

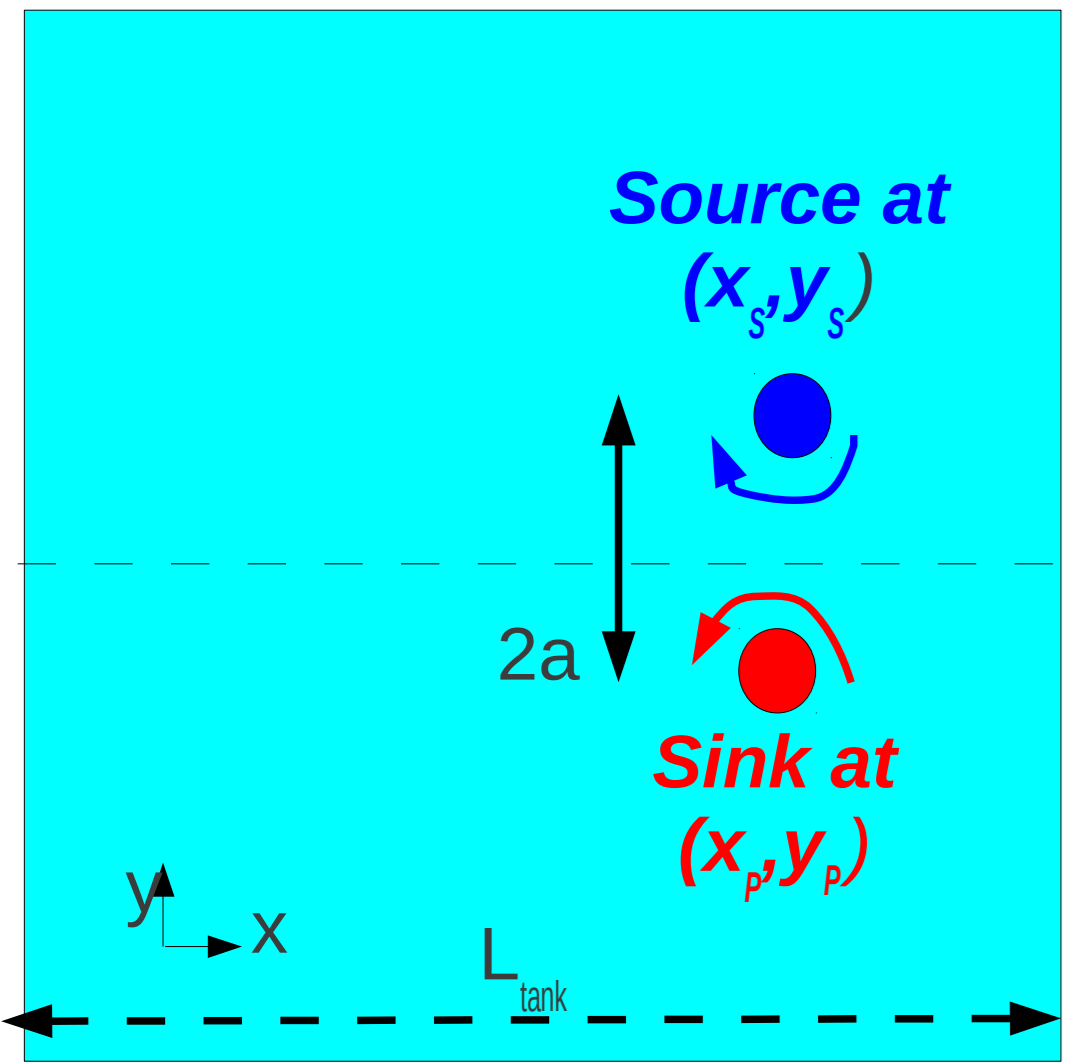
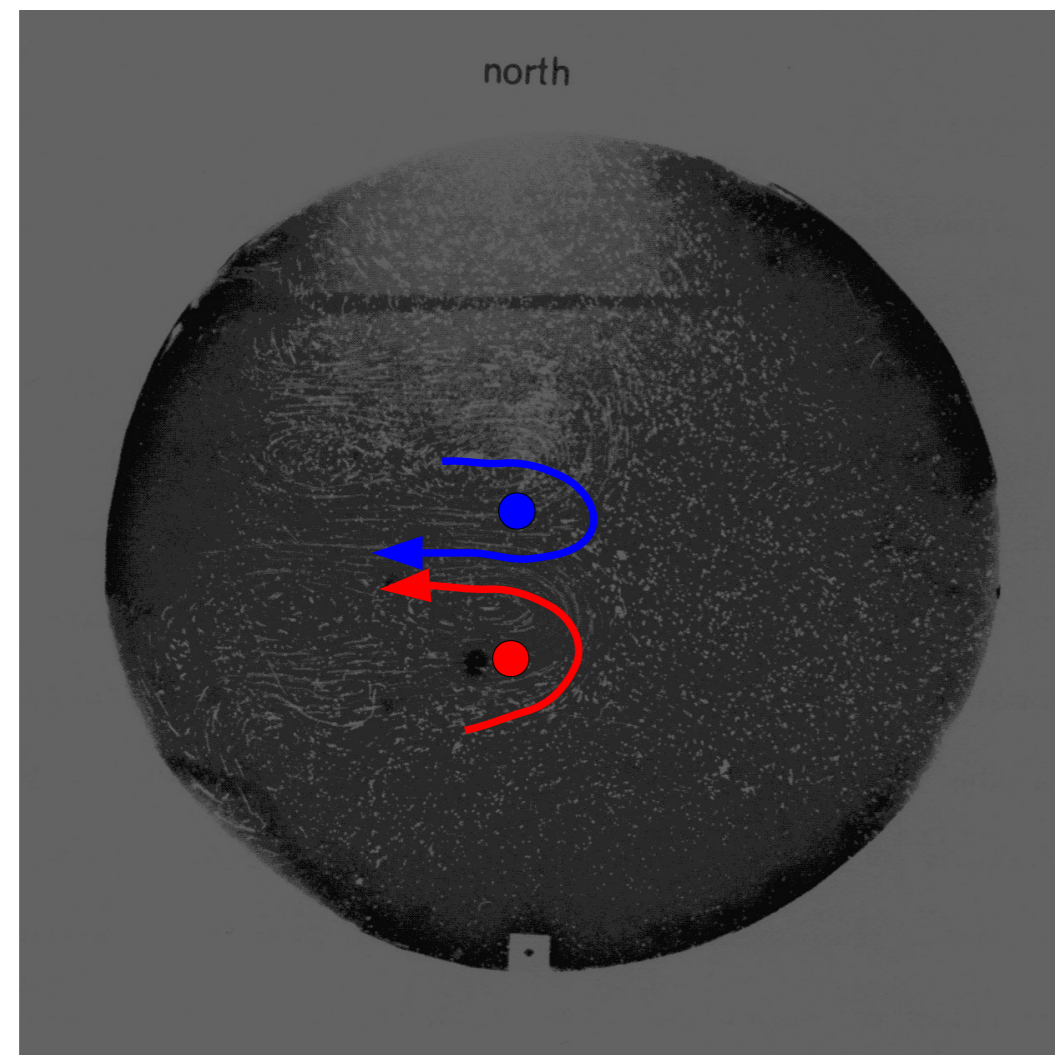
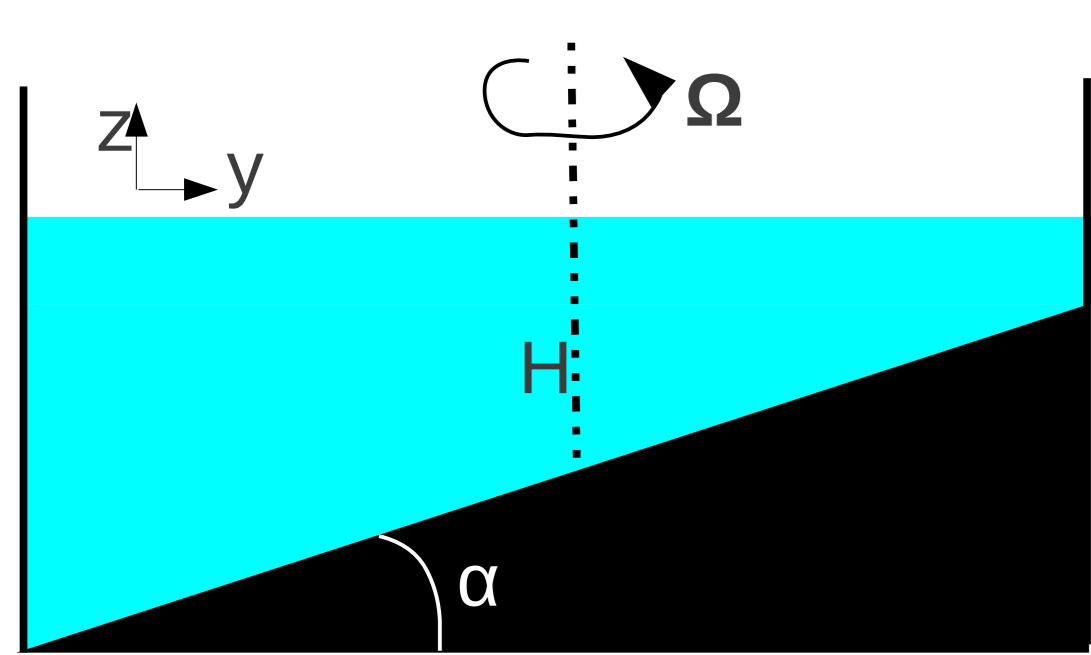
Dynamics of a dipolar gyre on a beta-plane

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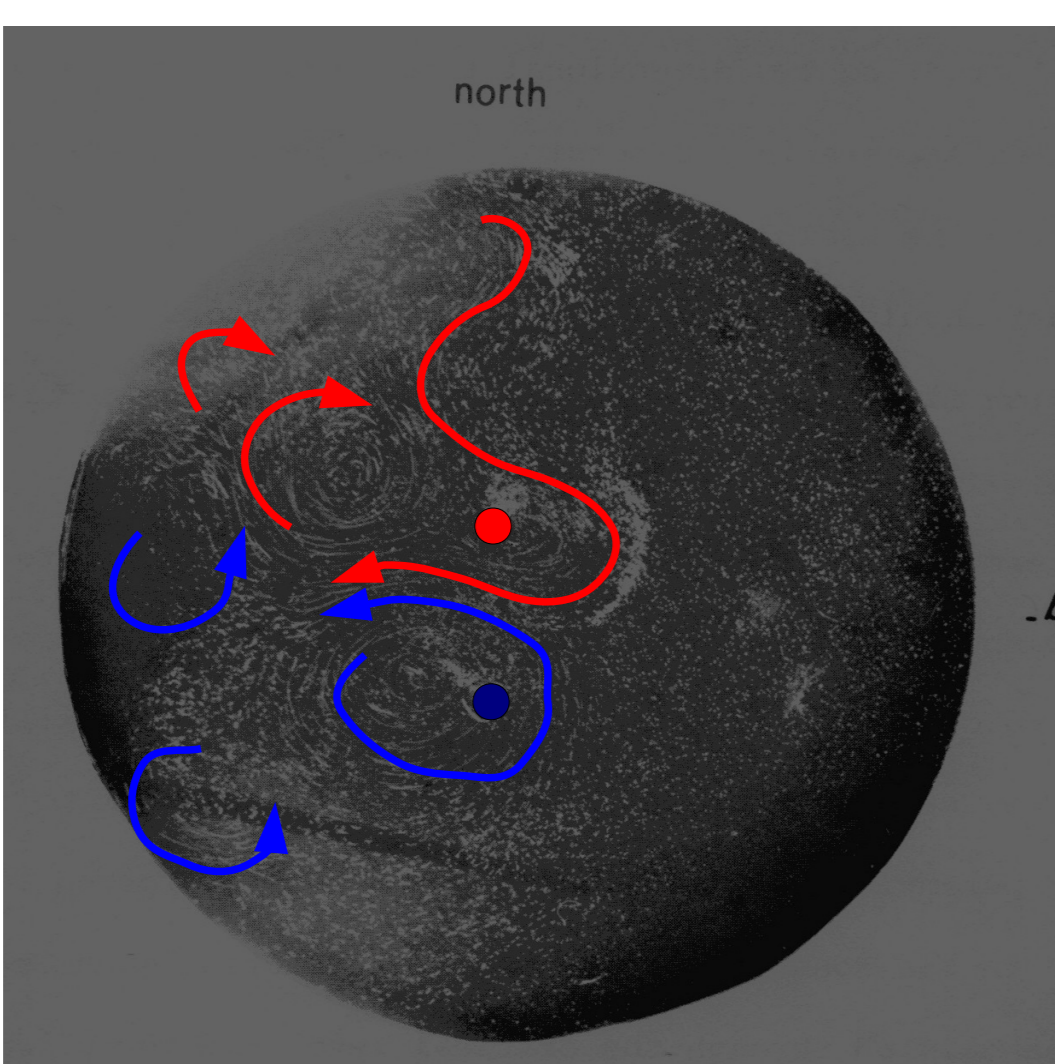
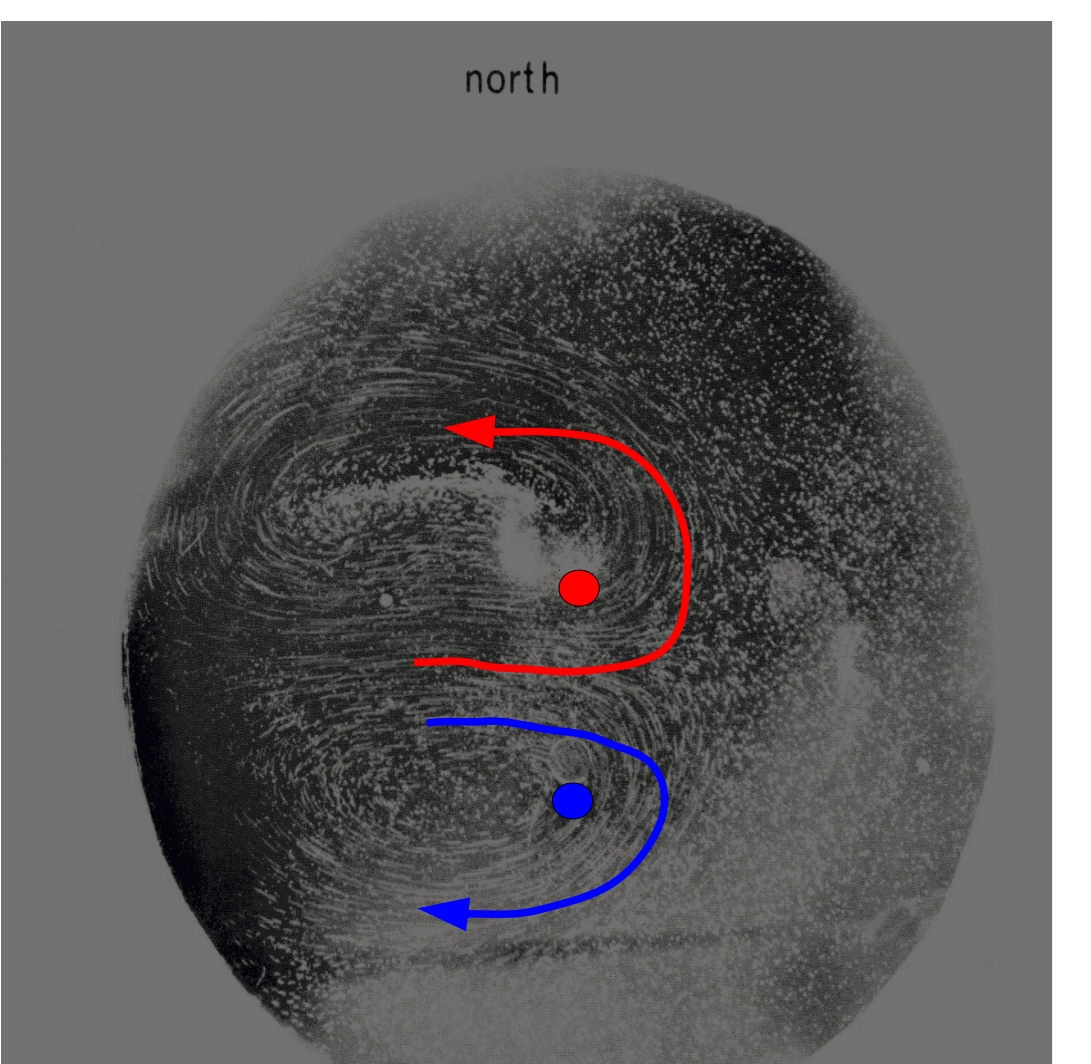
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Rotating tank experiments

The present study seeks for the understanding of a surprising phenomenon observed when using a source and a sink to force a dipolar gyre within a rotating homogeneous flow on an inclined plane. Fluid is injected (pumped out) at a source (sink) at rate F ($\text{m}^3 \cdot \text{s}^{-1}$). An anticyclonic (cyclonic) gyre is forced around the source (sink), with strong zonal jets on their external side connected through intensified western boundary currents. Increasing F leads to an instability when the source is north of the sink. When the source is south of the sink, the circulation stays stable, even at high forcing rates.



Unstable: $U/(\beta L^2)=1$. Western boundary currents overshoot and undulations appear when the western boundary current separates from the coast. Courtesy of A. Colin de Verdière.



Stable: $U/(\beta L^2)=10$, Courtesy of A. Colin de Verdière.

Unstable: $U/(\beta L^2)=10$, Courtesy of A. Colin de Verdière.

Numerics

Numerical simulations ($\delta x=2$ mm) are carried out using a barotropic shallow-water version of MICOM: $H=0.37$ cm, $\alpha=7^\circ$, $L_{\text{tank}}=0.5$ m, $\tau_{\text{tank}}=8$ s. Linear friction ($r=0.006$ s^{-1}) and biharmonic dissipation ($\nu=10^{-6}$ $\text{m}^2 \cdot \text{s}^{-1}$) are added to the momentum equation. All relevant boundary layers are resolved.

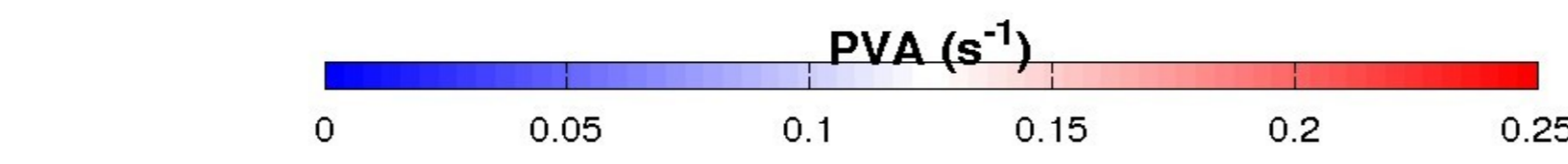
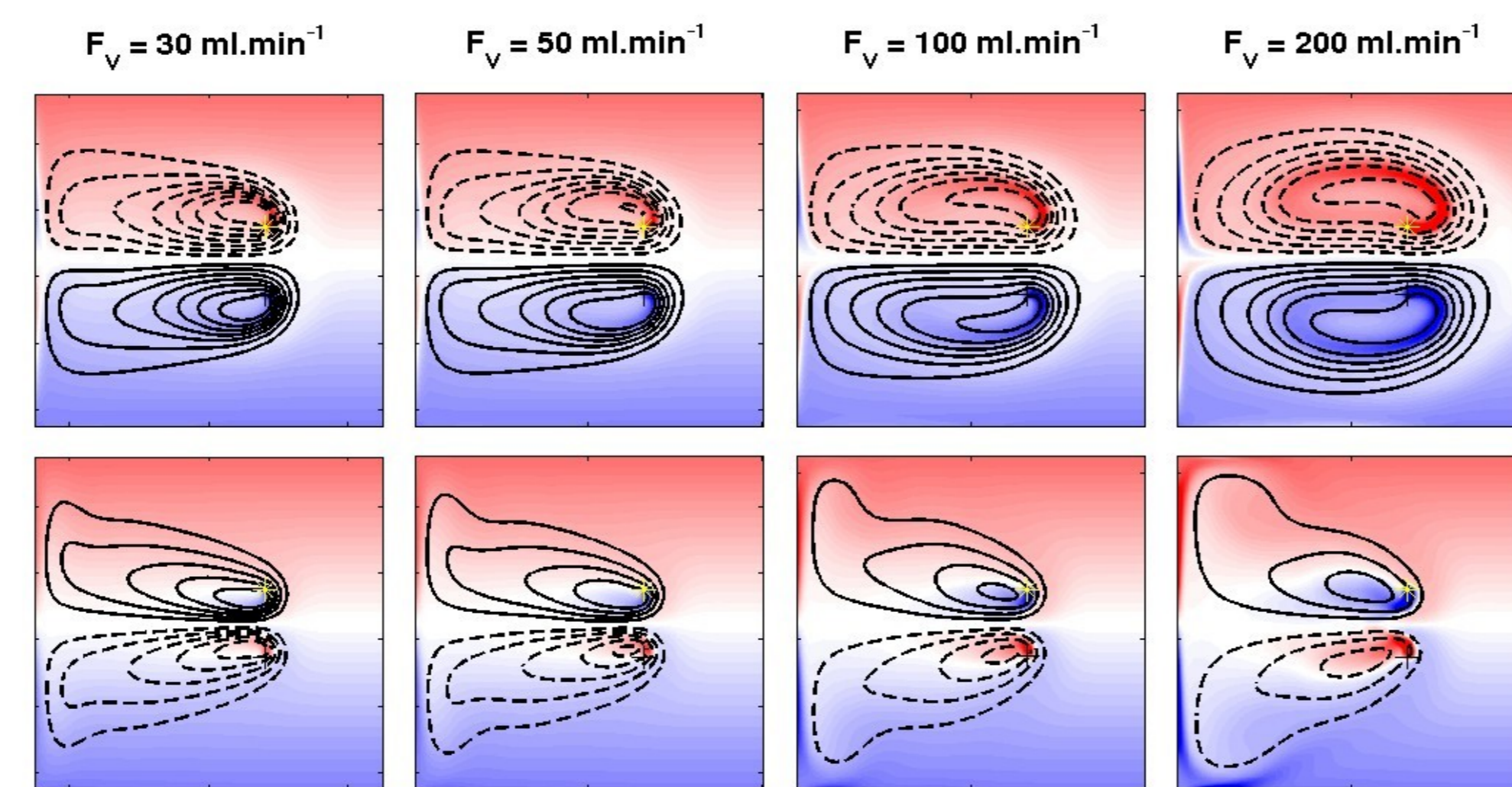
Inertial $\delta_I = \sqrt{\frac{U}{\beta}}$ Stommel $\delta_S = \frac{r}{\beta}$ Munk $\delta_M = \left(\frac{\nu}{\beta}\right)^{\frac{1}{3}}$

~ 1.24 cm

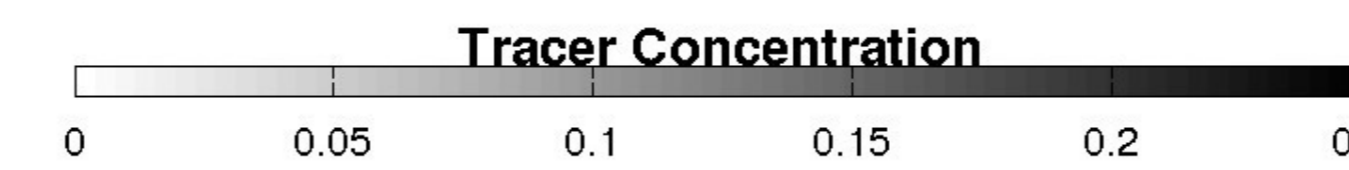
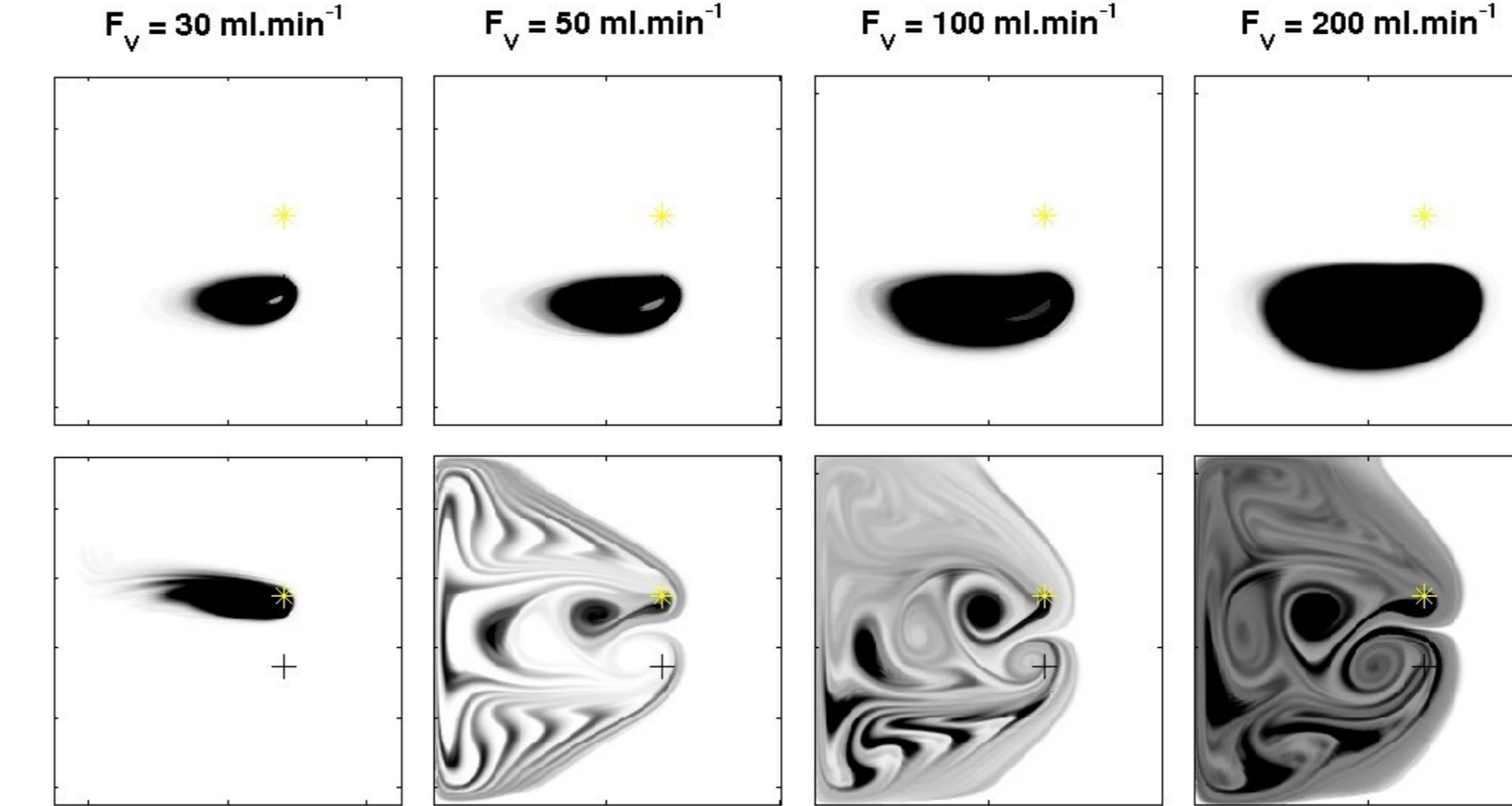
$s = \tan(\alpha)$
 $\beta = \frac{f_0 s}{H}$
 $f_0 = \frac{4\pi}{\tau_{\text{tank}}}$

Linear versus non-Linear dynamics

Increasing the forcing rate leads to the instability of the westward jet configuration: many eddies grow inside the zonal central jet, propagate towards the west and then get dissipated as they enter the western boundary currents, causing exchange of tracer between the two gyres.



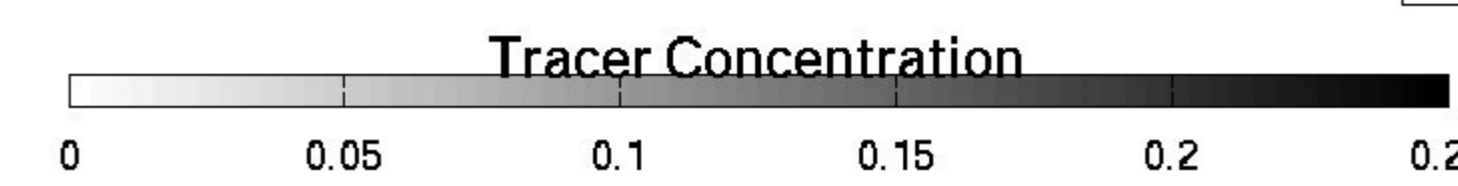
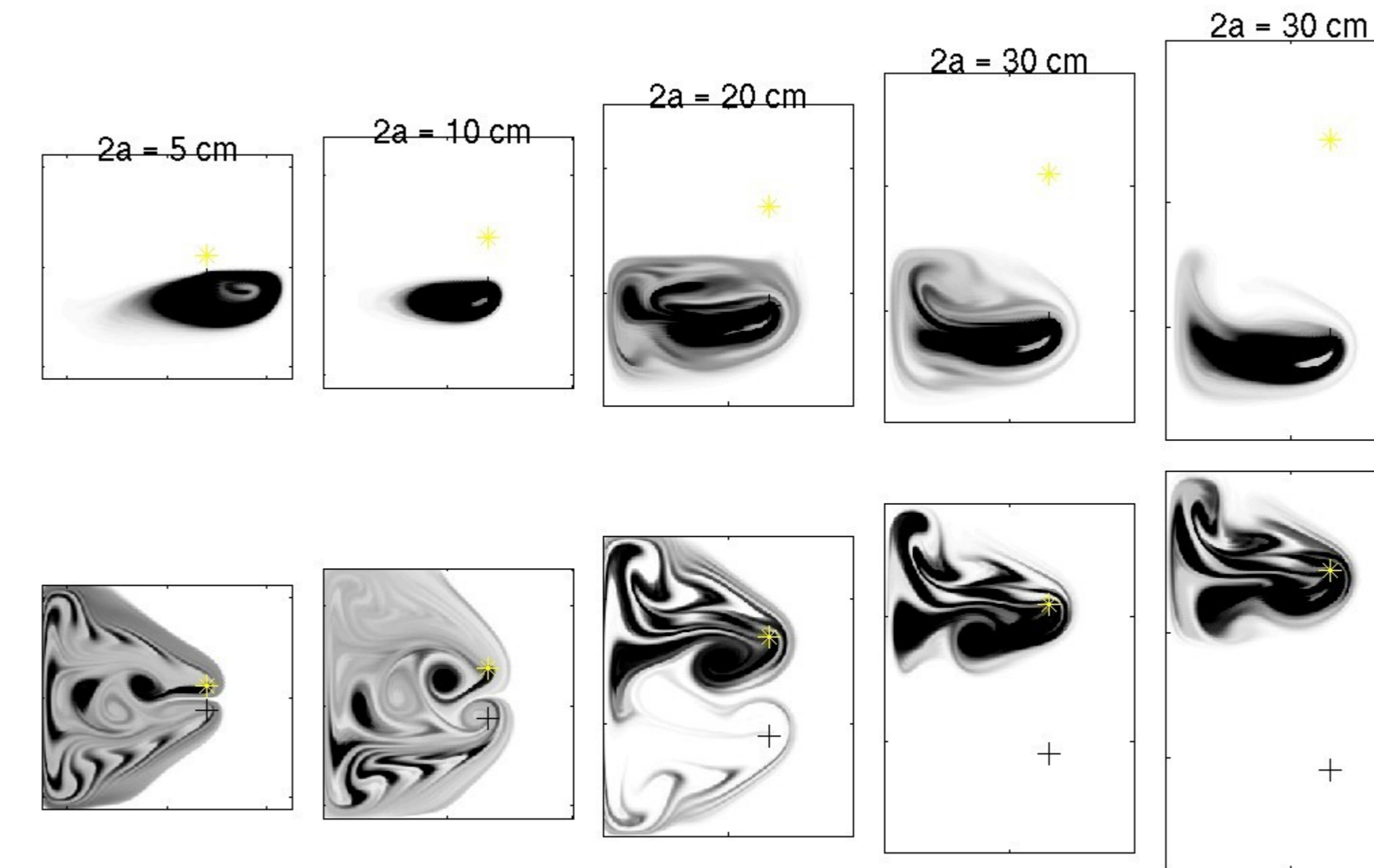
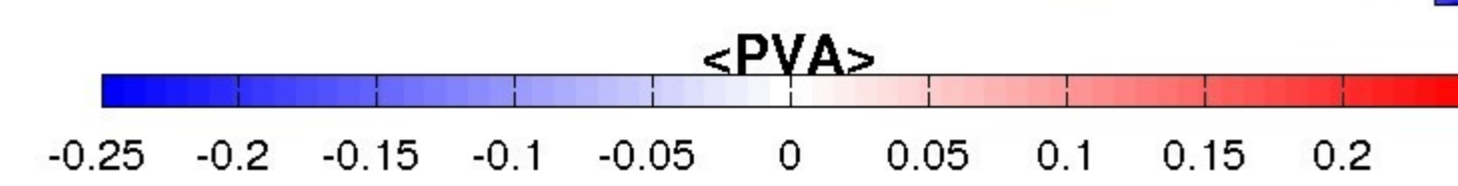
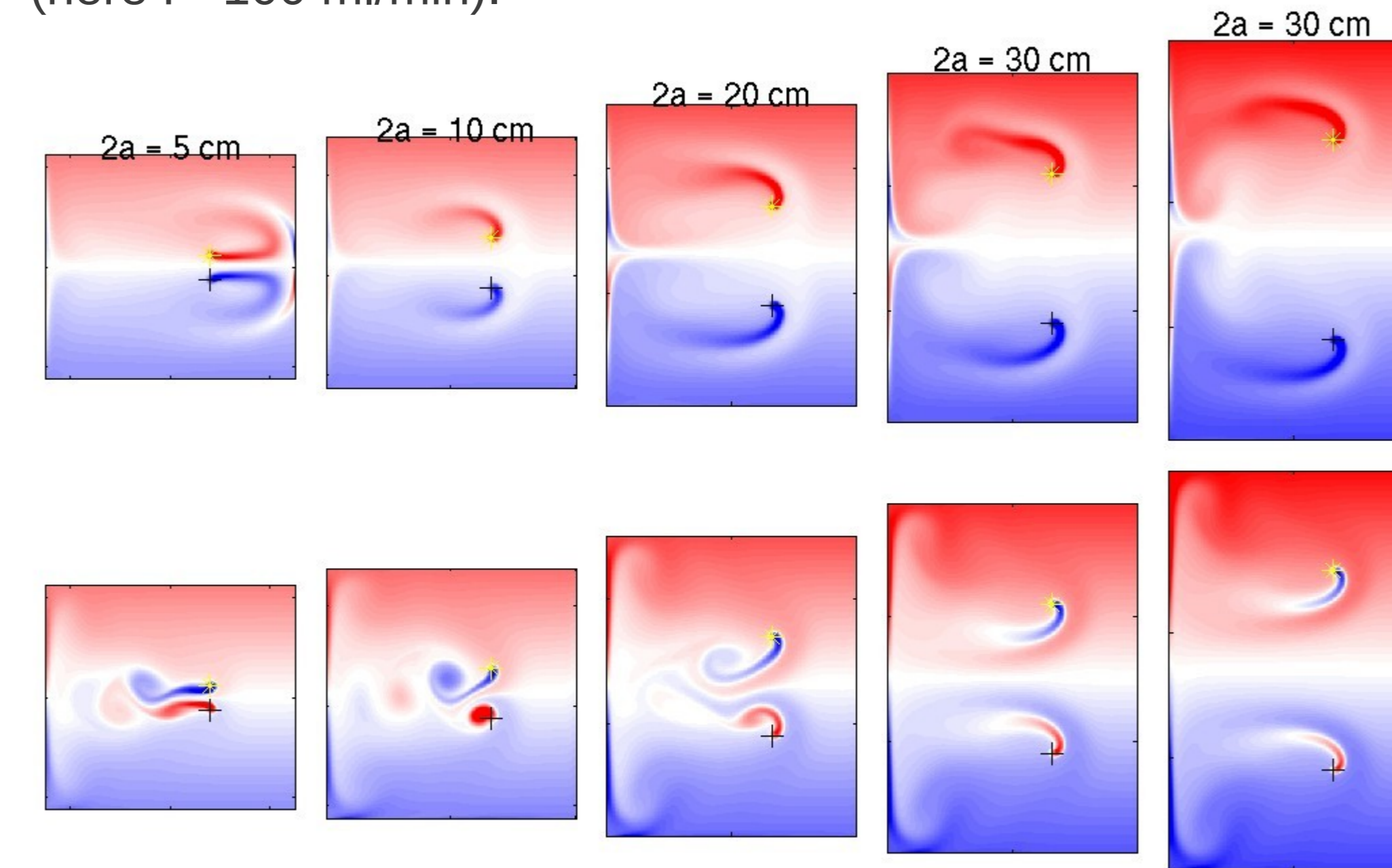
$\frac{U}{\beta L^2} = 0.3$ $\frac{U}{\beta L^2} = 0.4$ $\frac{U}{\beta L^2} = 0.5$ $\frac{U}{\beta L^2} = 0.7$



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Sensitivity to the distance separating the source/sink

The instability of the westward jet configuration can be inhibited by increasing the distance between the source and the sink (here $F=100$ ml/min).



Non dimensional vorticity equation

$$\frac{\delta_I^2}{L^2} \times \left(\partial_t \zeta + u \cdot \partial_x \zeta + v \cdot \partial_y \zeta \right) + v = -\frac{\delta_S \zeta}{L} + \frac{\delta_M^3}{L^3} \nabla^2 \zeta \dots$$

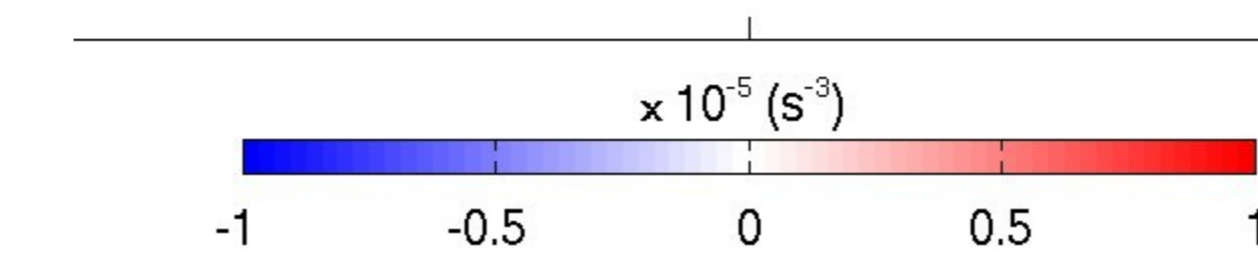
$$-\frac{F}{\beta L H} \times \left(\delta(\mathbf{x} - \mathbf{x}_{\text{source}}) - \delta(\mathbf{x} - \mathbf{x}_{\text{sink}}) \right).$$

Origin of the instability

The spatial origin of the instability may either be in the western boundary current, in the region near the source and sink, or in the westward jet. Counter mean potential vorticity gradient eddy fluxes suggest that the instability grows within the westward jet.

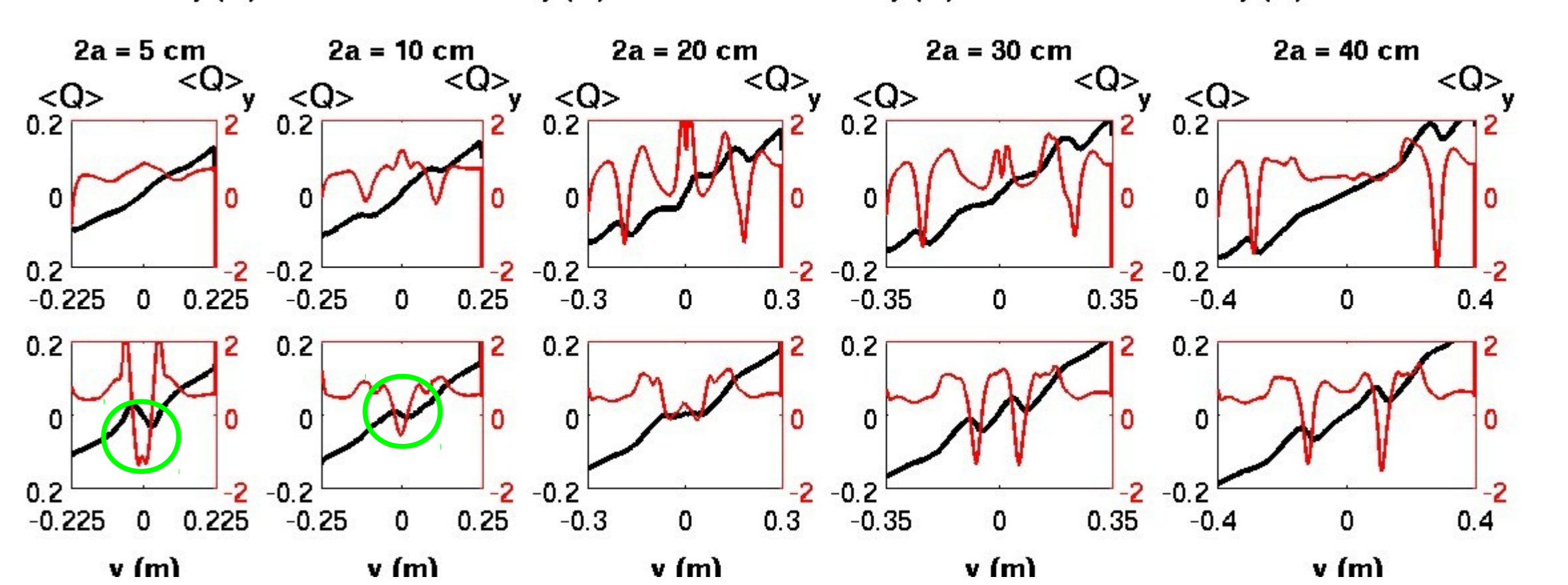
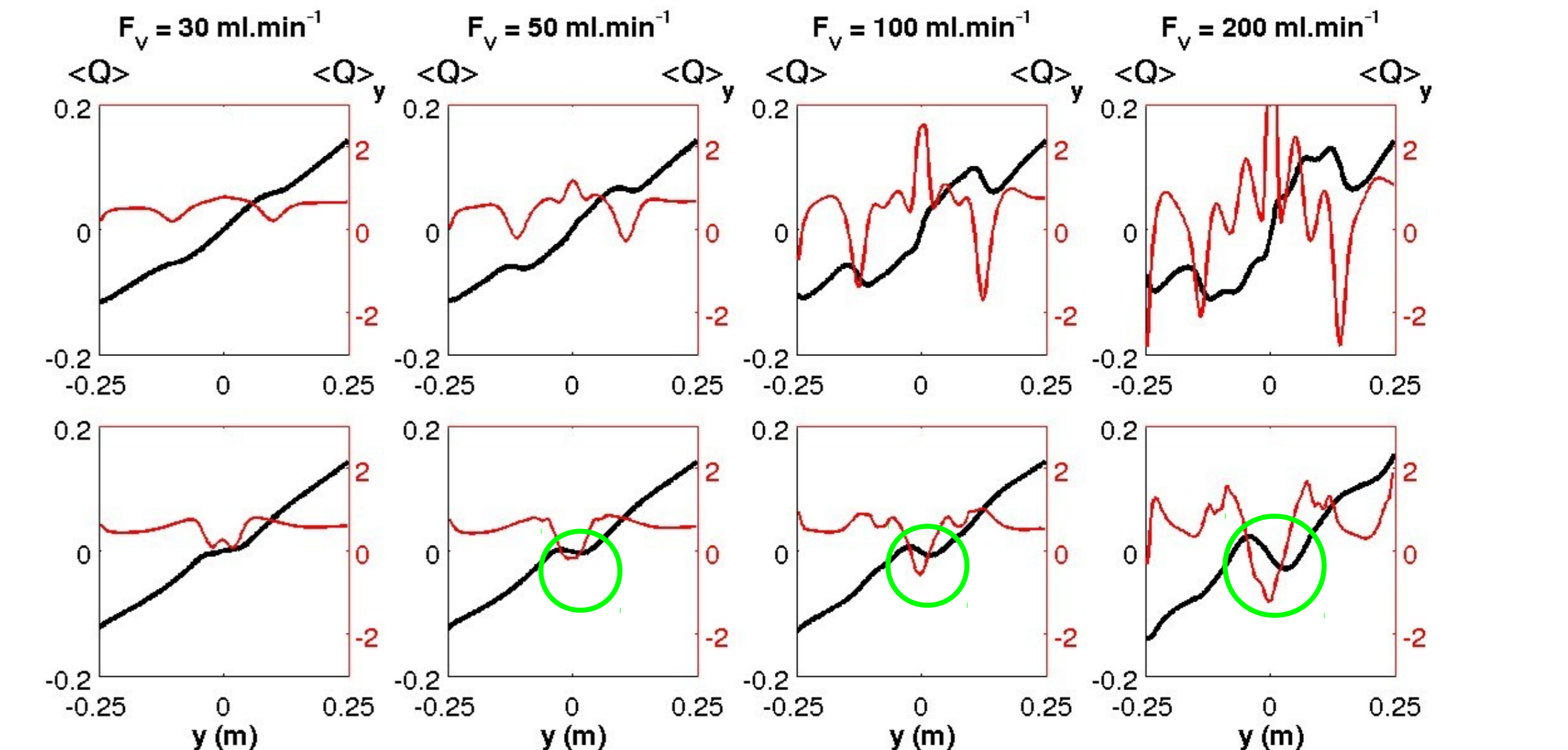
$$\left\langle \frac{\partial(q')^2}{\partial t} \right\rangle + \left\langle q'(\mathbf{u}' \cdot \nabla) \bar{Q} \right\rangle + \left\langle (\mathbf{U} \cdot \nabla) q'^2 \right\rangle + \dots$$

$$\left\langle (\mathbf{u}' \cdot \nabla) q'^2 \right\rangle = \left\langle q' D'_{\text{fric}} \right\rangle + \left\langle q' D'_{\text{vis}} \right\rangle + \left\langle q' F' \right\rangle$$



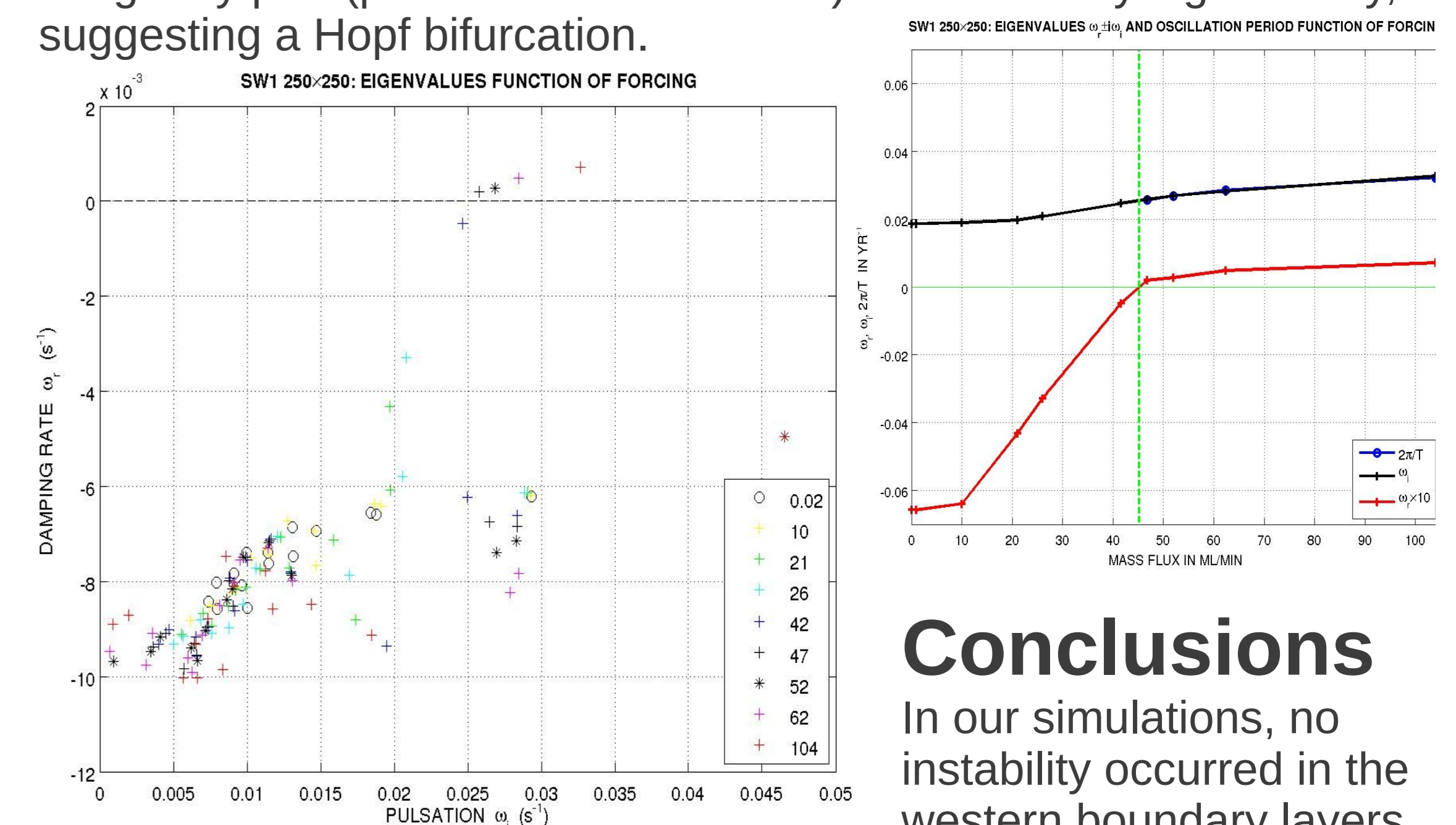
Mechanism of the instability

The Charney-Stern criteria for barotropic instability proves to be verified within the westward zonal jet of our unstable circulations: Uyy-β changes sign.



Linear stability analysis

The eigen value spectrum confirms the presence of an unstable mode above a threshold forcing. As the mode becomes unstable, the real part of the complex eigen value changes sign. Nevertheless, its imaginary part (period of the oscillation) does not vary significantly, suggesting a Hopf bifurcation.



Conclusions

In our simulations, no instability occurred in the western boundary layers, although western boundary currents played a major role in the exchange of tracer between the two gyres. Having distributed sources and sinks would certainly better mimic the ocean circulation. Nevertheless, a prerequisite to rationalize the much more complex eddy wind driven gyres or abyssal circulation is to understand the dynamics of such simple systems.

According to theory, the amplitude of the oscillations should grow as $\sqrt{F-F_c}$ around the Hopf bifurcation.