# Horizontal structure of nonlinear basin modes due to large scale baroclinic instability: A prototype for interannual variability

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# Motivation

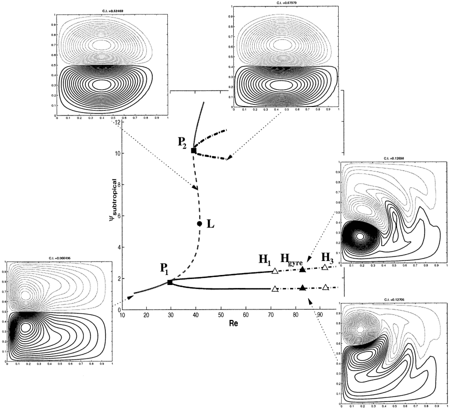
- In Thierry's keywords: Ocean intrinsic variability, Dynamical System Theories, Process studies - idealized models, Energetics
- interannual ocean variability: atmospheric forcing but also intrinsic!
- spontaneous multidecadal variability in PE/PG models idealized configurations under constant buoyancy forcing (Huck&al. 1999 JPO, Colin de Verdière&Huck 1999 JPO, te Raa&Dijkstra 2002 JPO...), but not in 2-layer PG shallow-water models (Ferjani&al. 2013, 2014 JMR)
- large-scale baroclinic instability $>$ potential source of low-frequency oceanic variability (Hochet&al. 2015 JPO)
- in realistic GCMs, difficult analysis of nonlinear oscillations, variance budget, eddy fluxes, linear instability (Jamet&al. 2015 CD)

$\Rightarrow$ **simplest framework with large-scale baroclinic instability: 2.5-layer shallow water model** $>$ easy analysis of local and global instability, comparison with nonlinear oscillation and eddy fluxes

# Variability of the wind-driven circulation in idealized models

interannual variability arises when Reynolds number increases $\mathrm{Re} = UL/\nu$

**dynamical system approach** > bifurcation structure in QG ou SW models: sensitivity to details like (free/no slip) boundary conditions, forcing symmetry... $\Rightarrow$ **gyre mode** (Nauw&Dijkstra 2001 JMR, Simonnet&Dijkstra 2002 JPO, Dijkstra&Ghil 2005 RG...)




**turbulent oscillator**: positive feedback due to eddy rectification (Berloff&al. 2007 JPO)

# 2.5-Layer Shallow-Water Model Equations

$$\partial_t h_i + \nabla \cdot (h_i \mathbf{u_i}) = 0$$

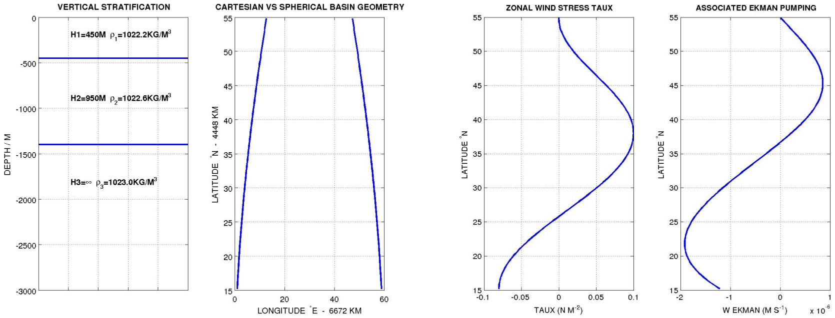
$$\partial_t \mathbf{u_i} + \mathbf{u_i} \cdot \nabla \mathbf{u_i} + f \mathbf{k} \times \mathbf{u_i} = -\nabla \phi_i + \nu \nabla^2 \mathbf{u_i} + \delta_{i1} \frac{\tau}{\rho_i h_i}$$

$$\phi_1 = g'_{13} h_1 + g'_{23} h_2 \; ; \; \phi_2 = g'_{23} h_1 + g'_{23} h_2$$

$$g'_{ij} = g \frac{\rho_j - \rho_i}{\rho_j} > 0$$

$h_i$ layer thickness, $\phi_i$ hydrostatic pressure/$\rho_0$
**reduced gravity** $g'_{ij}$: assumes 3$^{\text{rd}}$ layer has infinite depth, null circulation/pressure gradient
**forcing**: zonal wind stress $\tau$ fct(latitude), constant in time
**numerical resolution** in Cartesian or spherical geometry: Arakawa C-grid with energy conserving scheme (Sadourny 1975) aka "ENE"
**1/3°** (1/6°) resolution $\Rightarrow$ Laplacian viscosity $\nu$ 3200 (400) m$^2$ s$^{-1}$

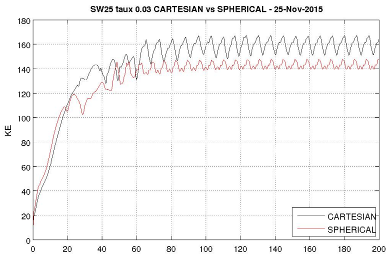
# Idealized configuration and forcing

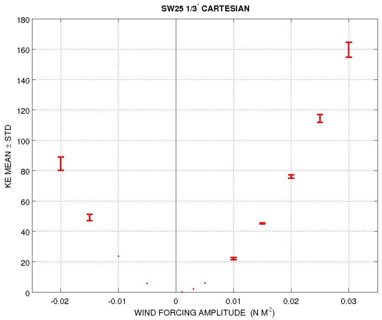


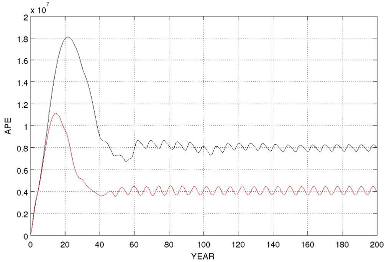

Vertical stratification, basin geometry and analytical wind forcing (zonal stress and Ekman pumping), identical to Sirven&al. 2015 JMR.

**Control parameter: wind forcing amplitude, limited by upper layer outcropping for linearization**

# A flavor of the variability: oscillations with period O(7–8 yr)



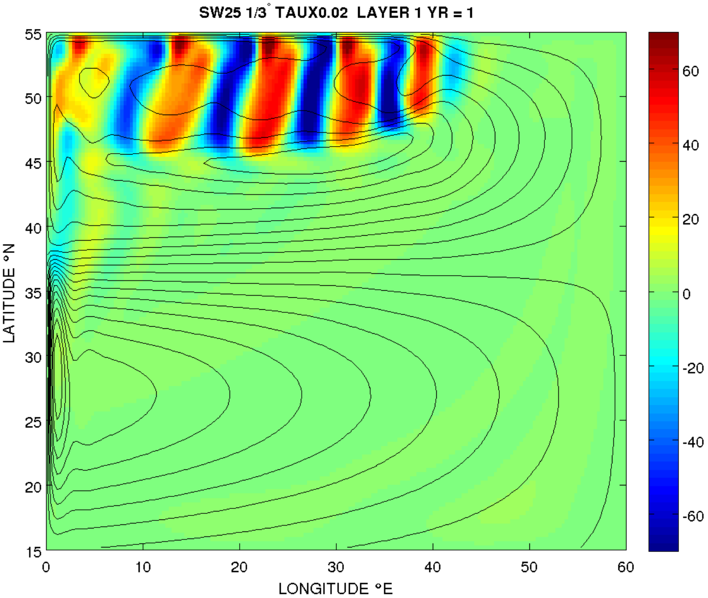



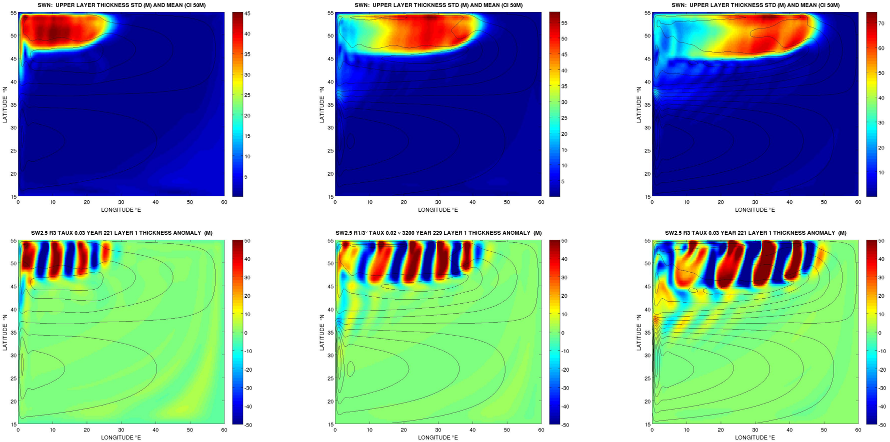

▲ KE mean and standard deviation function of wind forcing amplitude > Hopf bifurcation

◀ Evolution of KE and APE for spherical and cartesian geometry

# Animation of h1' anomalies: westward propagation



# Influence of increasing wind forcing amplitude




(top) Standard deviation and (bottom) anomaly of $h_1$ (color), superposed on the mean state (contours).

**Variability increases and extends further east with increasing wind forcing: link with unstable regions?**

# Regions of unstable waves propagation

$$\mathrm{PV}_i \approx \frac{f}{h_i}$$

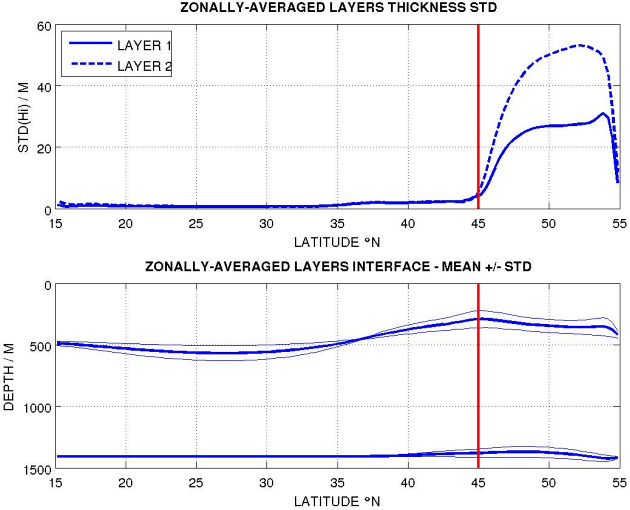
North/45°N:

$\partial_y h_1 > 0 \Rightarrow$
$\partial_y \mathrm{PV}_1 < 0$

$\partial_y h_2 > 0 \Rightarrow$
$\partial_y \mathrm{PV}_2 > 0$

**conditions for**

baroclinic instability



# Conditions for baroclinic instability in SW2.5

Linear stability analysis of the 2.5 layer QG equations (Liu 1999 QJRMS)
- zonal mean flow U1 U2
- long wave approximation

⇒ analytical solutions

**conditions for instability**

U1<0 !

Hochet&al. 2015 JPO →

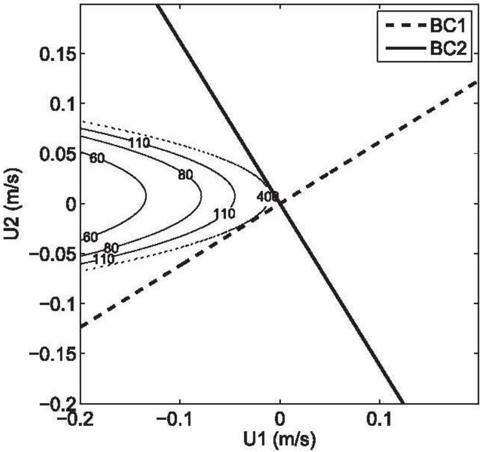
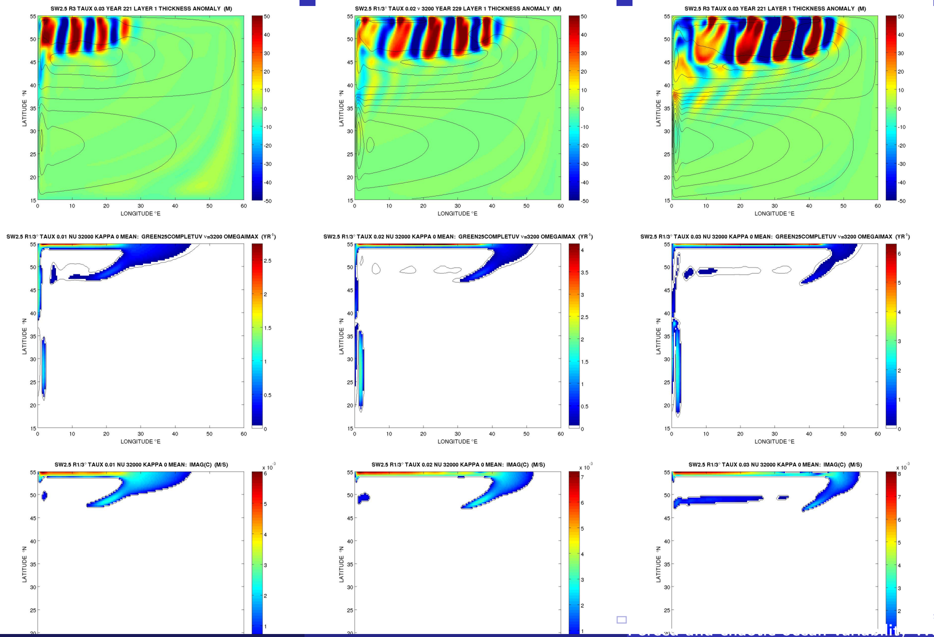
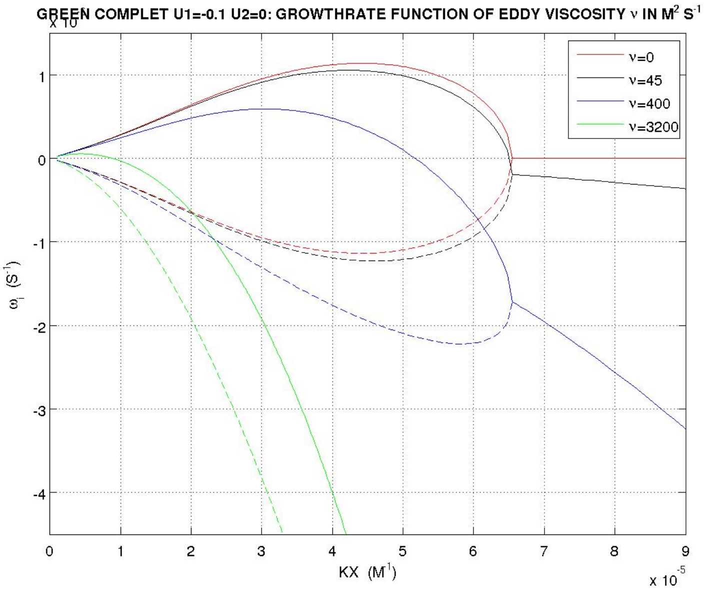



FIG. 1. Growth time (days) of large-scale modes in a 2.5-layer quasigeostrophic model as a function of the zonal velocities in layer 1 ($U_1$) and layer 2 ($U_2$). The zonal wavelength is 1000 km, and the latitude is 30°N. The two lines are the first (dashed) and second (solid) baroclinic modes; $H$ = 200 m.

# Relation with local baroclinic instability growthrate

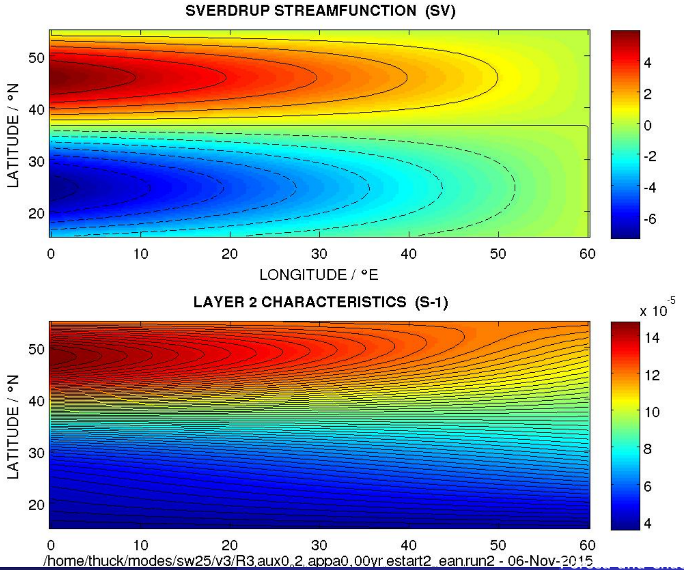


# Selection of most unstable wavelength... by eddy viscosity

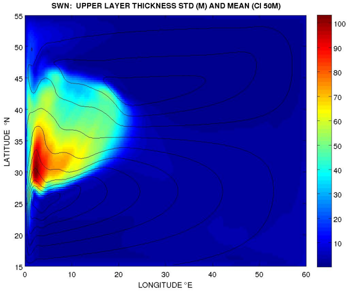


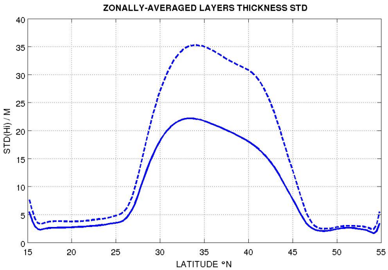
# Closed PV contours in layer 2 define instability region

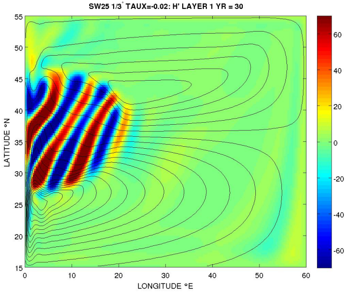
## Rhines&Schopp 1991 JPO

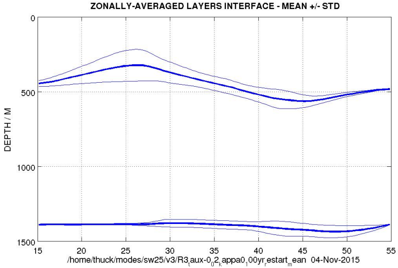


# Reversed double gyre: instability at intergyre

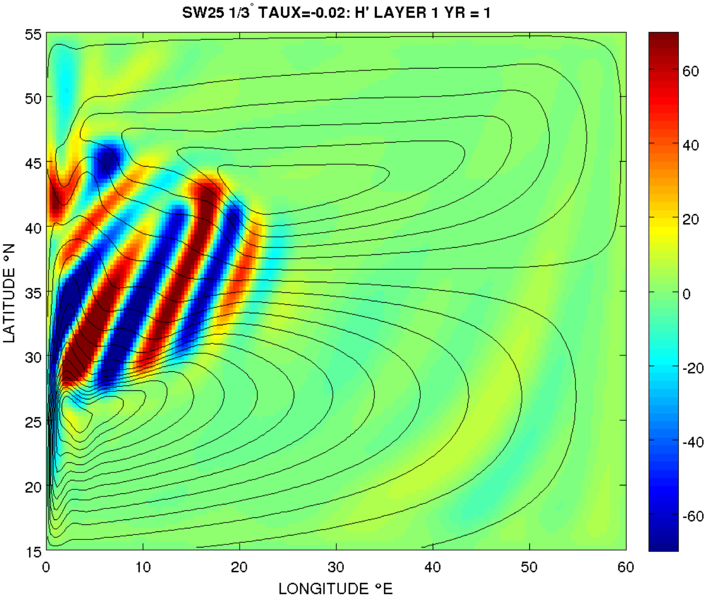


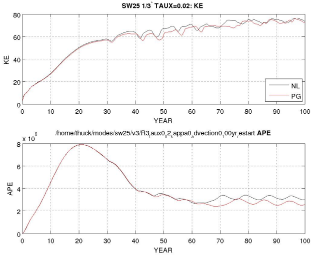



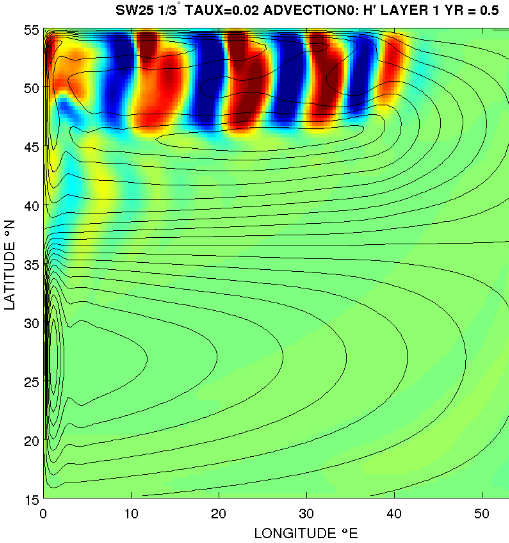



# Reversed double gyre: Animation of h1'



# Classical double gyre with no momentum advection "PG"




at 1/3°, planetary geostrophic dynamics makes almost no difference with full nonlinear equations

# 3D linear stability analysis > technical part

• linearized version of 2.5-layer SW model (no outcropping):
$X = \overline{X} + X'$, where $\overline{X}$ is steady or time-mean state

$$\partial_t h'_i + \nabla \cdot (\overline{h_i}\, \mathbf{u'_i}) + \nabla \cdot (h'_i\, \overline{\mathbf{u_i}}) = 0$$

$$\partial_t \mathbf{u'_i} + \mathbf{u'_i} \cdot \nabla \overline{\mathbf{u_i}} + \overline{\mathbf{u_i}} \cdot \nabla \mathbf{u'_i} + f\mathbf{k} \times \mathbf{u'_i} = -\nabla \phi'_i + \nu \nabla^2 \mathbf{u'_i} - \delta_{i1} \frac{h'_i\, \tau}{\rho_i \overline{h_i}^2}$$

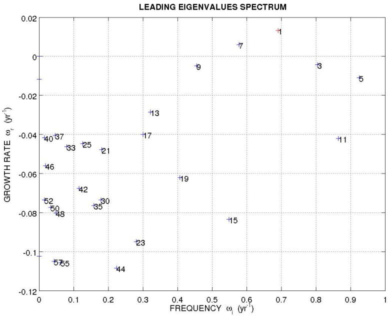
$$\phi'_1 = g'_{13} h'_1 + g'_{23} h'_2 \; ; \; \phi'_2 = g'_{23} h'_1 + g'_{23} h'_2$$

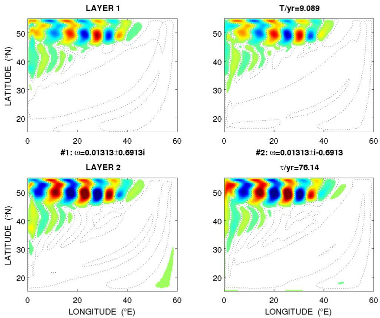
• computation of eigenvalues of Jacobian $J$ through those of propagator $M(\tau)$ with ARPACK: $\partial_t X = J\, X = \omega X$

$$X(t=0) \rightarrow X(t=\tau) = M(\tau)\, X(t=0) = e^{J\tau}\, X(t=0),$$

- initial perturbations $X(t=0)$ provided by iterative Arnoldi method,
- tangent linear model integrated for 'short' time, typically $\tau$=0.1 yr,
- Jacobian eigenvalues $\omega$ deduced from propagator eigenvalues $e^{\omega\tau}$.

# 3D linear stability analysis: results




- most unstable eigenmode with growth time scale $1/\omega_r$ 76.1 yr has period 9.1 yr and wavenumber 4 similar to nonlinear oscillation

# Energy budget

- linear framework clearly separates perturbations from mean flow
- allows precise estimation of energy source and sink for modes
- sucessfully used in barotropic shallow water model (Herbette&al. 2014 GAFD) to determine regions of barotropic instability sustaining oscillation
- extended here to address baroclinic modes where APE » KE

# Energy budget > natural definition of APE

Multiplying momentum equations by $(h_i \mathbf{u_i})$:

$$\partial_t \left[ h_i \frac{|\mathbf{u_i}|^2}{2} \right] + \nabla \cdot \left[ h_i \mathbf{u_i} \frac{|\mathbf{u_i}|^2}{2} \right] = -h_i \mathbf{u_i} \cdot \nabla \phi_i = -\nabla (h_i \mathbf{u_i} \phi_i) - \phi_i \, \partial_t h_i$$

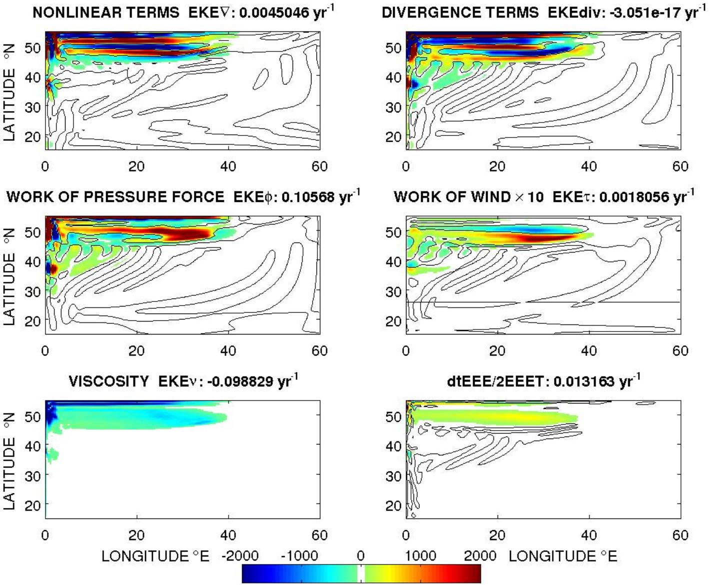
Summing last rhs term for the 2 layers leads to natural definition of APE:

$$\begin{aligned} \sum_i \phi_i \, \partial_t h_i &= (g'_{13} h_1 + g'_{23} h_2) \, \partial_t h_1 + (g'_{23} h_1 + g'_{23} h_2) \, \partial_t h_2 \\ &= \partial_t \left( g'_1 \frac{h_1^2}{2} + g'_2 \frac{h_2^2}{2} + g'_2 h_1 h_2 \right) \end{aligned}$$

In terms of interface deviations $\eta_i$

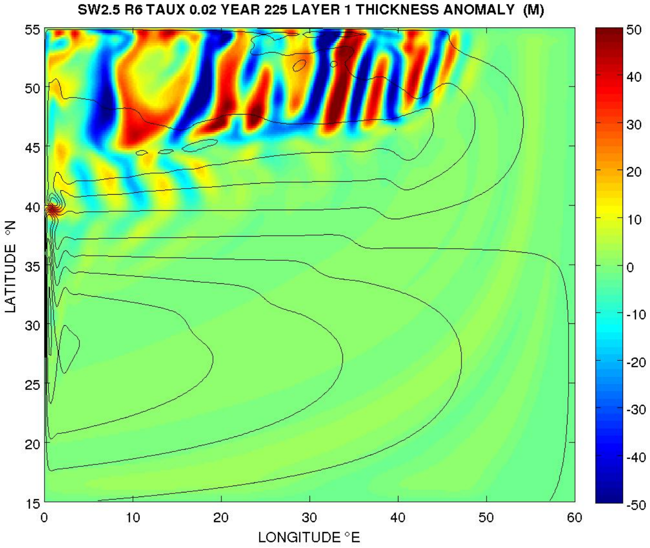
$$\mathrm{APE} = g \frac{\eta_1^2}{2} + g'_{12} \frac{\eta_2^2}{2} + g'_{23} \frac{\eta_3^2}{2}$$

# Energy budget for leading mode > exact growth rate



# Influence of resolution and dissipation: 1/6°, $\nu$=400 m$^2$ s$^{-1}$

shorter wavelength and more irregular oscillations



# The limits of crude vertical resolution

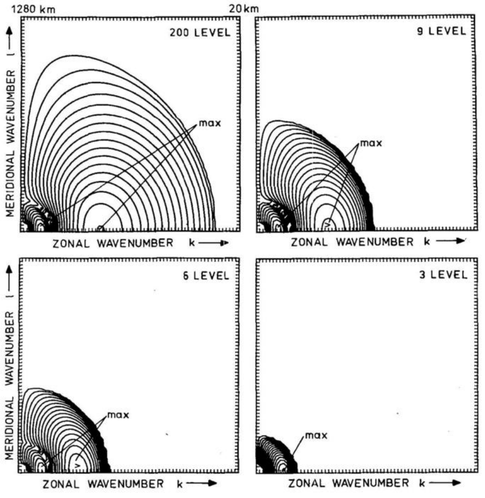



FIG. 5. Spectral distribution of maximum growth rate of unstable shear modes depending on the vertical resolution of the model. The maximum lies on the $k$-axis. Contour interval: $10^{-7}$ s$^{-1}$.

- in 2.5 layers, single vertical structure of unstable mode mix longwave Green type and mesoscale Charney type instability (Beckmann 1988 JPO)

# Discussion & Conclusions

- Peculiar regime of wind-forced ocean circulation: weak forcing (1/3) and moderate dissipation ⇒ Reynolds number O(1–1000), not described in the litterature as far as I know?
- 'textbook case' for the analysis of local and global instability, energetics of the unstable mode...
- but... unstable mode wavelength set by eddy viscosity

**Stronger forcing?** leads to upper layer outcropping, variability regions shift to intergyre outcropping, waves become more nonlinear

**Signature of these modes in oceanic variability?** to be compared to response to varying wind forcing (Sirven&al. 2015 JMR)

**Higher horizontal and vertical resolution?** see my poster!

THANK YOU FOR YOUR ATTENTION