Optimal surface salinity perturbations of the thermohaline circulation I: 2D model.

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ABSTRACT

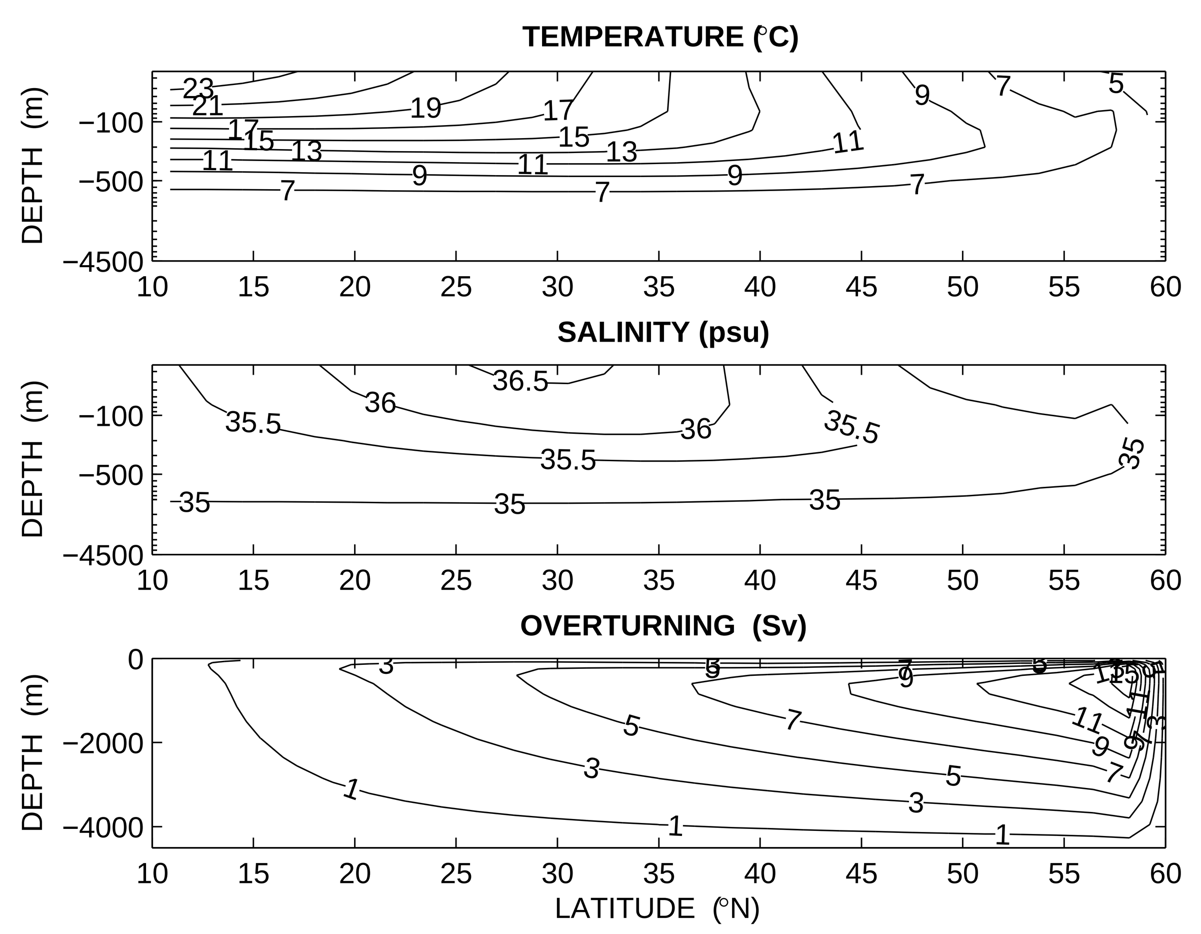
Optimal surface salinity perturbations influencing the meridional overturning circulation maximum are exhibited and interpreted on a stable steady-state of a 2D latitude-depth ocean thermohaline circulation model. In spite of the stability of the steady state the non-normality of the dynamics is able to create some transient growth and variability through stimulation by optimal perturbations. It is found that the response to the optimal initial sea surface salinity perturbation involves a transient growth mechanism leading to a maximum modification of the circulation intensity after 67 yr. Then, looking for the optimal stochastic surface salinity flux perturbation, it is established that the variance of the circulation intensity is controlled by an oscillation of 150 yr period. These optimal initial perturbations leading to explicit solutions (*vs.* eigenvalue problems), their determination in more realistic 3D models should be straightforward.

INTRODUCTION

Recent observations and modeling studies have stressed the influence of surface salinity perturbations on the North Atlantic circulation. The sensitivity of the meridional overturning circulation to initial, constant or stochastic perturbations can be estimated objectively through the analysis of optimal perturbations. As a first methodological step, we look for these optimal perturbations in a 2D model of the thermohaline circulation.

MODEL AND STEADY STATE

The latitude-depth 2D model (Sévellec et al., 2006) is based on the zonal-average approximation of Marotzke et al. (1988); Wright and Stocker (1991), for a single hemisphere North Atlantic size basin. Mixed boundary conditions are used at the surface, i.e. restoring sea surface temperature and prescribed freshwater flux.



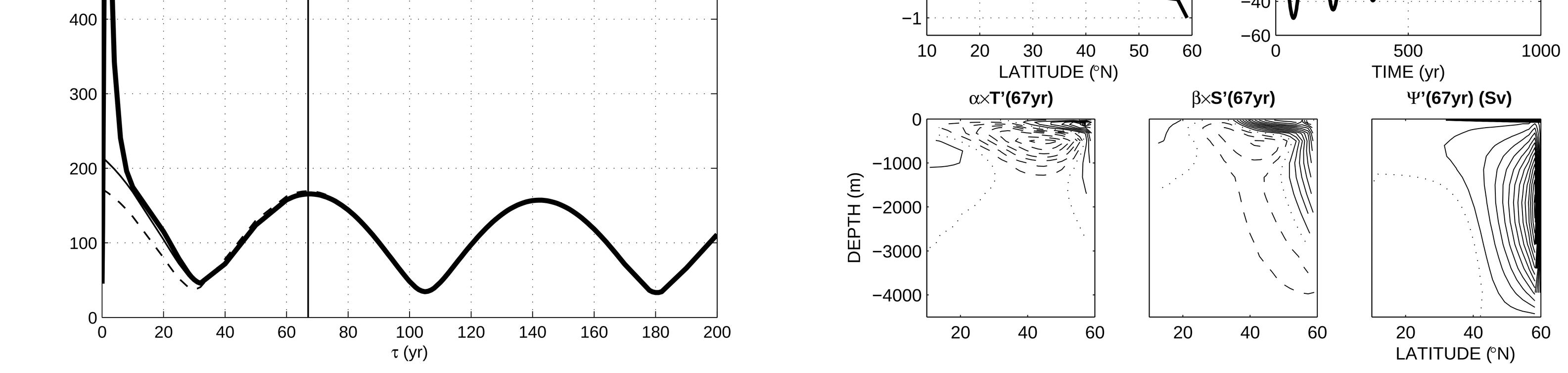
Temperature, Salinity and Meridional Overturning Circulation at steady state of the 2D model.

OPTIMAL INITIAL SSS PERTURBATION

We look for the optimal sea surface salinity perturbation influencing the oceanic circulation in a linearized approach. Optimality is defined with respect to the MOC maximum – a discussion on the choice of the norm is available in Sévellec et al. (2007). The optimal SSS for this measure is obtained by propagation of the cost function relative to the measure, through the adjoint of the linear tangent model. In addition we constrain the solution (1) to be SSS only, (2) to conserve salt and (3) to be normed (in such optimal linear problem a quadratic norm must be used). The optimal solution can be explicitly written in terms of the two constraints, the adjoint of the linear tangent model and the cost function.

$$\text{Maximisation problem:} \quad \text{Max}\left(\langle F|u(\tau)\rangle, |u(0)\rangle\right).$$
$$\text{Constraints: (1) \& (2)} \ |u(0)\rangle = \mathbf{P}\,|u'\rangle \ \text{and (3)} \ \langle u(0)|\mathbf{S}|u(0)\rangle = 1.$$
$$\text{Perturbation propagation:} \quad |u(\tau)\rangle = \mathbf{M}(\tau)\,|u(0)\rangle.$$
$$\Rightarrow \textit{Explicit solution:} \quad |u'\rangle = (2\gamma)^{-1}\left(\mathbf{N}^{-1}\mathbf{P}^\dagger\mathbf{M}^\dagger(\tau)\,|F\rangle\right),$$
$$\text{with } \mathbf{N} = \mathbf{P}^\dagger\mathbf{S}\mathbf{P} \text{ and } \gamma = \text{fct}\left(\mathbf{M}(\tau), |F\rangle, \mathbf{N}, \mathbf{P}\right).$$

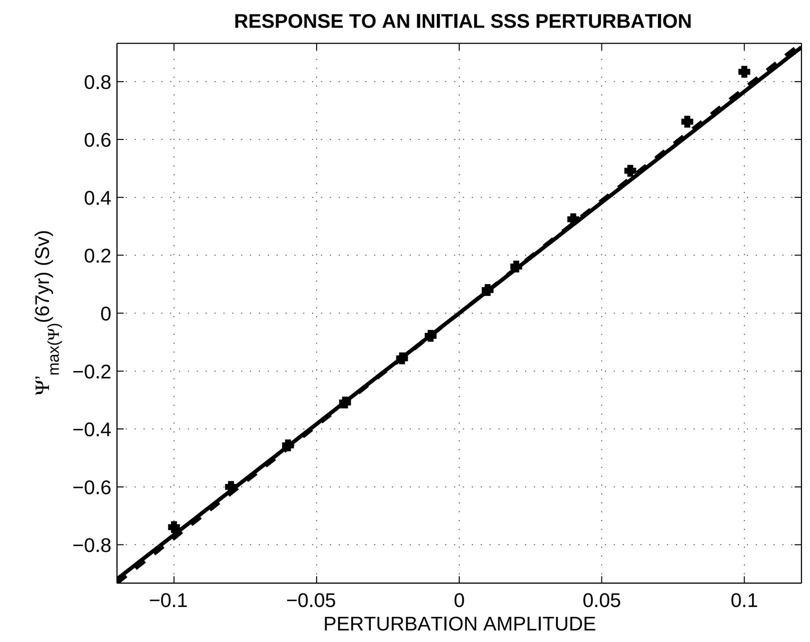
The solution depends on the integration time of the adjoint model: we choose to examine the most efficient growth in time in agreement with our 2D model 'adjusted' dynamics, found for a delay of 67 yr. The corresponding pattern is a large-scale dipole perturbation. The time integration of the linear model perturbed by this optimal SSS triggers an oscillation of 150 yr (Sévellec et al., 2006) leading to a transient growth of the MOC intensity after 67 yr.



Optimal initial SSS structure modifying the MOC (upper center). The response has a maximum at 67 yr (left), confirmed by the integration of the linear tangent model with the optimal structure for initial conditions (upper right): temperature, salinity and MOC anomalies after 67 yr (lower right).

LINEAR VS NONLINEAR TIME INTEGRATIONS

In order to validate the linear approach, we have compared the linear and nonlinear time integrations initialized by the optimal SSS perturbation for different amplitudes ranging from -0.1 to 0.1 psu (confined in the upper 50 m). A strong correspondance exists between the linear and nonlinear integrations, although the latter show a weak asymmetry between positive and negative perturbations.



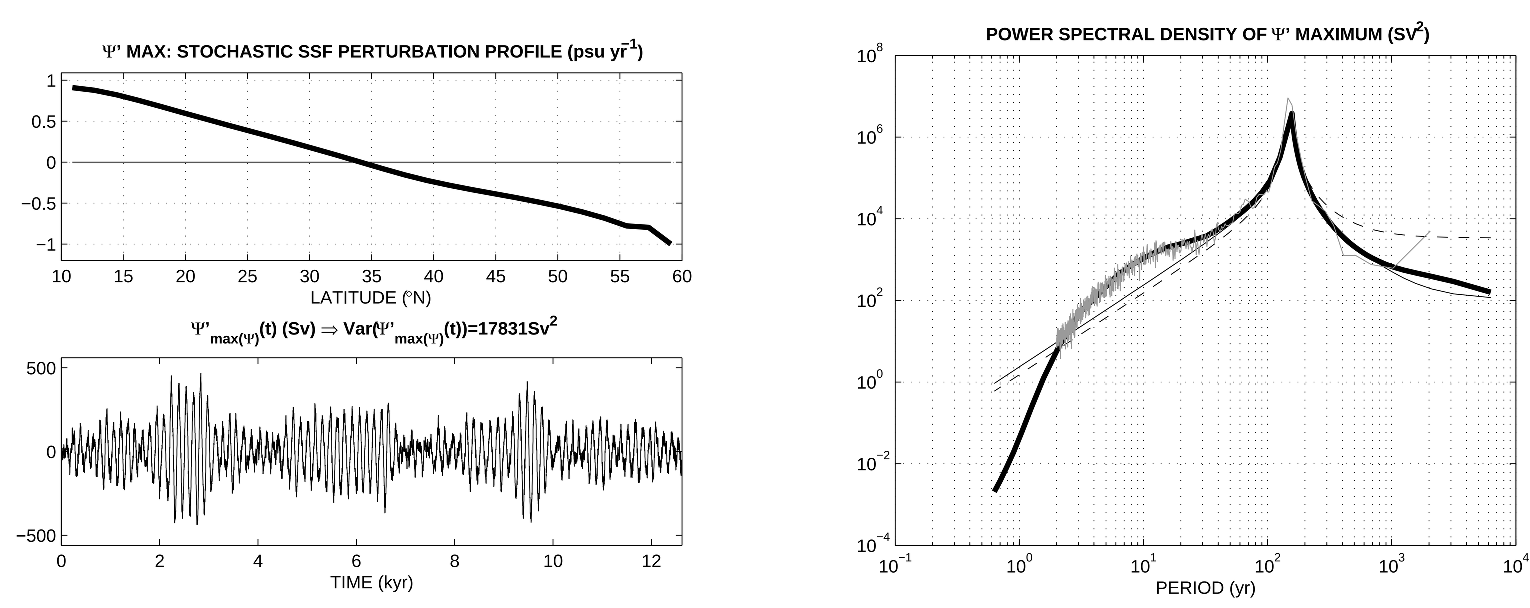
Optimal response of the meridional overturning circulation intensity, as a function of the optimal initial perturbation amplitude (maximum SSS anomaly in psu), according to the theoretical computation (solid line), the linear (dashed line) and the nonlinear (crosses) time integrations.

OPTIMAL STOCHASTIC SSF PERTURBATION

We look for the optimal stochastic surface salinity flux influencing the oceanic circulation in a linearized approach (the optimality is defined as previously and the constraints remain the same). The optimal solution can be explicitly written in terms of the two constraints, the cost function, the eigenvectors of the linear tangent model and those of its adjoint.

$$\text{Maximisation problem:} \quad \text{Max}\left(\text{var}(\langle F|u(\tau)\rangle), |g\rangle\right).$$
$$\text{Constraints: (1) \& (2)} \ |g\rangle = \mathbf{P}\,|g'\rangle \ \text{and (3)} \ \langle g|\mathbf{S}|g\rangle = 1.$$
$$\text{Perturbation propagation:} \quad |u(\tau)\rangle = \int_0^\tau ds\,\mathbf{M}(\tau - s)a(s)\,|g\rangle.$$
$$\Rightarrow \textit{Eigenvalue problem:} \quad \mathbf{N}^{-1}\mathbf{P}^\dagger\sum_{ij}\frac{-1}{\lambda_i^* + \lambda_j}\,|u_i^\dagger\rangle\,\langle u_i|F\rangle\,\langle F|u_j\rangle\,\langle u_j^\dagger|\,\mathbf{P}\,|g'\rangle = \gamma\,|g'\rangle.$$
$$\Rightarrow \textit{Explicit psd solution:} \quad \text{psd}\,(\omega) = \sum_{ij}\langle g|u_j^\dagger\rangle\,\frac{1}{-i\omega - \lambda_i^*}\,\langle u_i|F\rangle\,\langle F|u_j\rangle\,\frac{1}{i\omega - \lambda_j}\,\langle u_j^\dagger|g\rangle.$$

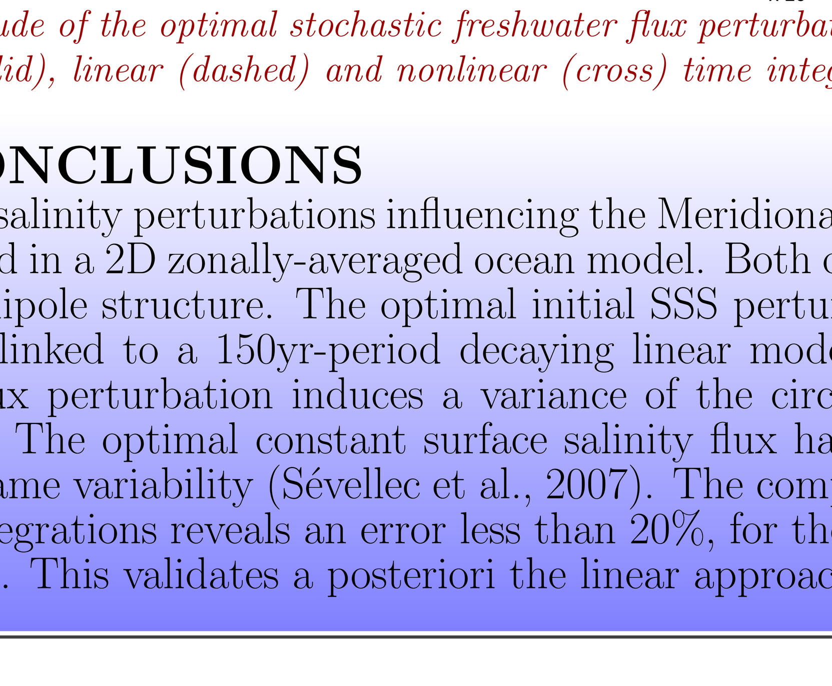
The optimal SSF corresponds to a large-scale dipole. The linear integration with this perturbation, as the power spectral density, shows a variance dominated by a 150 yr oscillation.



(left) Optimal stochastic flux structure modifying the MOC and the corresponding linear time integration. (right) Power spectral density of the overturning circulation intensity. The theoretical spectrum (thick line) is compared with the truncation at 50 (solid line) and 2 (dashed line) eigenmodes and to the perturbed linear time integration spectrum (gray line).

LINEAR VS NONLINEAR TIME INTEGRATIONS

As previously, to validate the linear approach, we have compared the linear and nonlinear time integrations for different amplitudes ranging from -10^{-3} to 10^{-3} psu yr^{-1} of the stochastic freshwater flux. The relative error between linear and nonlinear time integration is stronger than the one of the optimal initial experiment but remains lower than 20%.



MOC variance as a function of the amplitude of the optimal stochastic freshwater flux perturbation, according to the theoretical computation (solid), linear (dashed) and nonlinear (cross) time integrations.

CONCLUSIONS

Optimal initial and stochastic surface salinity perturbations influencing the Meridional Overturning Circulation have been obtained in a 2D zonally-averaged ocean model. Both optimal patterns correspond to a large-scale dipole structure. The optimal initial SSS perturbation induces a transient growth at 67 yr linked to a 150yr-period decaying linear mode. The optimal stochastic surface salinity flux perturbation induces a variance of the circulation linked to the same oscillatory mode. The optimal constant surface salinity flux have also been studied and do not lead to the same variability (Sévellec et al., 2007). The comparison between linear and nonlinear time integrations reveals an error less than 20%, for the range of perturbation amplitude tested here. This validates a posteriori the linear approach.

References

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