F}(u)$ the function to maximize

$\mathbf{C_i}(u) = 0$ les contraintes

$\Rightarrow \mathbf{G}(u, \gamma_i) = \mathbf{F}(u) - \sum_i \gamma_i \mathbf{C_i}(u)$ (γ_i Lagrange parameter)

the optimum u maximize $\mathbf{F}$ under the $\mathbf{C_i}$ constraints if

$d\mathbf{G}(u, \gamma_i) = 0 \Rightarrow d\mathbf{F}(u) - \sum_i \gamma_i d\mathbf{C_i}(u) = 0$

Oceanic circulation application

- Functions to maximize ($|F\rangle$):
  - Meridional Overturning Circulation (MOC)
  - Meridional Heat Transport (MHT)
- Constraints
  1. Norm: $\langle u(0)|\mathbf{S}|u(0)\rangle = 1$
  2. Salt conservation: $\langle C|u(0)\rangle = 0$
  3. Only surface salinity perturbation: $|u(0)\rangle = |u'\rangle$

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Computation

Perturbation evolution:

$$\partial_t |u\rangle = \mathbf{A} |u\rangle$$

$$\Rightarrow |u(\tau)\rangle = \mathbf{M}(\tau) |u(0)\rangle = \sum_i |u_i\rangle e^{\lambda_i \tau} \langle u_i^\dagger | u(0)\rangle .$$

Explicit solution of the optimal initial perturbation:

$$\Rightarrow |u(0)\rangle = \mathbf{P} |u'\rangle$$

$$|u'\rangle = \frac{\mathbf{N}^{-1}\mathbf{P}^\dagger \mathbf{M}^\dagger(\tau) |F\rangle - \gamma_2 \mathbf{N}^{-1}\mathbf{P}^\dagger |C\rangle}{2\gamma_1}, \text{ with } \mathbf{N} = \mathbf{P}^\dagger \mathbf{S} \mathbf{P},$$

$$\gamma_1 = \text{fct}\left(\mathbf{M}(\tau), |F\rangle, |C\rangle, \mathbf{N}, \mathbf{P}, \gamma_2\right) \text{ and}$$

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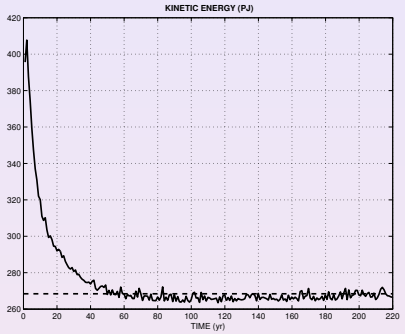
$\Rightarrow$ Solution function of the maximisation delay τ

OPA ORCA2



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- Initial condition: Levitus climatology
- 200 yr run without seasonal cycle
- Rigid lid
- Surface restoring term in Red and Mediterranean sea (T, S)

Reaches a steady state

$\Rightarrow$ Variability of 4 yr period in the equatorial Pacific

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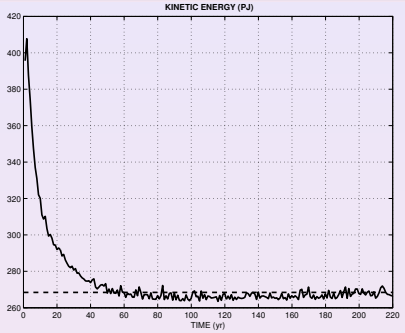

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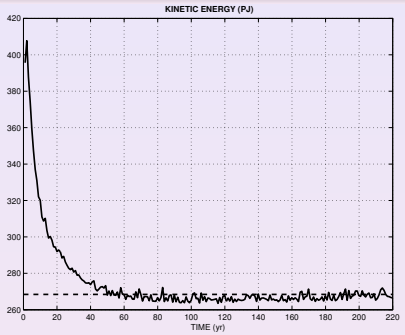

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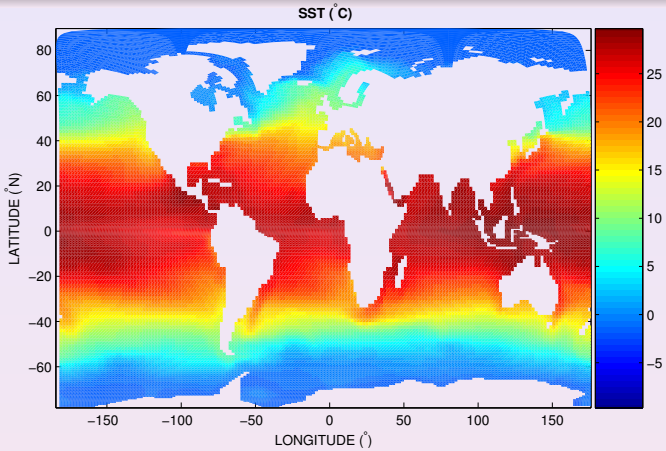

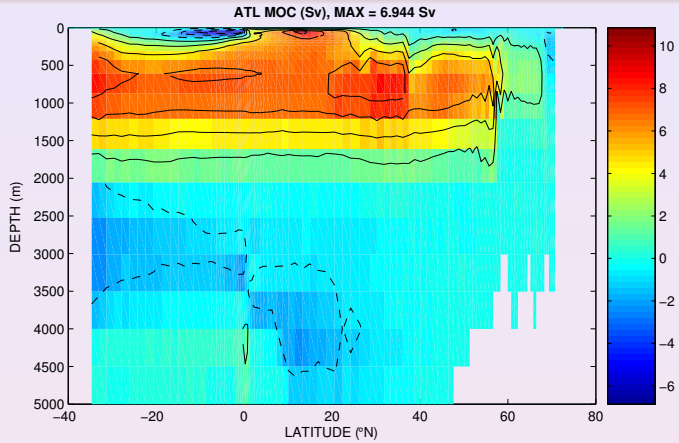

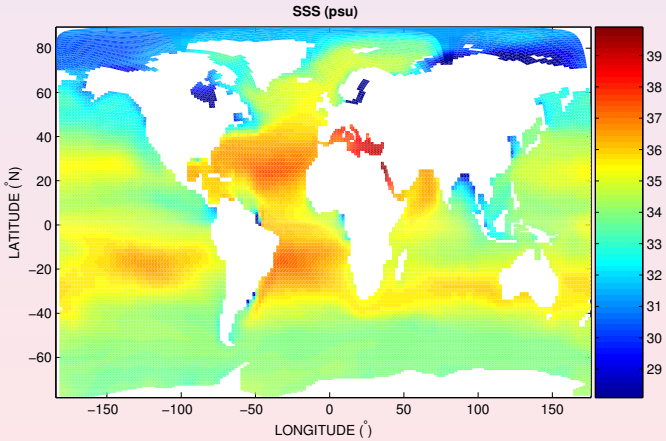

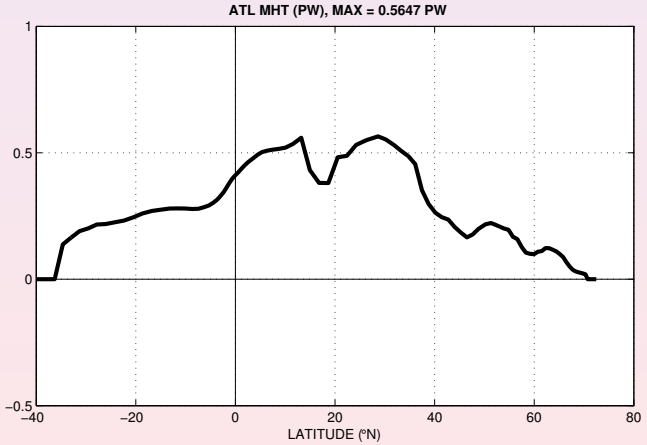

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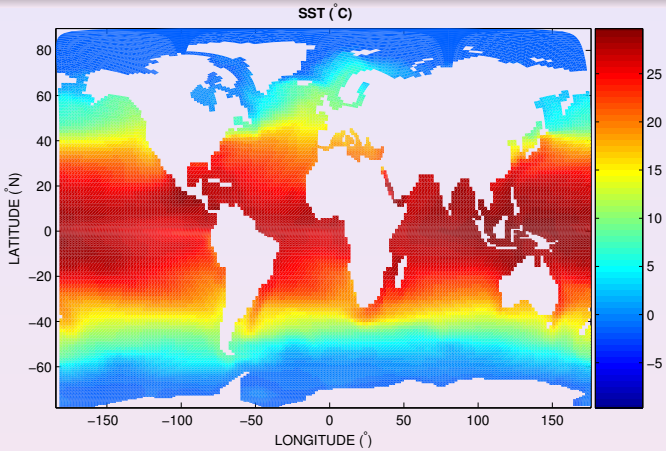
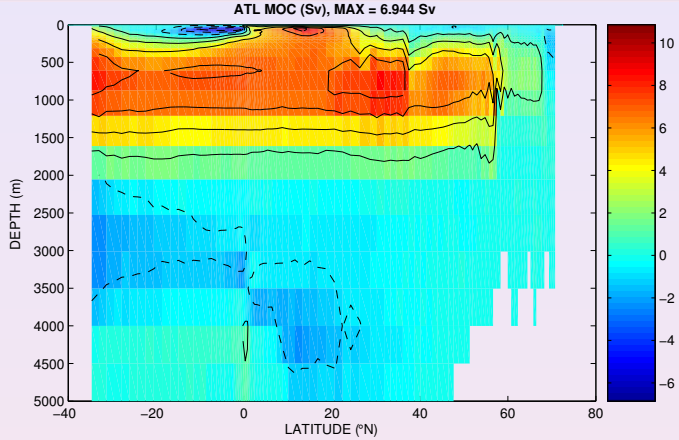
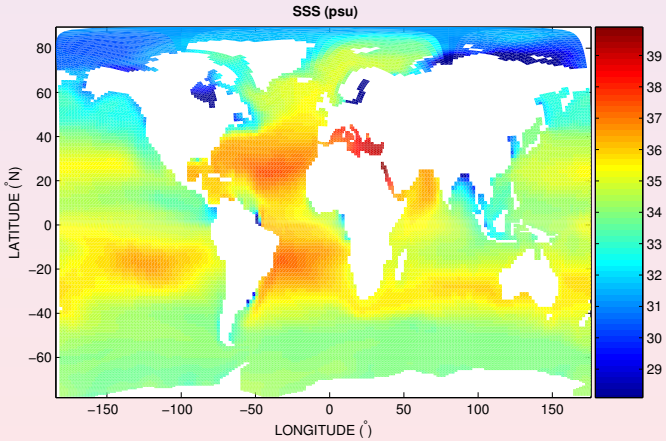
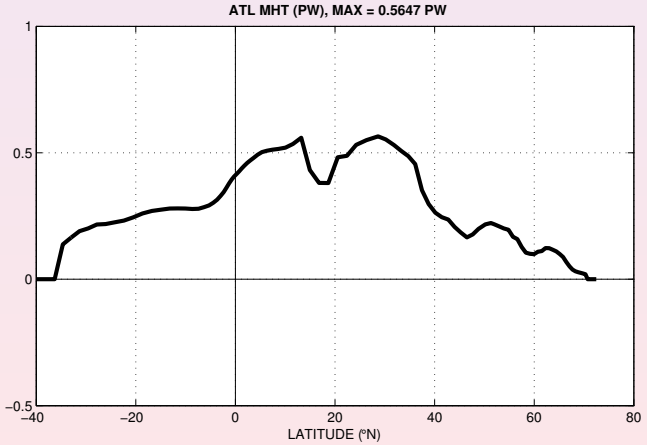
- MAX(MHT)=0.6 PW (24°N)

OPA ORCA2: steady state









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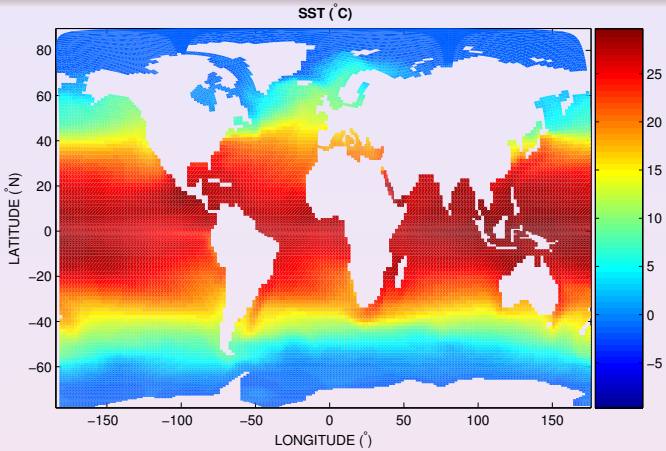

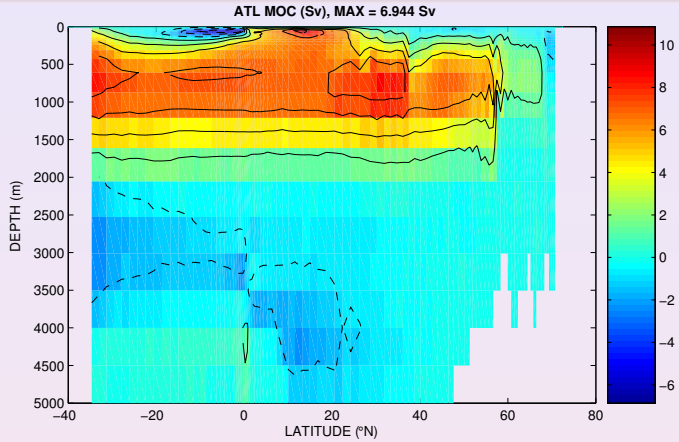

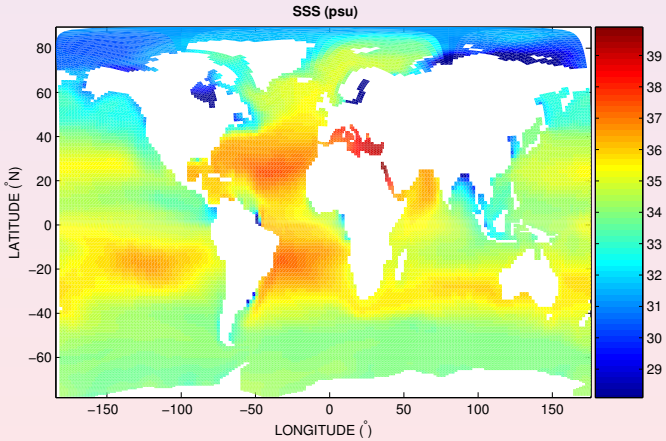

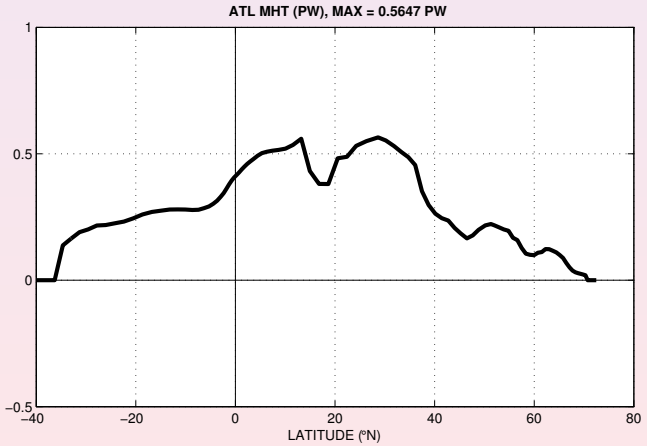

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OPATAM: linear and adjoint model

Modification for linear and adjoint run:

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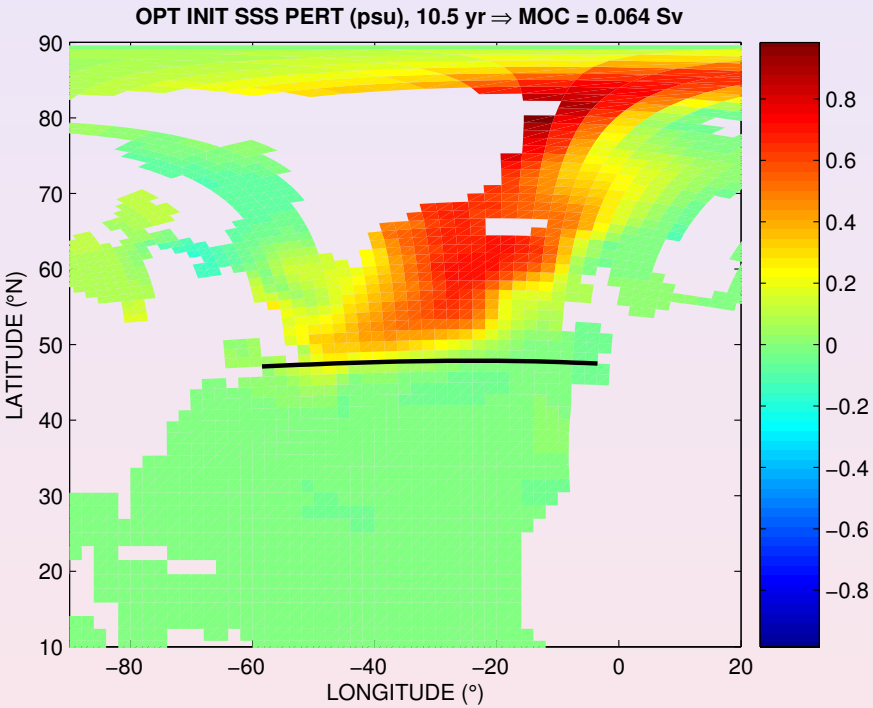

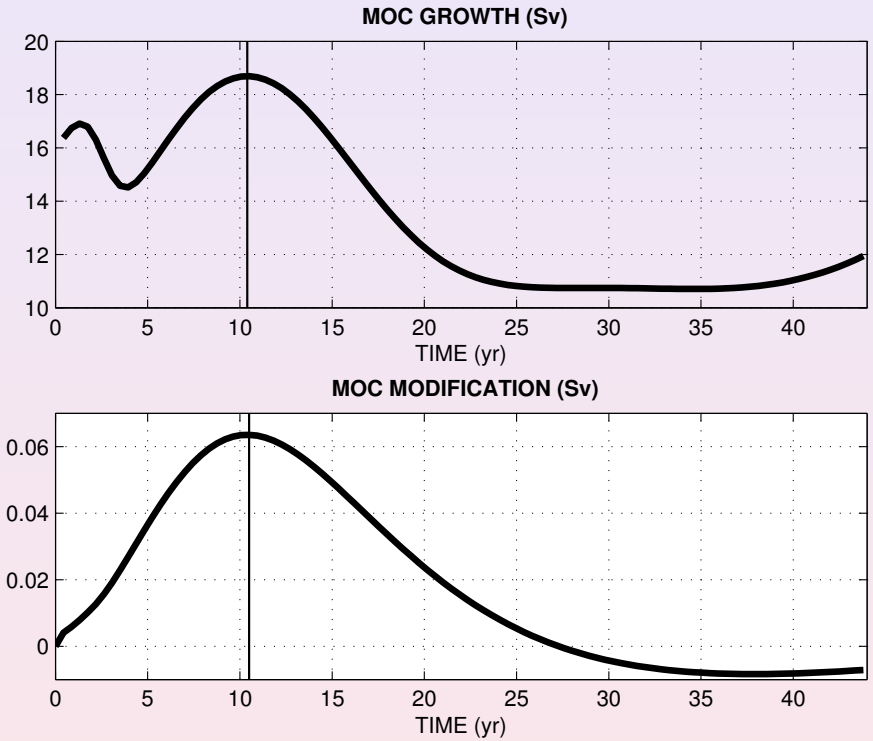

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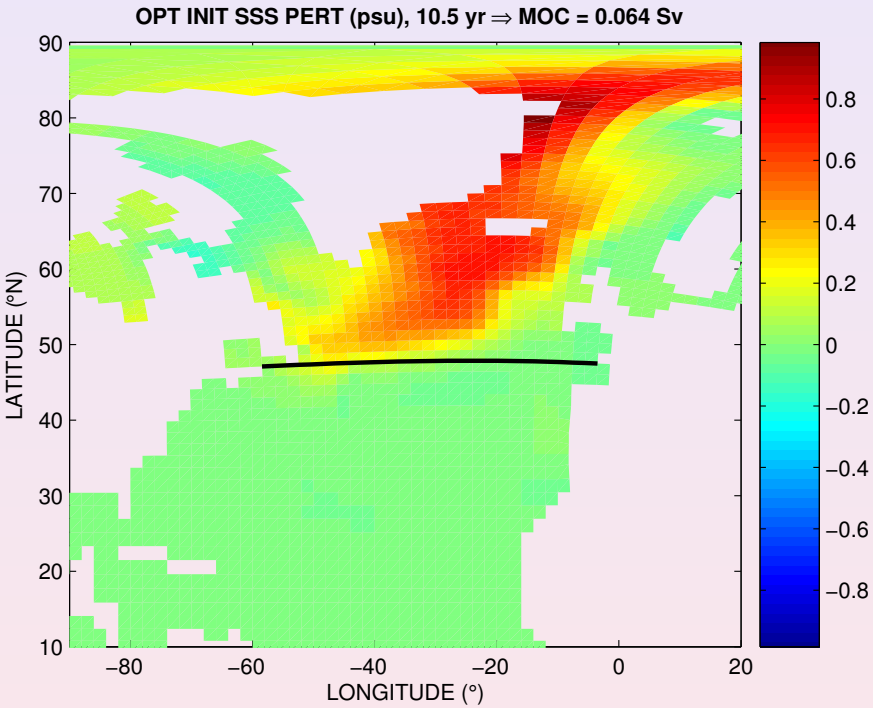

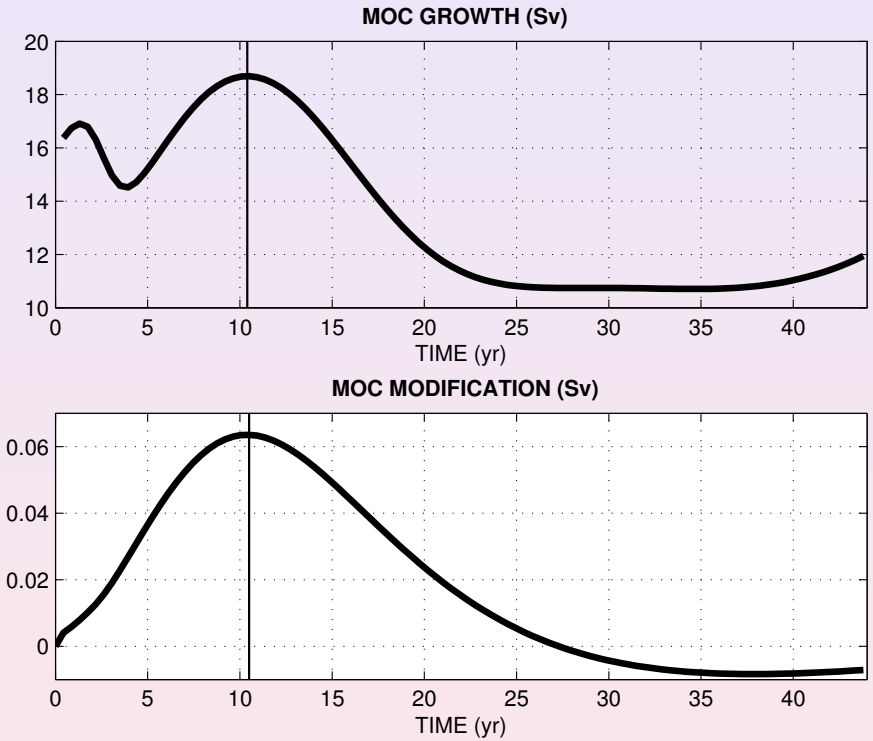

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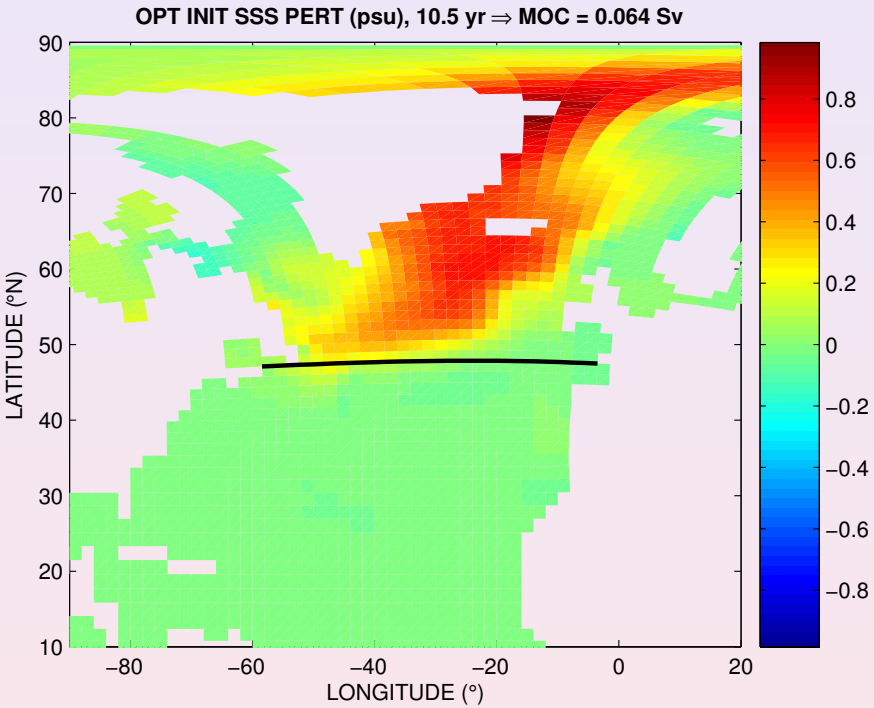

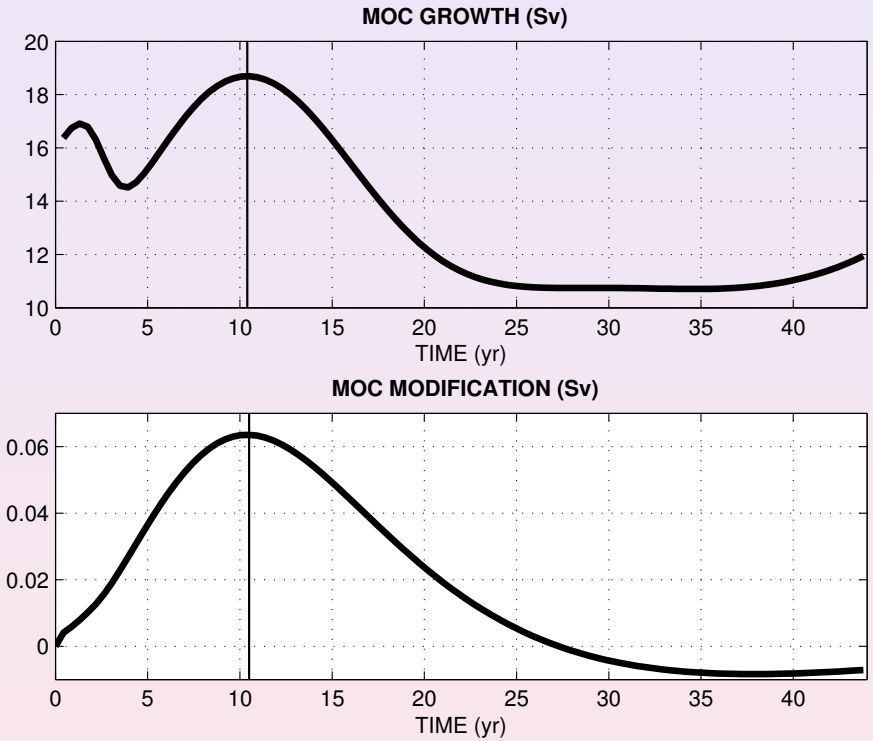

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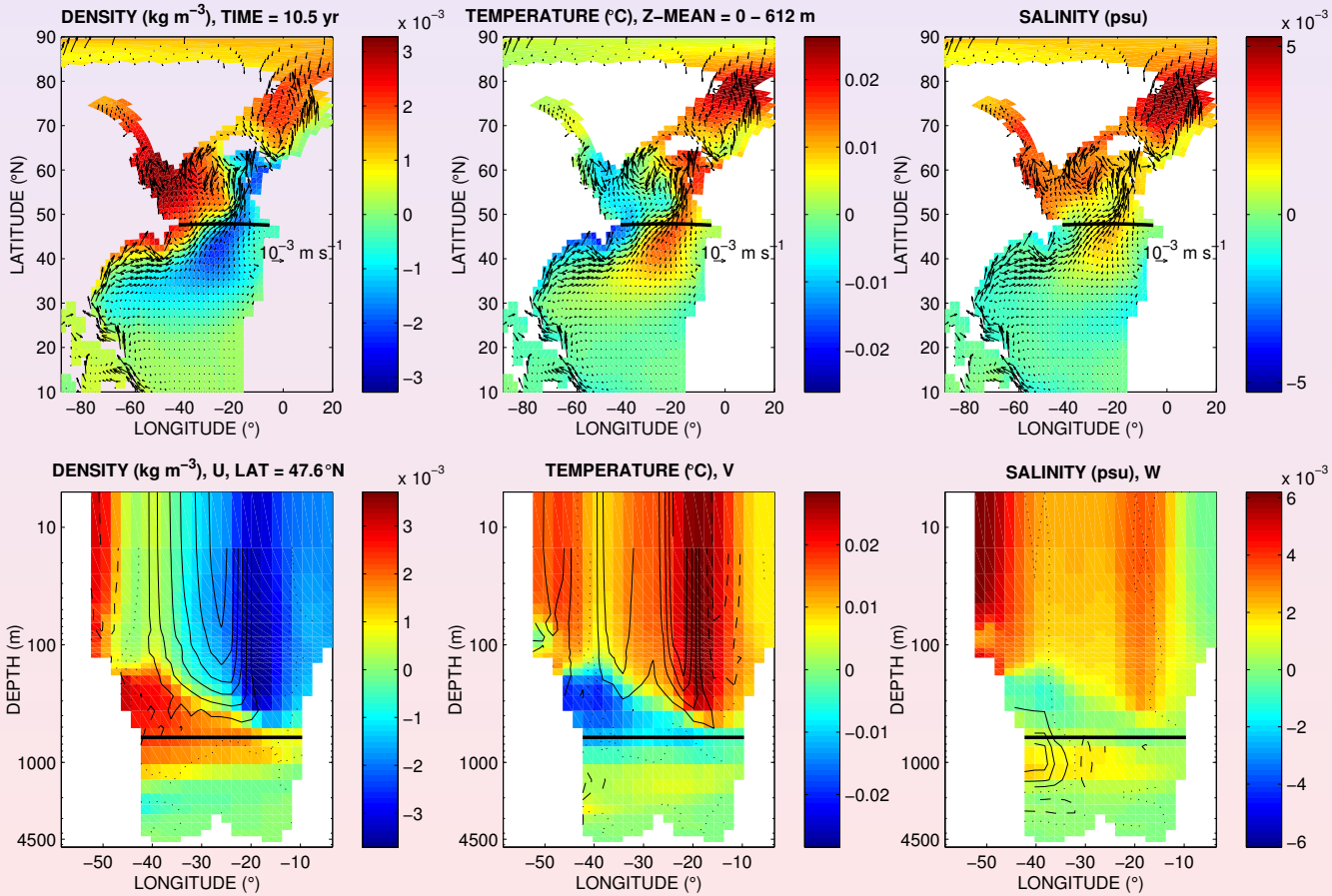

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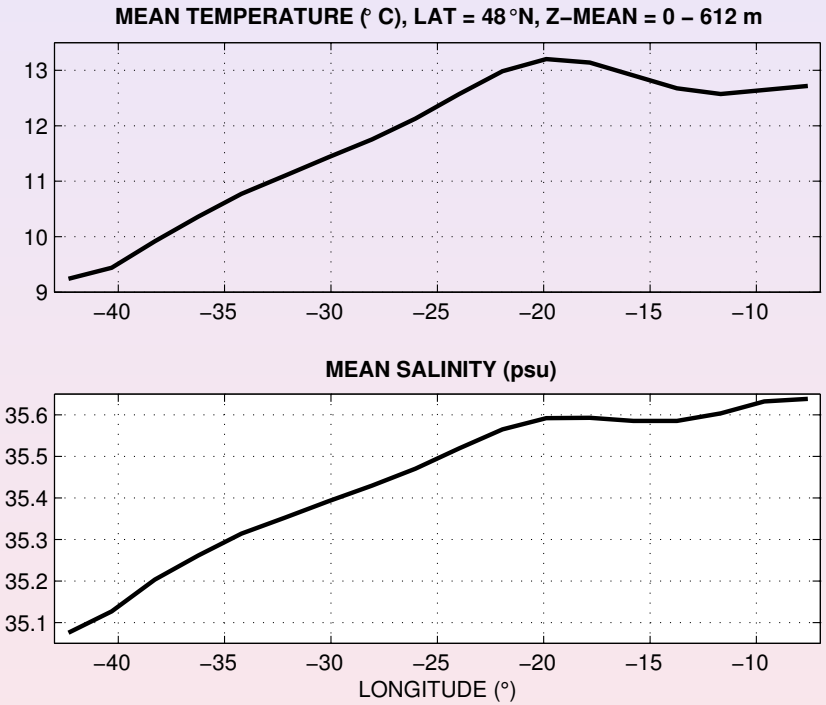
Finite time growth mechanism



$\alpha \partial_\phi \bar{T} \gg \beta \partial_\phi \bar{S}$

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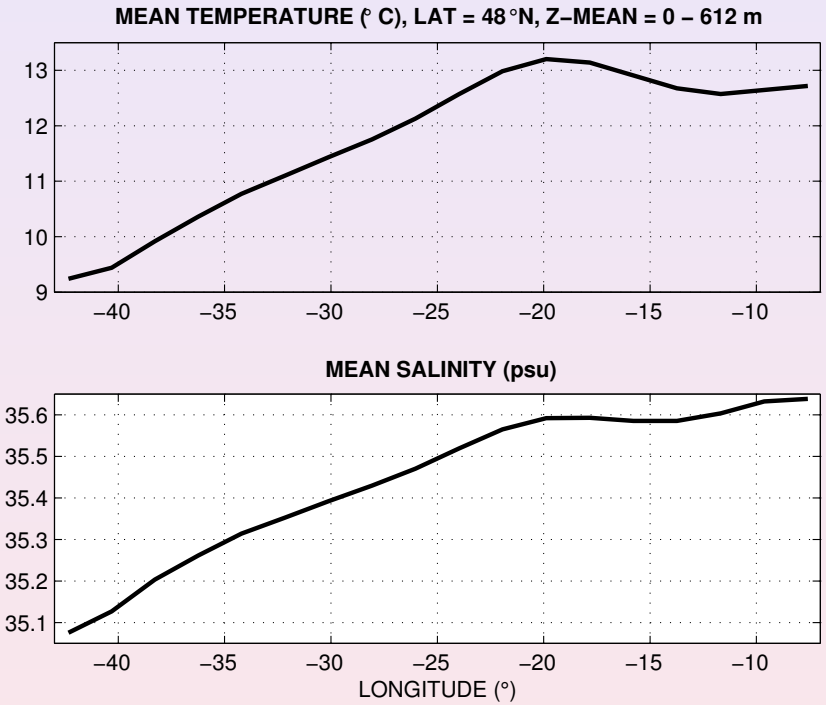

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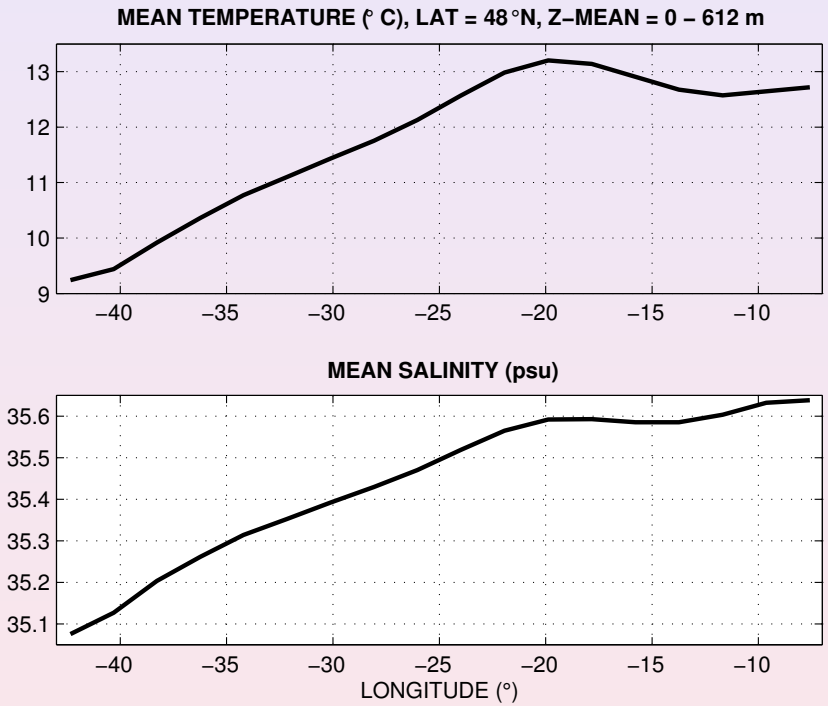

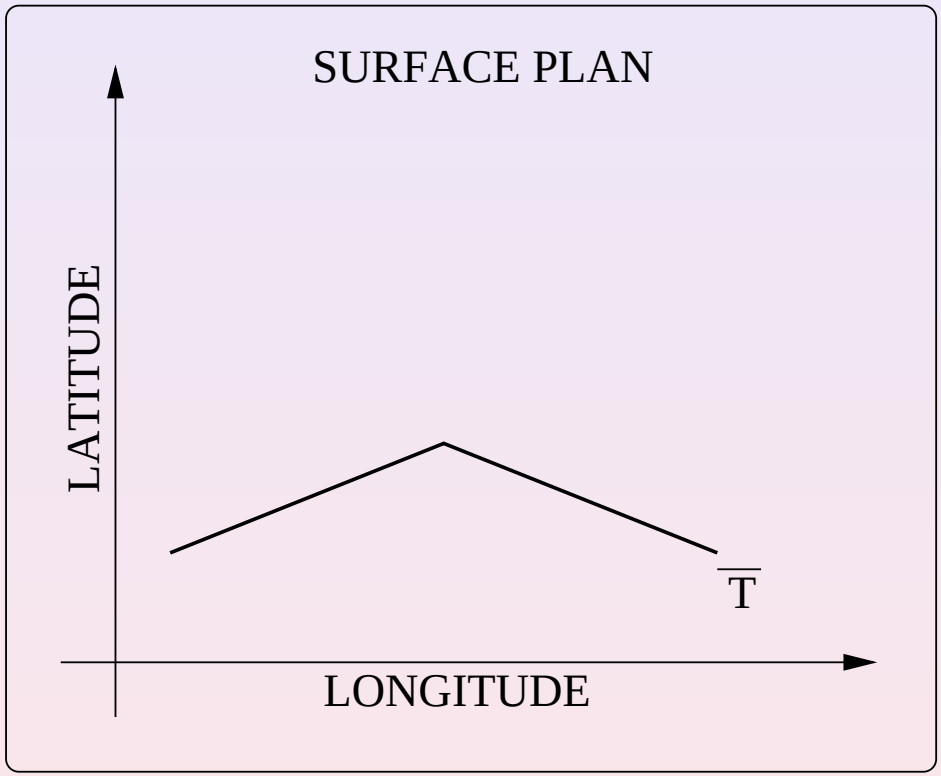

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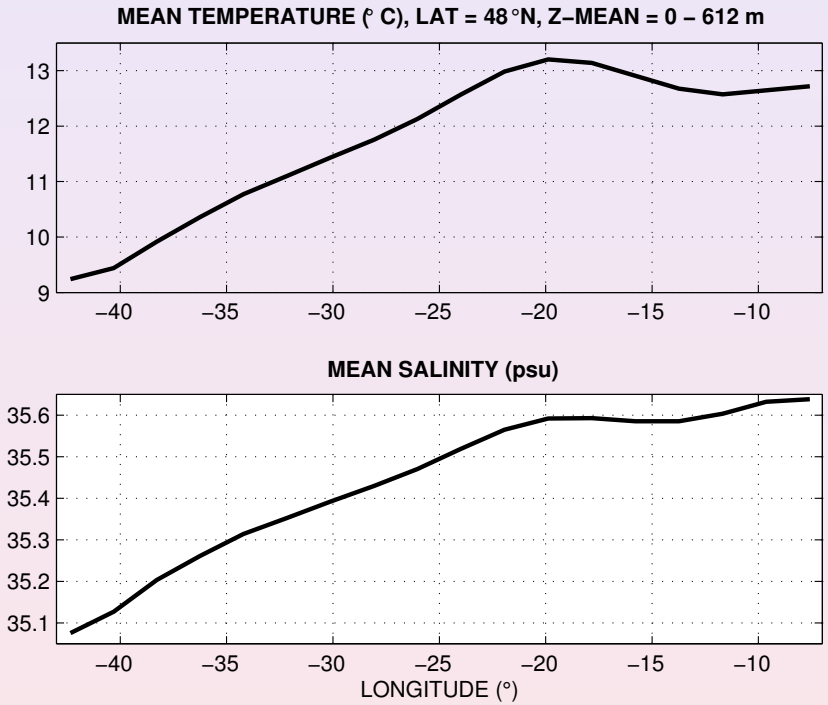

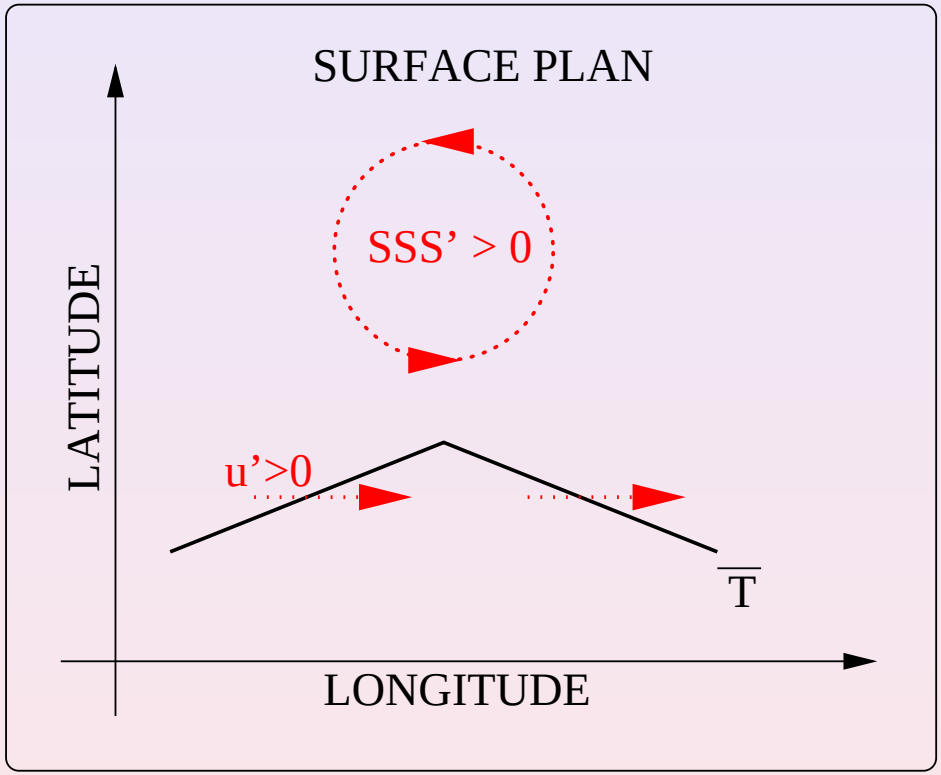

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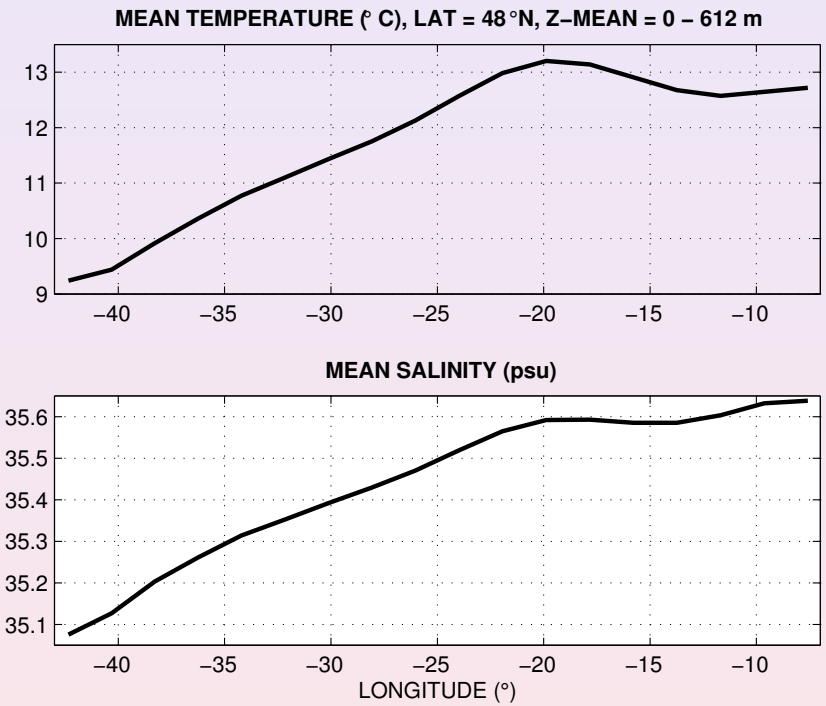

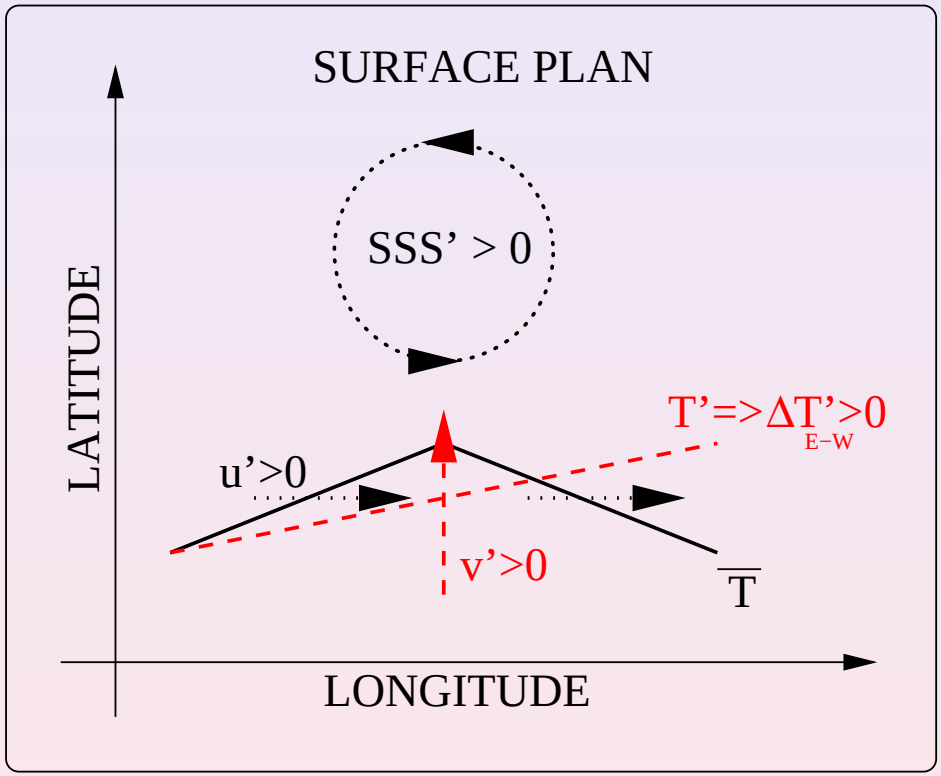

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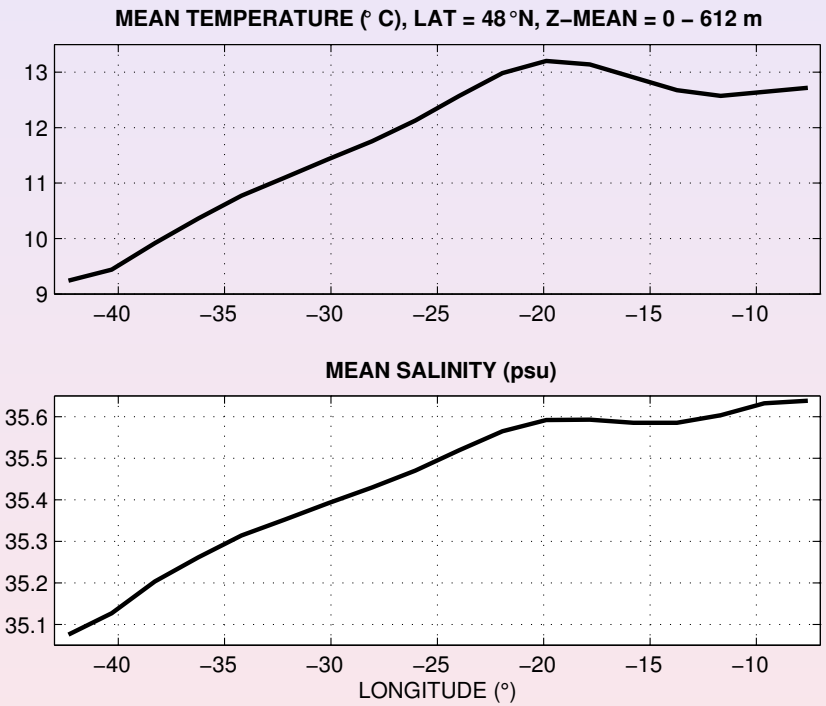

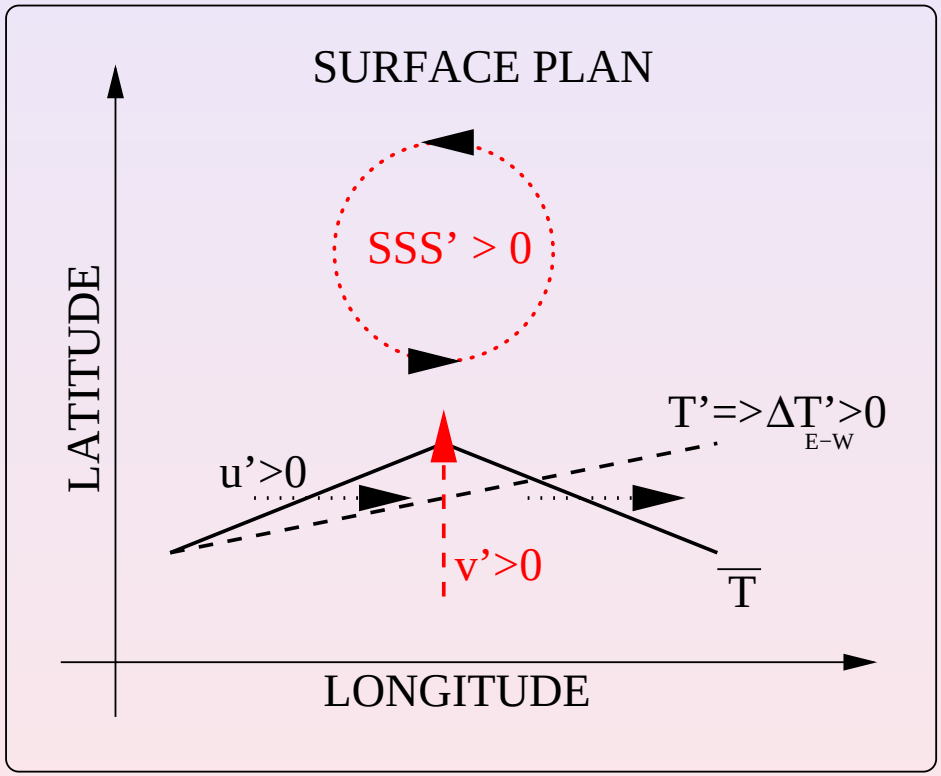

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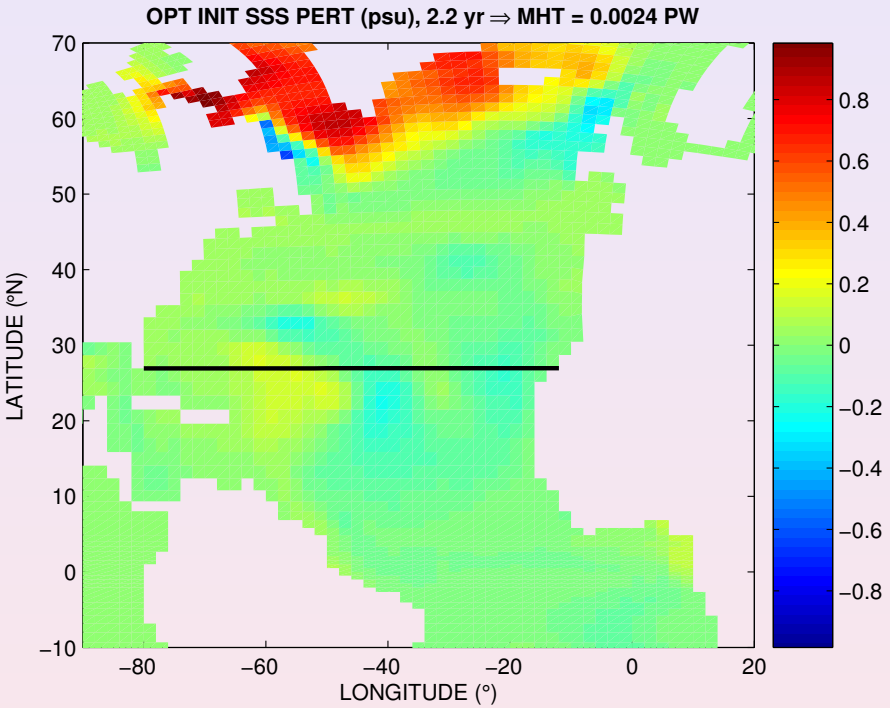

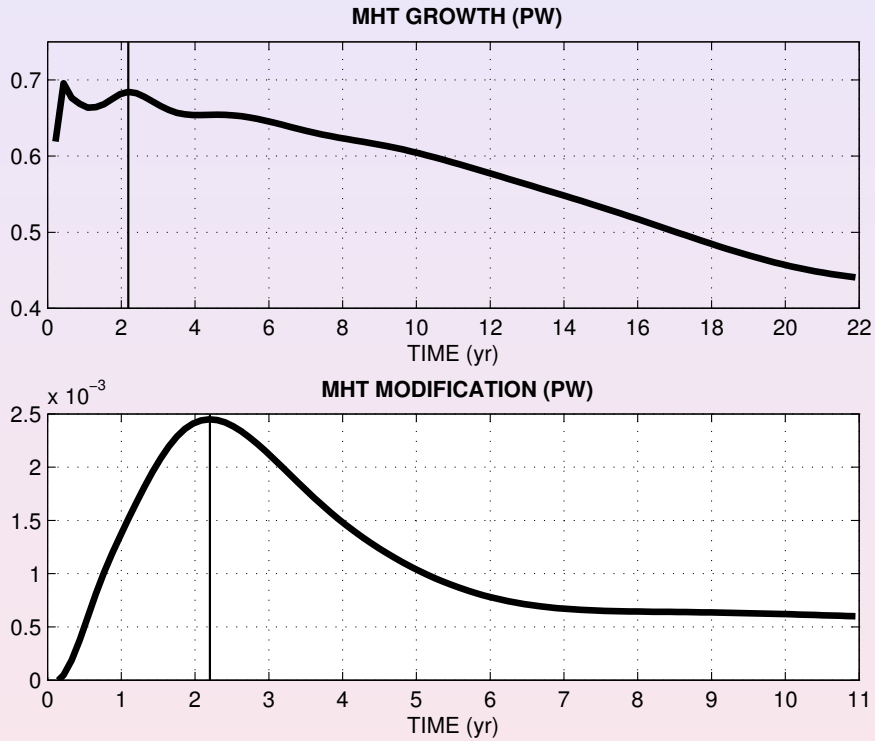

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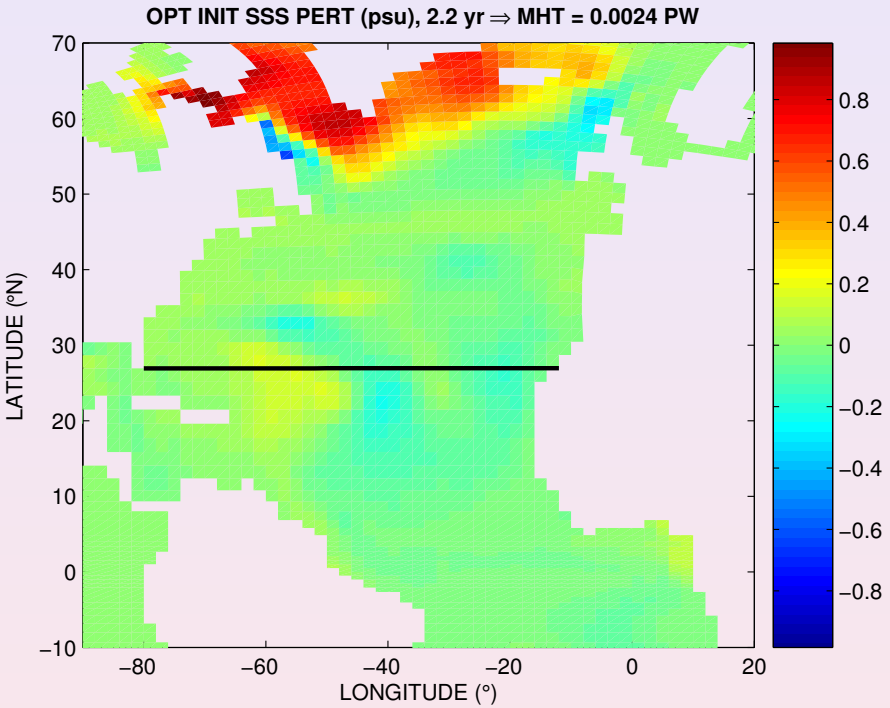

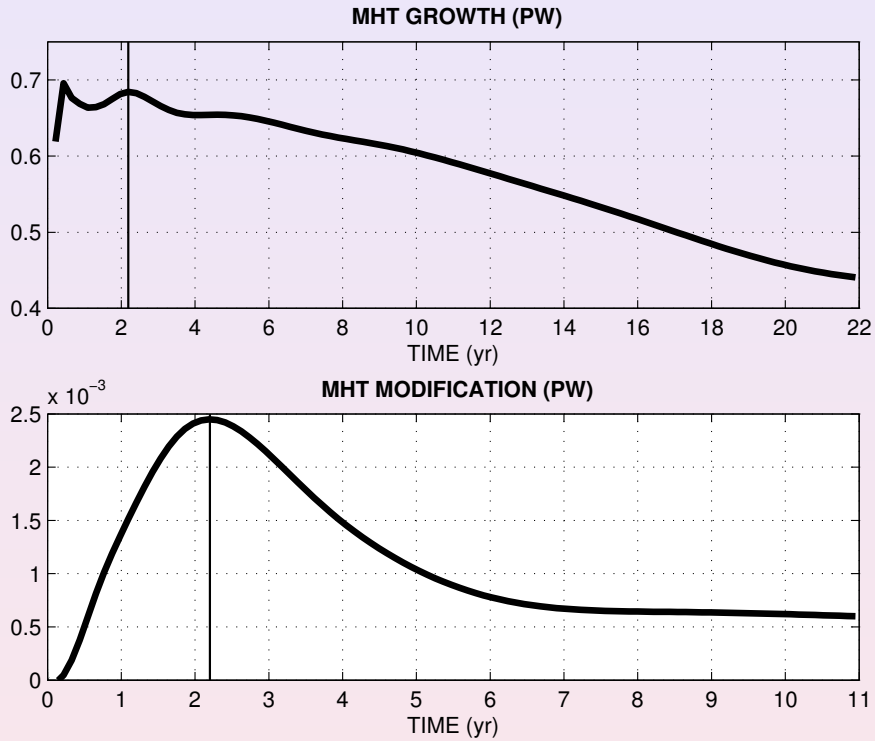

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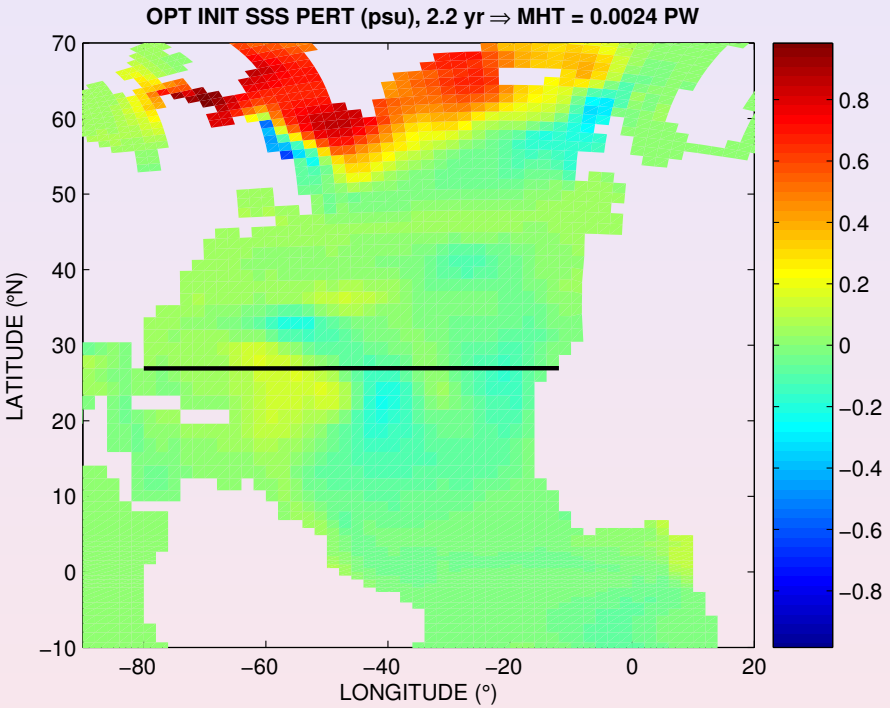

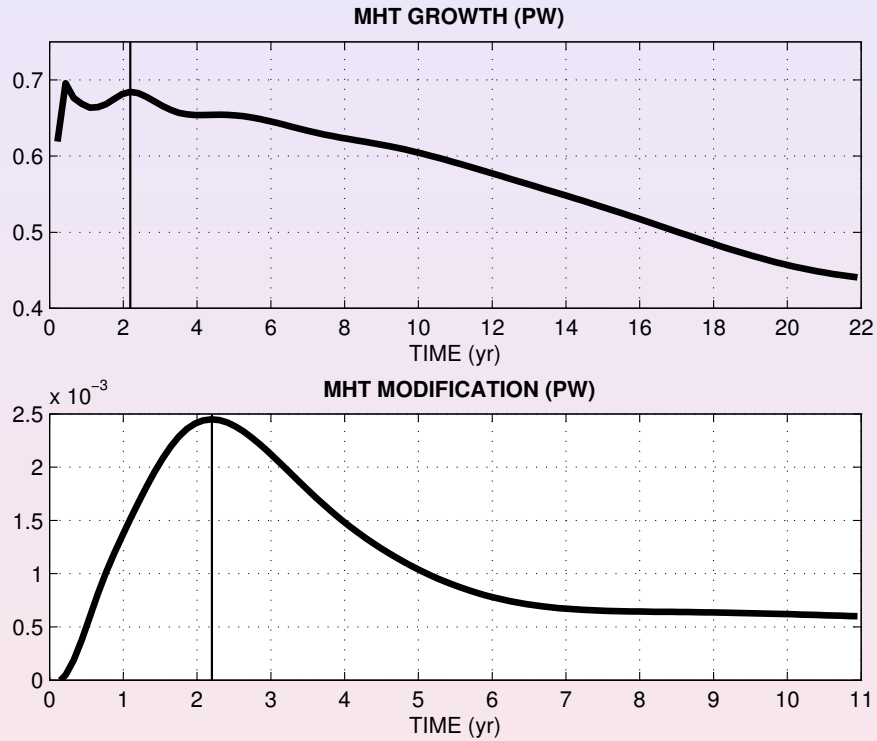

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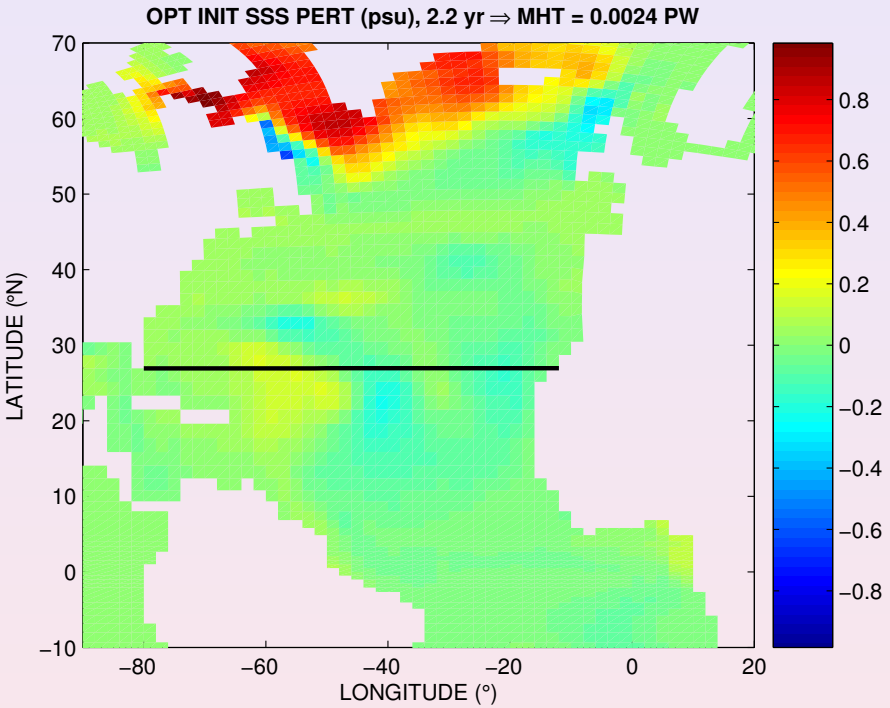

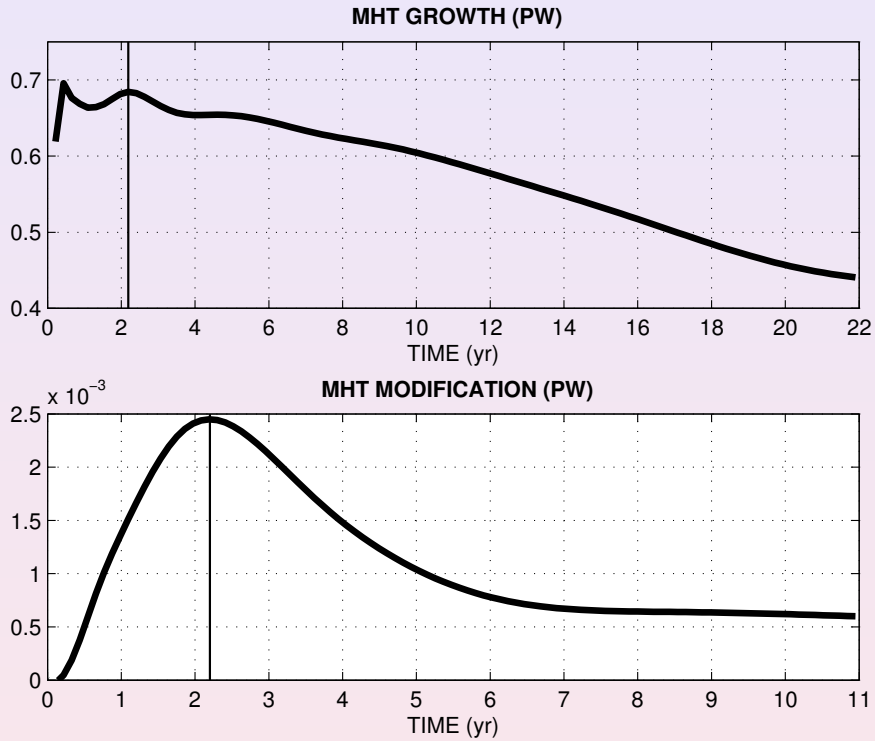

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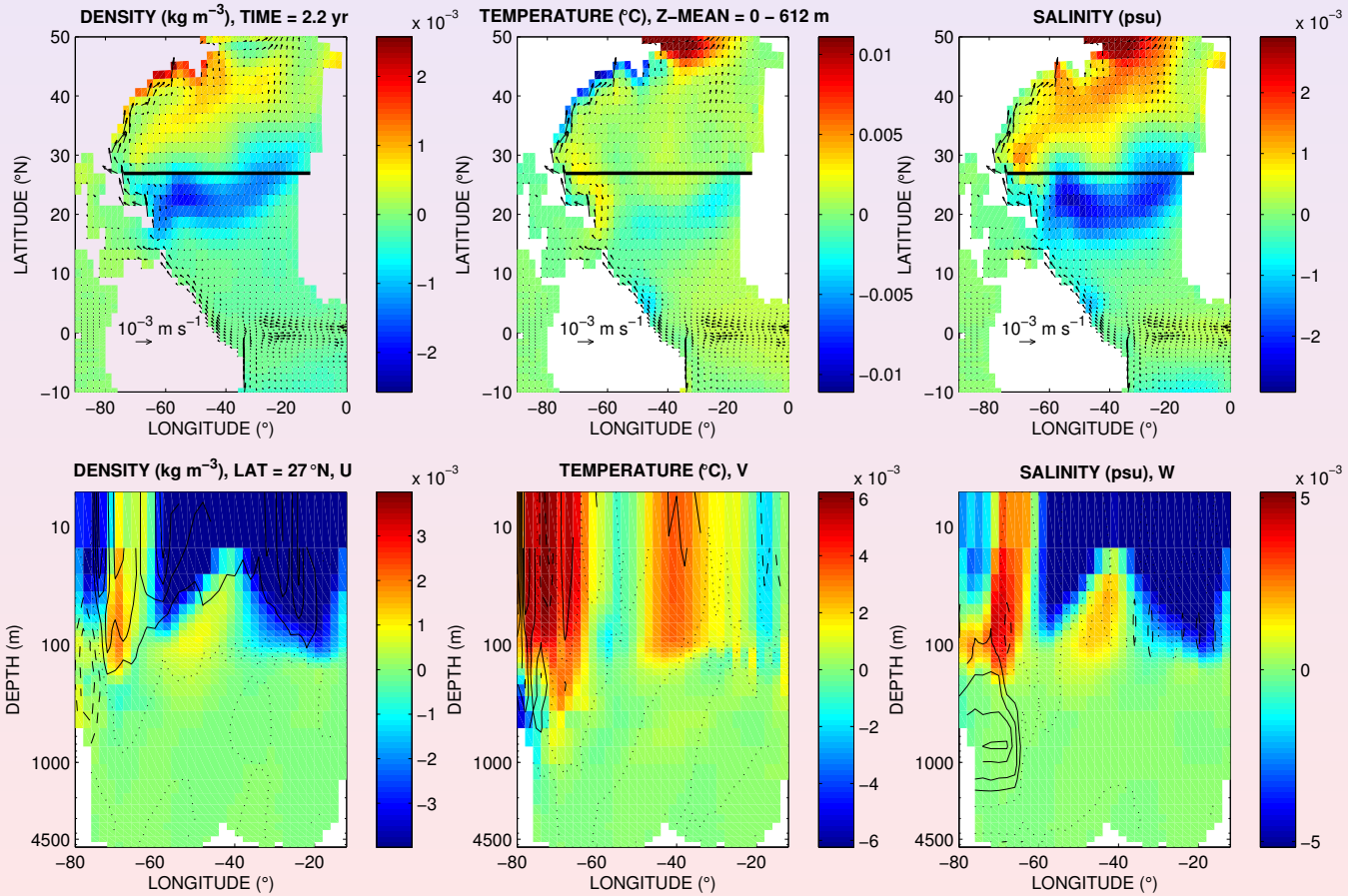

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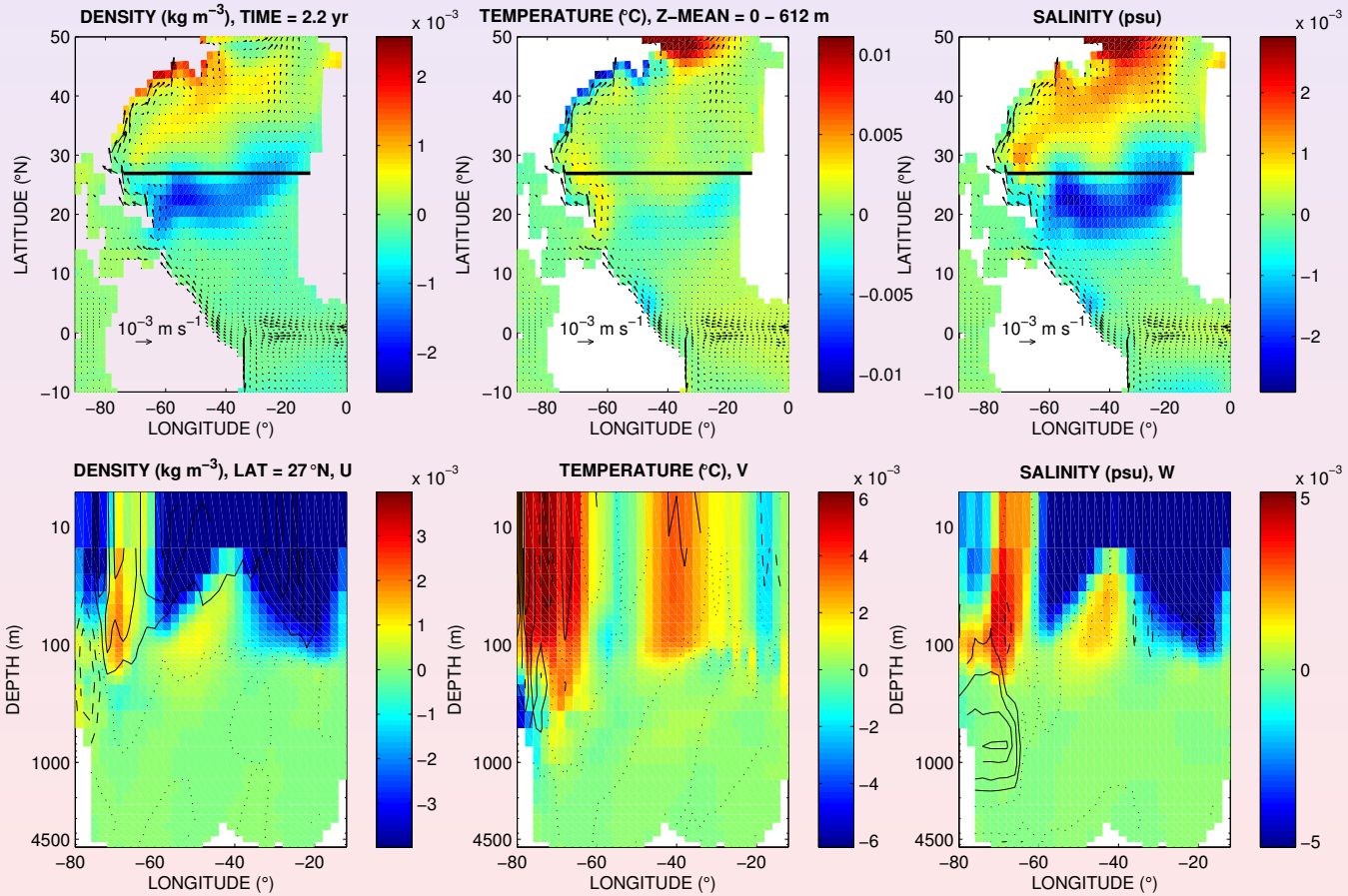

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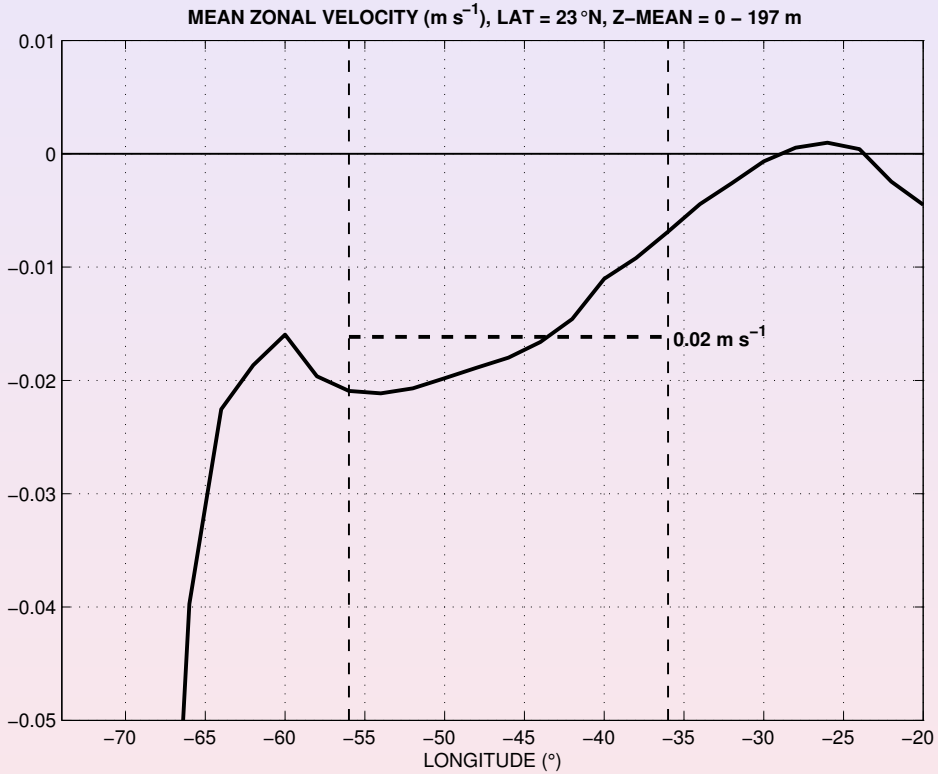

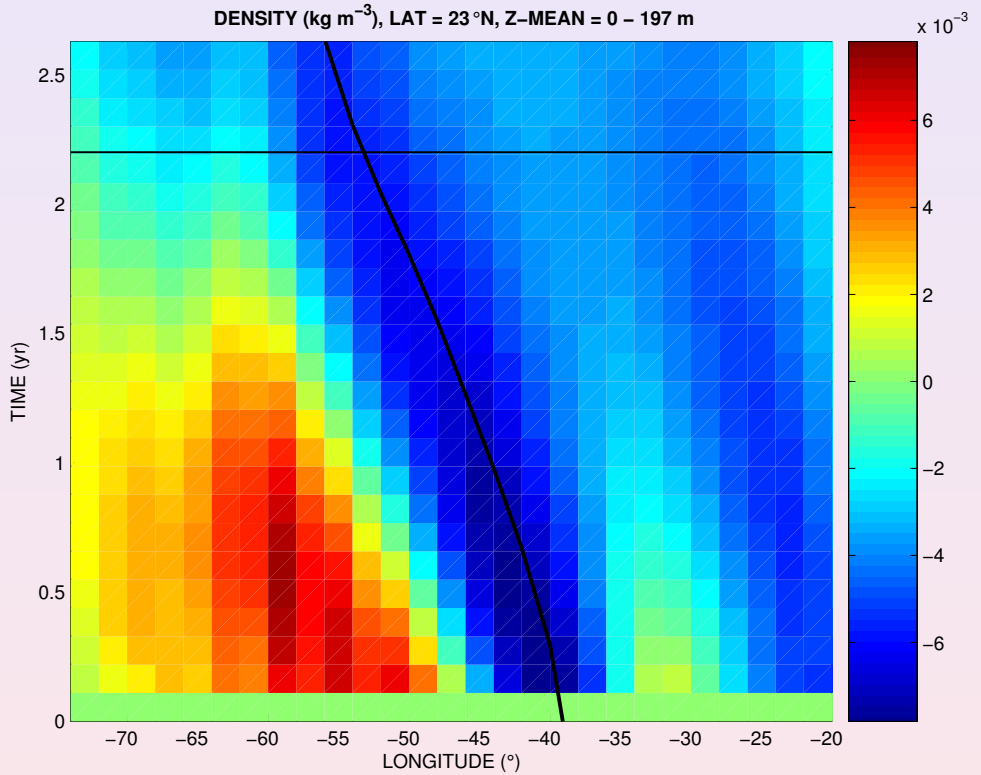

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$\partial_\phi S' < 0$ (t=0 yr) $\Rightarrow \partial_\theta S' > 0$ (t=2.2 yr)

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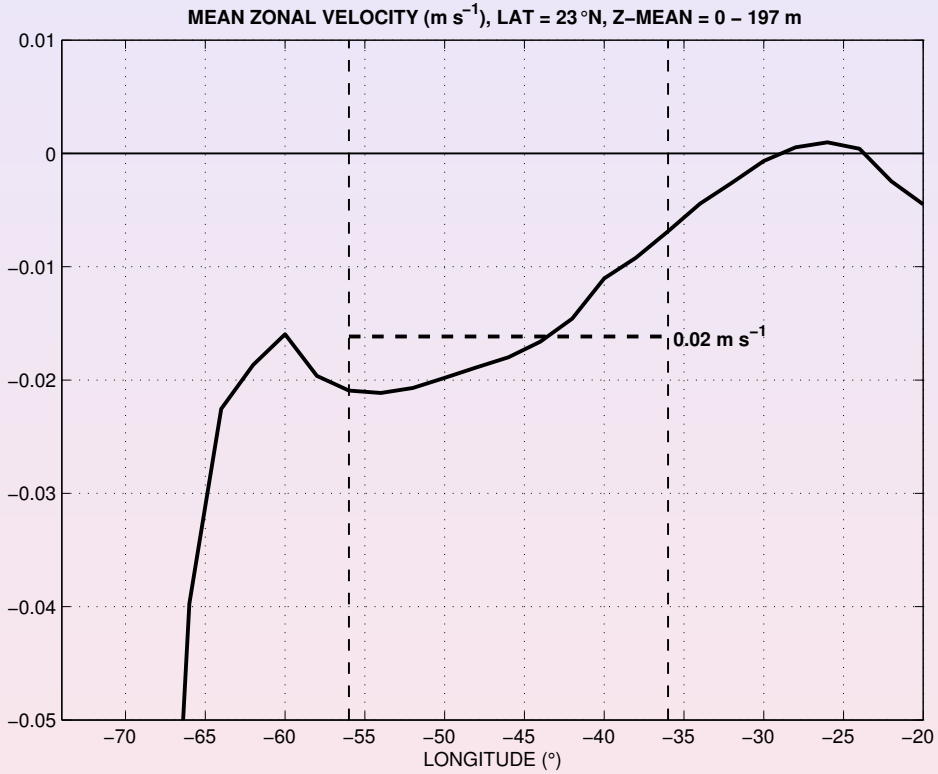

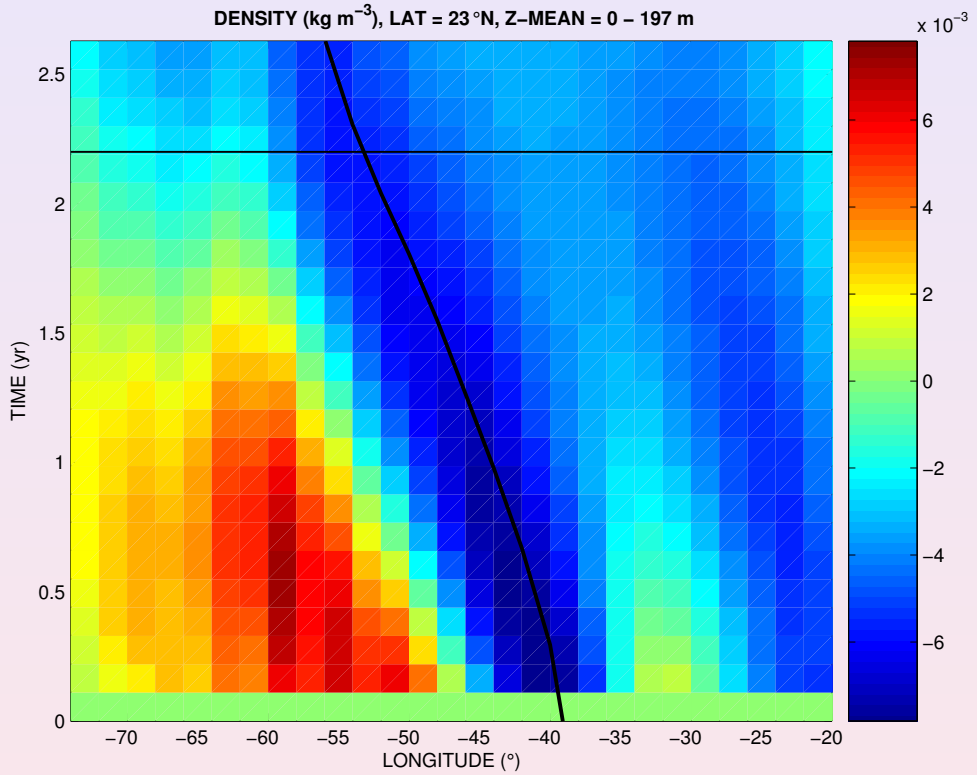

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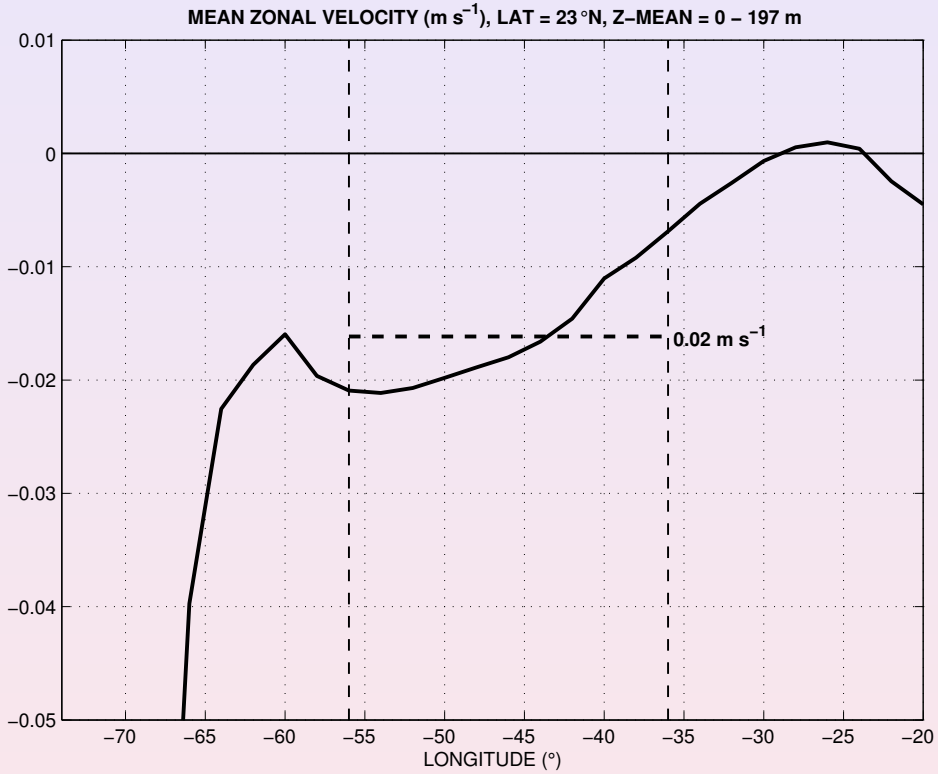

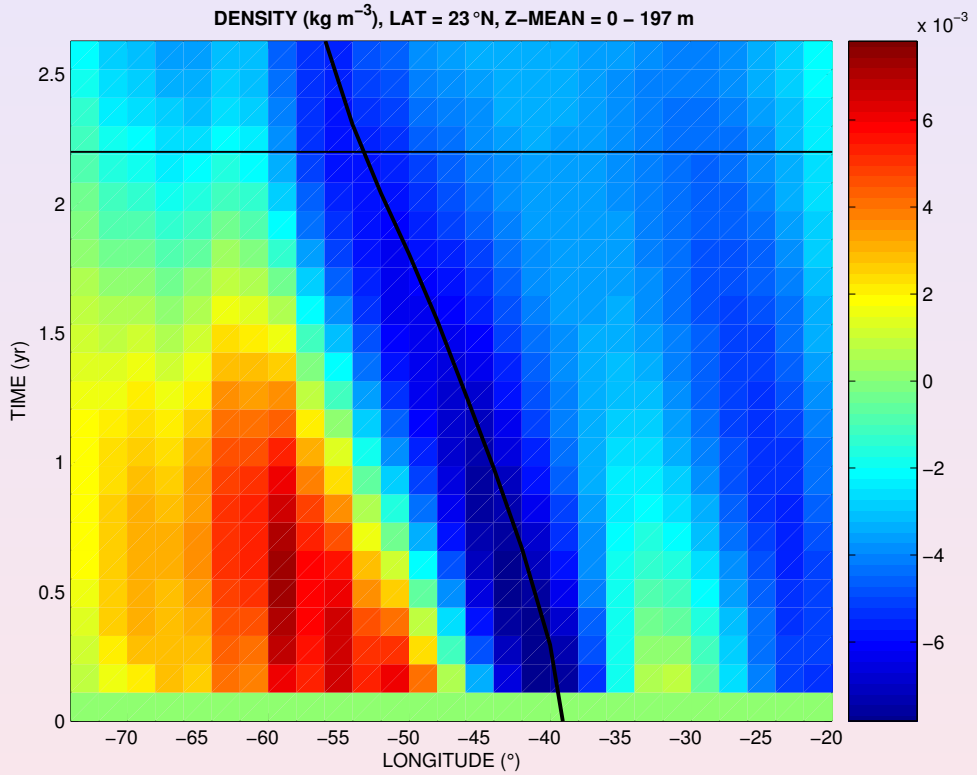

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