Interdecadal modes of variability through a hierarchy of ocean models

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ABSTRACT

The fundamental mechanisms of interdecadal variability of the thermohaline circulation are investigated through a hierarchy of ocean models, from quasigeostrophic (QG), shallow water (SW), to planetary geostrophic (PG) and primitive equations (PE). In idealized geometry, this variability occurs spontaneously under constant (thermal or haline) forcing, and hence does not rely on so-called mixed boundary conditions. Linear stability analyses show it appears through an unstable or weakly damped linear mode. This mode is tracked through the hierarchy of ocean models, from the classical Rossby basin modes in QG models, to the deformed and recirculating baroclinic modes when a mean gyre circulation is included, to the unstable modes when large-scale baroclinic instability is enabled (SW, PG). Here the results from a 2.5-layer shallow-water model are presented, and show the emergence of large-scale low-frequency basin modes, the most weakly damped being trapped in the closed geostrophic contour pool when the mean circulation has a realistic amplitude.

1. INTRODUCTION

Numerous analysis of historical and proxy climatological time series show significant climate variability on interdecadal timescales, especially around the North Atlantic. The Atlantic Multidecadal Oscillation has been described both in observations and coupled models (Delworth and Mann 2000). On such timescales, the ocean circulation is likely a major player, given its large heat capacity and long adjustment: In fact, baroclinic planetary waves, through which the ocean circulation adjusts, do cross the Atlantic basin in a few decades at mid-latitude. Hence interdecadal variability could be linked to baroclinic basin modes, which structure has been investigated in various dynamics and configurations.

The emergence of low-frequency basin modes in reduced-gravity QG and SW dynamics was found to rely both on baroclinic Rossby waves propagation across ocean basins and their resonance through equatorial and boundary waves fast adjustment (Cessi and Primeau 2001, Primeau 2002, Ben Jelloul and Huck 2003). We investigate further if these modes coudl become unstable when the vertical structure enables large-scale baroclinic instability, and the mean flow is energetic enough and/or the dissipation small enough, as found in planetary geostrophic dynamics (Colin de Verdière and Huck 1999, Huck and Vallis 2001, te Raa and Dijkstra 2002).

The spectrum of baroclinic basin modes has been investigated in two-layer wind-driven quasigeostrophic models, the baroclinic basin modes being advected by a barotropic steady Sverdrup flow (Spydell and Cessi 2003, Ben Jelloul and Huck 2005). In the large-scale limit, i.e. for basin scale considerably larger than the Rossby radius of deformation, all the basin modes are neutral. Numerical investigations lead to three types of modes arising for wind forcing strong enough to produce a recirculating gyre with closed geostrophic contours: classical Rossby basin modes deformed by the mean flow (shadow modes), stationary modes and recirculating pool modes, the two latter trapped in the closed-contours pool. Recirculating modes could have very low frequencies for moderate recirculating gyre.

This work is extended here to the shallow water equations, with two active layers on top of a resting abyss. The mean circulation is prescribed, and the equations are linearized for perturbations. The eigenanalysis of the Jacobian matrix provides the most unstable modes structure, frequency, and growth rate. The same typology as in the 2-layer quasi-geostrophic model is found.

2. THE 2.5-LAYER SHALLOW WATER EQUATIONS

The dimensional reduced-gravity shallow water equations for two active layers of fluid (thickness h_i, density ρ_i, velocity $\mathbf{u_i}$, $i = 1$ for the upper layer and $i = 2$ for the lower layer), on top of a resting abyss (density ρ_3), read:

$$\partial_t h_1 + \nabla \cdot (h_1 \mathbf{u_1}) = F_1 \; ; \; \partial_t \mathbf{u_1} + f\mathbf{k} \times \mathbf{u_1} = -g_1' \nabla h_1 - g_2' \nabla h_2 - r\mathbf{u_1} + \nu \nabla^2 \mathbf{u_1} \quad (1)$$
$$\partial_t h_2 + \nabla \cdot (h_2 \mathbf{u_2}) = F_2 \; ; \; \partial_t \mathbf{u_2} + f\mathbf{k} \times \mathbf{u_2} = -g_2' \nabla h_1 - g_2' \nabla h_2 - r\mathbf{u_2} + \nu \nabla^2 \mathbf{u_2} \quad (2)$$

where $\mathbf{k}$ is a unit vector pointing upward, the reduced gravity $g_i' = g(\rho_3 - \rho_i)/\rho_3$, F_i the forcing (Ekman pumping, heat and freshwater fluxes) in terms of mass flux (m s^{-1}), f the Coriolis parameter (s^{-1}) varying with latitude and β its meridional gradient $\partial_y f$ (m^{-1} s^{-1}), Dissipation is parameterized through a linear (Rayleigh) friction r or a Laplacian eddy viscosity ν. Boundary conditions of no normal flow are imposed on the solid walls. Nonlinear terms in the momentum equations are neglected for the large basin-scales considered (even the time derivatives could be neglected, as in the planetary geostrophic equations). These equations are solved for a rectangular Cartesian β-plane basin, of longitudinal (meridional) extent L_x (L_y), away from the equator.

The mean state

The steady-state is imposed through the upper layer thickness (from a Stommel type single gyre analytical expression), the mean velocities are then computed from the momentum equations, and the implied forcing F_i can be diagnosed.

$$f\mathbf{k} \times \mathbf{U_1} + r\mathbf{U_1} - \nu \nabla^2 \mathbf{U_1} = -g_1' \nabla H_1 - g_2' \nabla H_2 \; ; \; F_1 = \nabla \cdot (H_1 \mathbf{U_1}) \quad (3)$$
$$f\mathbf{k} \times \mathbf{U_2} + r\mathbf{U_2} - \nu \nabla^2 \mathbf{U_2} = -g_2' \nabla H_1 - g_2' \nabla H_2 \; ; \; F_2 = \nabla \cdot (H_2 \mathbf{U_2}) \quad (4)$$

Perturbations

The linearized equations for the perturbations from the steady-state (the primes have been dropped for convenience) read:

$$\begin{aligned}
\partial_t h_1 &= -\nabla \cdot (H_1 \mathbf{u_1}) - \nabla \cdot (h_1 \mathbf{U_1}) && (5)\\
\partial_t \mathbf{u_1} &= -f\mathbf{k} \times \mathbf{u_1} - g_1' \nabla h_1 - g_2' \nabla h_2 - r\mathbf{u_1} + \nu \nabla^2 \mathbf{u_1} && (6)\\
\partial_t h_2 &= -\nabla \cdot (H_2 \mathbf{u_2}) - \nabla \cdot (h_2 \mathbf{U_2}) && (7)\\
\partial_t \mathbf{u_2} &= -f\mathbf{k} \times \mathbf{u_2} - g_2' \nabla h_1 - g_2' \nabla h_2 - r\mathbf{u_2} + \nu \nabla^2 \mathbf{u_2} && (8)
\end{aligned}$$

Associating all the variables into the state variable $X = \{h_1, u_1, v_1, h_2, u_2, v_2\}$, the linearized equations for the perturbations simply read: $\partial_t X = JX$, where J is the Jacobian operator computed for the given mean circulation. Looking for solutions of the form $X = X_0(x, y) \exp(\omega t)$, one obtains a classical eigenvalue problem: $\omega X = JX$.

Numerically, the problem is discretized using finite differencing on an Arakawa C-grid. The eigenvalue problem is solved using the Arnoldi package ARPACK available in both matlab and fortran (Lehoucq et al. 1994).

Typical parameters

$L_x = L_y = 6 \times 10^6$ m, $f_0 = 10^{-4}$ s^{-1}, $\beta = 1.6 \times 10^{-11}$ m^{-1} s^{-1}, $< H_1 >= 500$ m, $< H_2 >= 1000$ m, $g_1' = 0.03$ m s^{-2}, $g_2' = 0.01$ m s^{-2}, $r = 0$, $\nu = 3 \times 10^4$ m^2 s^{-1}.

The baroclinic modes velocity at mid-basin is then 3.2 and 0.8 cm s^{-1}, but this varies considerably with latitude.

3. THE OCEAN BASIN MODES

Baroclinic Rossby basin modes

When the mean circulation is weak (flat layer interfaces), the most weakly damped modes are the (deformed) Rossby basin modes for the first and second baroclinic modes, as found in 1.5 layer (Cessi and Louazel 2001, Primeau 2002). Because of the large variations in baroclinic wave velocity as a function of the Coriolis parameter, wave fronts are strongly tilted and the lowest frequencies are imposed along the poleward boundary.

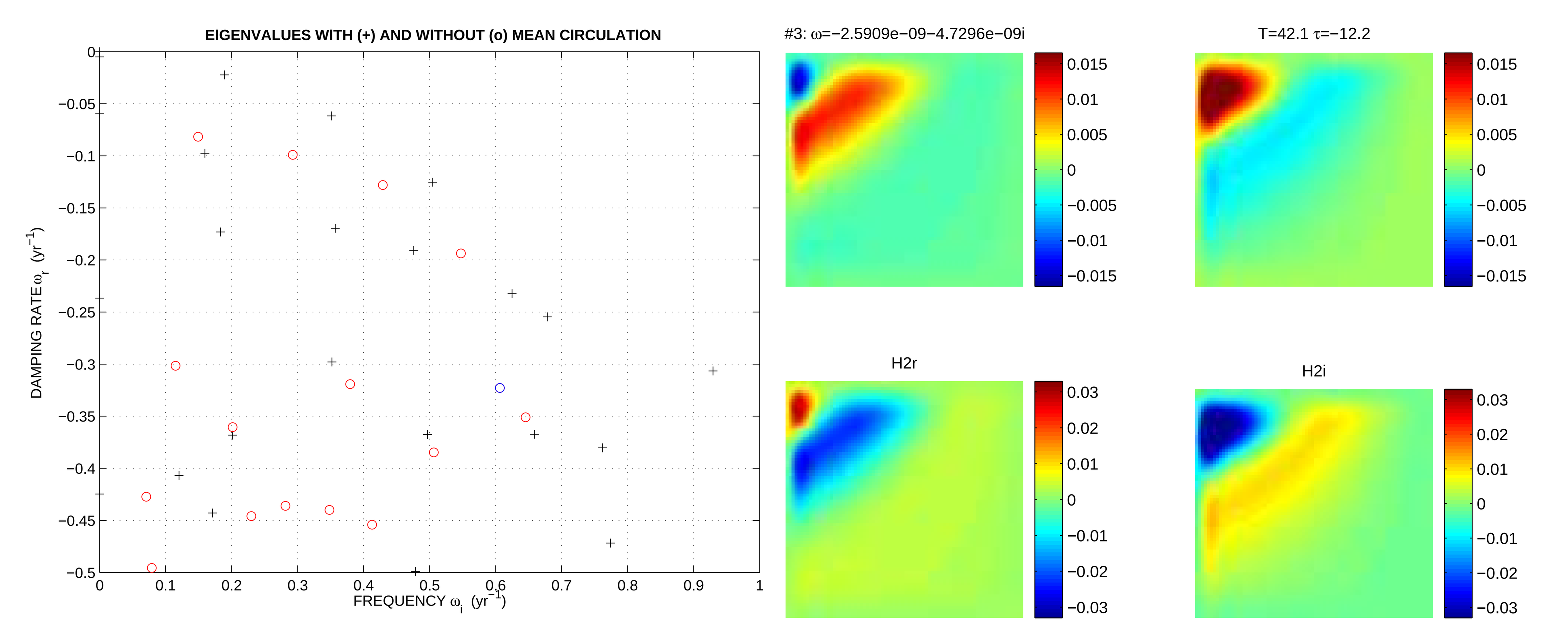

Figure 1: *(left) Eigenvalues closest to zero for a steady mean state (circle, blue for first baroclinic, red for second baroclinic mode structure), and for an upper layer deviation amplitude of 150 m (black +). Note the low damping rate of the latter recirculating pool modes compared to the former classical Rossby basin modes. (right) Most weakly damped second baroclinic mode structure for a steady mean state, with period 43 yr and damping time scale 12.2 yr.*

Recirculating pool modes

When the mean circulation is increased, the baroclinic Rossby waves propagation is influenced by the mean current (accelerated in the tropical region but slowed in the poleward half of the basin), until the second baroclinic mode can no longer propagate westward (equivalent to the closed geostrophic contours in the 2-layer QG case). Recirculating pool modes then appear trapped in the pool, alternatively advected eastward by the mean flow in the poleward region, and propagated westward as Rossby wave in the equatorward region of the pool. These modes achieve both the lowest frequencies and the lowest damping rate.

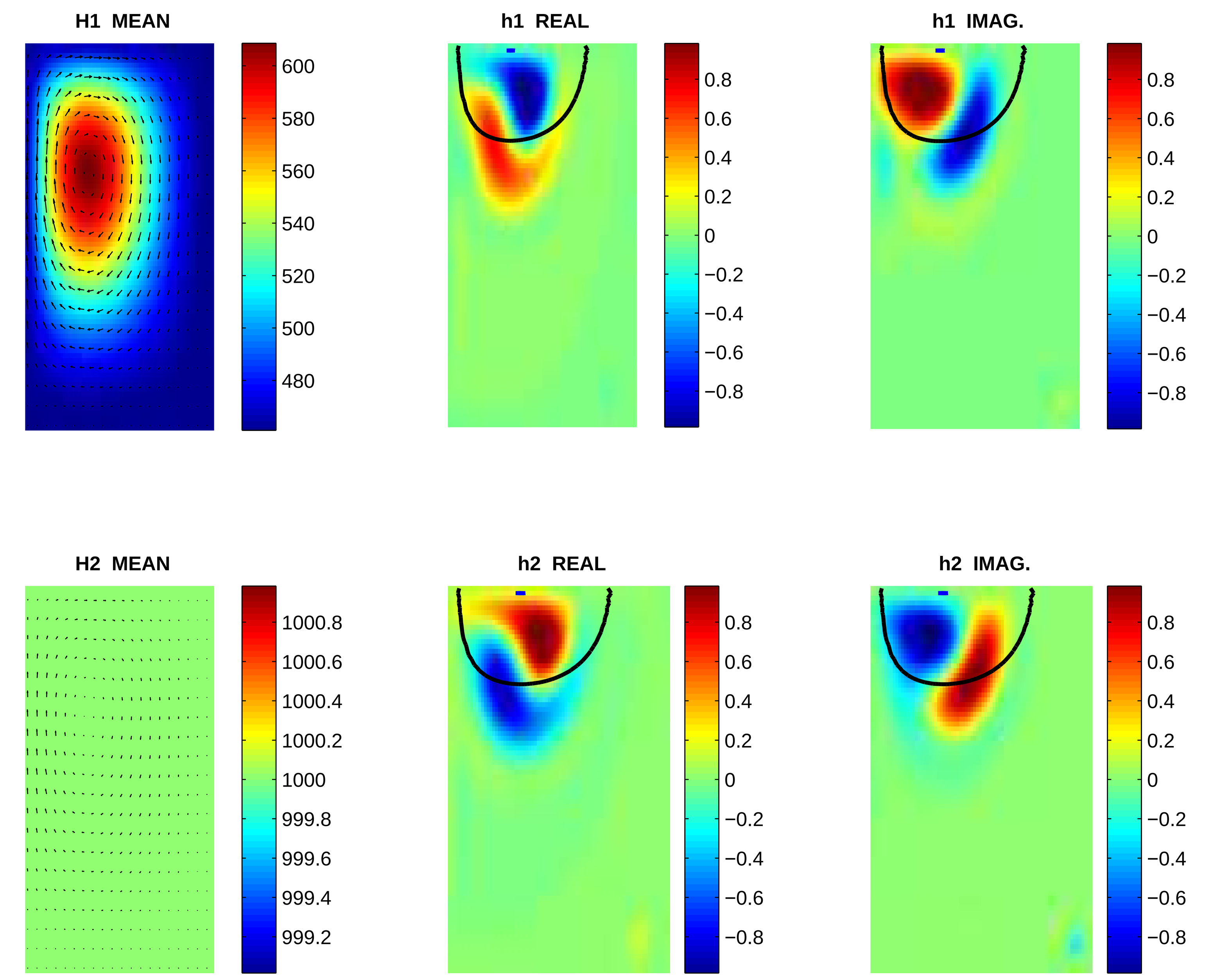

Figure 2: *Mean-state (left, in m), and recirculating pool modes real and imaginary part (a quarter period later), for the upper (top) and lower (bottom) layer (arbitrary unit), as a function of longitude and latitude. The mode period is 33 yr and its damping time scale 45 yr. The upper layer deviation amplitude is 150 m, corresponding to a single subtropical gyre flow of about 34 Sv. The thick black contour is an estimation of the recirculation pool where the second baroclinic mode westward velocity is arrested by the barotropic eastward mean flow.*

4. DISCUSSION AND CONCLUSION

For a realistic ocean stratification and mean subtropical gyre circulation imposed in a 2.5-layer shallow water model, the least damped baroclinic basin modes are large scale, low frequency modes trapped in the recirculation pool, where the second baroclinic mode is arrested by the eastward mean flow. Their oscillating period is in the interdecadal range, but is quite different from the classical Rossby basin modes westward propagation across the basin, since the mean circulation strongly influences their propagation. Huck and Vallis (2001) found that in planetary geostrophic dynamics, a linear mode, unstable through large-scale baroclinic instability, is responsible for the interdecadal variability found in ocean models forced by constant fluxes. It was expected that such unstable mode could be found in such a 2.5-layer shallow water model. Unfortunately, numerical constraints limit the range of dissipation that can be used for affordable resolution. In fact, unstable modes are associated here with small scale features that are not well resolved. We were not able to reduce dissipation enough to see if these modes eigenvalues would cross the imaginary axis and achieve positive growth rate.

These interdecadal modes may be weakly damped in realistic configurations, especially through interaction with bottom topography, and thus require stochastic noise in the atmospheric forcing to be excited. This work is presently extended to the stability analysis of the ocean general circulation in realistic configuration to provide the interdecadal mode signature in the North Atlantic.

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