Session 3309

Climate Dynamics

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Nonlinear interaction between Sverdrup flow and basin modes in the quasi-geostrophic wind driven circulation

Mahdi Ben Jelloul and Thierry Huck

Laboratoire d'Océanographie Dynamique, Univ. Paris VI, France

Laboratoire de Physique des Océans, Univ. Bretagne Occidentale, Brest, France

benjello@lodyc.jussieu.fr, thuck@univ-brest.fr.

ABSTRACT

The linear solution of the barotropic and baroclinic quasi-geostrophic wind driven circulation are decomposed in a steady forced solution and a time dependent component. The steady wind forced solution consists in a classical Sverdrup flow dissipated in the western boundary layer where the viscosity is active while the homogeneous time dependent solution is a sum of oscillatory modes with arbitrary amplitudes.
The effect of the nonlinear terms is handled through a weakly nonlinear analysis providing a set of evolution equations for the modes amplitudes. We treat here the barotropic case. It can be proven that mode stability is related to the wind stress symmetry. Pure basin modes interactions yields triads with cycling energy and sub-harmonic instabilities.

1. INTRODUCTION

From numerous time series of climate variability , it is now clear that there is significant variability on interannual to interdecadal time scales. Several paradigms apply to this low frequency variability of the climate system, from external forcing variability (solar cycles, volcanic eruption, atmospheric composition), to integration of atmospheric white noise by the ocean into a red spectrum frankignoul77:-stoch-climat-model, to intrinsic modes of variability of the atmospheric james94:-wave-zonal-flow-inter-ultra oceanic, or coupled systems. To the extent that the ocean intrinsic modes play an important role, identifying their dynamical nature is crucial for climate prediction. As a first step, we study the time dependent wind driven quasi-geostrophic circulation.

2. ONE LAYER QUASI-GEOSTROPHIC CASE

One layer quasi-geostrophic dynamics

The nondimensionnal one layer quasi-geostrophic evolution equation reads

$$\partial_t\left(\nabla^2\psi - \mathsf{Bu}^{-1}\psi\right) + \beta\partial_x\psi + \epsilon J\left(\psi, \nabla^2\psi\right) = W_E + D, \qquad (1)$$

where W_E is a function with maximum amplitude of one and D the weak dissipative processes. Additional nondimensional parameters are the Burger number and a small parameter $\epsilon \ll 1$ controlling the inertial nonlinearities,

$$\mathsf{Bu} = \frac{R_d^2}{L^2}, \quad \epsilon = \frac{\tau_0(\rho\beta_0)^{-1}}{(\beta L_x^3)},$$

where $R_d = \sqrt{g'H}/f_0$ is the Rossby deformation radius, g' the reduced gravity and τ_0 is a characteristic value of wind stress amplitude. Note that the nondimensional parameter $\beta = O(1)$ is kept in (1) to tract the origin of the term involved in the following sections algebra.
No normal flow at the boundaries and mass conservation constraint are used

$$\forall\, \mathbf{x} \in \delta\mathcal{D},\ \psi(\mathbf{x}) = \psi_b(t), \quad \iint_{\mathcal{D}} dx\, dy\, \psi = 0. \qquad (2)$$

The spatial domain of integration $\mathcal{D}$ will be assumed rectangular with an aspect ratio denoted by r.

Weakly nonlinear expansion: Linear decomposition

We solve by weakly non-linear expansion for ψ and multiple time scale expansion using

$$\psi = \psi_0 + \epsilon\psi_1 + \epsilon^2\psi_2 + \ldots, \quad \partial_t = \partial_{t_0} + \epsilon\partial_{t_1} + \epsilon^2\partial_{t_2} + \ldots. \qquad (3)$$

The flow therefore is split into two components:

- A stationary wind-forced Sverdrup solution

$$\bar{v}_0 = \partial_x\bar{\psi}_0 = \beta^{-1}W_E - \beta^{-1}\delta(x)\int_0^1 dx\, W_E, \qquad (4)$$

where δ is the Dirac distribution and represents the strong localized western boundary current.

- A discrete spectrum of basin modes solutions of

$$\partial_{t_0}\left(\nabla^2\tilde{\psi}_0 - \mathsf{Bu}^{-1}\tilde{\psi}_0\right) + \beta\partial_x\tilde{\psi}_0 = 0, \qquad (5)$$

with the mass conservation constraint (2).
For the case with no surface deviation , i.e. $\mathsf{Bu}^{-1} = 0$, we get from pedlosky87-geoph classical textbook,

$$\Phi_\Omega = D_\Omega e^{\frac{i\beta x}{2\Omega}} \sin m\pi x \sin\frac{n\pi y}{r}, \quad \Omega = \frac{\beta}{2\pi\sqrt{m^2 + n^2r^{-2}}}, \quad D_\Omega = \frac{(4\Omega)^2}{\beta^2},$$

where D_Ω is computed such that the modes provides an orthonormal base for energy norm. Note that low frequency modes are those with large wavenumbers.
However, large scale baroclinic modes correspond to small values of Bu. Solving them numerically shows that the low frequency modes are those with low wavenumbers, in contrast with the previous case.

Bu	1	10^{-2}	10^{-4}	Bu	1	10^{-2}
1x1	68	1.38	1.04	1x2	155	1.56
2x1	247	1.57	0.53	2x2	1753	
3x1		1.73	0.36	3x2		1.96
4x1		2.00	0.28	4x2		2.29

Table: *Basin mode period as a function of the mode spatial structure for different Burger. Wavenumbers are indicated using the scheme (zonal x meridional).*

Weakly nonlinear expansion: Amplitude equations

The first order solution is then

$$\psi_0 = \bar{\psi}_0 + \tilde{\psi}_0 = \bar{\psi}_0 + \sum\left[A_0(\Omega, t_1, \ldots)\Phi_\Omega e^{i\Omega t_0} + \text{c.c.}\right],$$

where the slow evolution (over times t_i, $i \geq 1$) of the amplitudes $A_0(\Omega, t_1, \ldots)$ results from nonlinear dynamics. The amplitude equations are obtained as solvability conditions of the resonances elimination process. Three type of "three wave" resonances can occur in this system

- Self interaction of a mode via the Sverdrup flow ($\Omega_0 + 0 = \Omega_0$),
- Triad of distinct modes ($\Omega_1 + \Omega_2 = \Omega_0$),
- Biharmonic interaction ($\Omega_1 - \Omega_0 = 2\Omega_0 - \Omega_0 = \Omega_0$).

The evolution equation is then

$$\partial_{t_1}A_0(\Omega_0) = \underbrace{a_1A_0(\Omega_0)}_{\text{Sverdrup flow}} + \underbrace{a_0^{12}A_0(\Omega_1)A_0(\Omega_2)}_{\text{Triad}} + \underbrace{a_{2,-1}A_0(2\Omega_0)A_0^*(\Omega_0)}_{\text{Subharmonic}},$$

Since the triadic interaction, the other interactions being absent, will lead to nonlinear oscillations conserving both enstrophy and energy, we will focus on the other interactions. To lighten the notation we put $\Omega = \Omega_0$ in the following.

Sverdrup flow instability The key parameter to compute is

$$a_1 = (2r)^{-1}\iint_{\mathcal{D}} dx\, dy\left[\nabla^2\bar{\psi}_0 J\left(\Phi_\Omega^*, \Phi_\Omega\right) + \bar{\psi}_0 J\left(\nabla^2\Phi_\Omega, \Phi_\Omega^*\right)\right].$$

The imaginary part of the coefficient a_1 corresponds to a small correction to the mode frequency. The real part which is the growth rate of the modes is computed to be

$$2\Re(a_1) = a_1'' + \text{c.c.} = -i(2r\Omega)^{-1}\iint_{\mathcal{D}} dx\, dy\, J\left(\bar{\psi}_0, \beta y\right) J\left(\Phi_\Omega, \Phi_\Omega^*\right).$$

It can be easily proved that mirror-symmetric single gyre are stable. The instability takes its source in the antisymmetric component of the Sverdrup flow. Furthermore, it can be proved that for an antisymmetric gyre, only gyre with an anticyclonic gyre north of a cyclonic gyre are unstable. This latter case presents a latitudinal gradient of relative potential vorticity opposed to the planetary gradient.

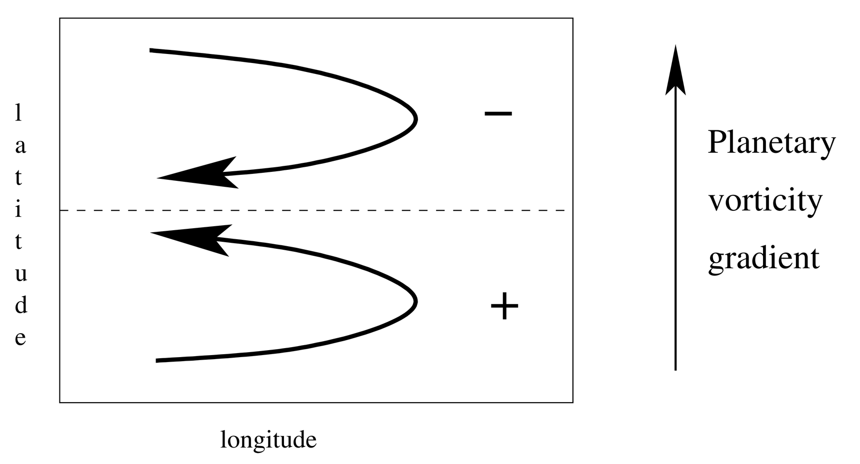

Figure 1: *Unstable double gyre. The plus and minus signs indicate the vorticity anomaly due to the Sverdrup gyre.*

Period doubling instability Every system of the following form

$$\partial_t A = \alpha A^* + i\nu A, \qquad \alpha \in \mathbb{C}^*, \nu \in \mathbb{R}$$

is know to be unstable for sufficiently low $\nu/|\alpha|$. Thus if the coefficient $a_{2,-1}$ is non zero, every mode with frequency 2Ω will undergo a period doubling instability by feeding the mode with frequency Ω. This phenomena have been numerically observed by cessi01:-excit-basin-modes-ocean-atmos-coupl and provides a nonlinear mechanism for climate spectra reddening. Note that a complete spectrum can be computed from the amplitude equations once all the resonances are computed.

3. TWO-LAYER QUASI-GEOSTROPHIC CASE

Two layers quasi-geostrophic dynamics

We decompose the flow in its barotropic and baroclinic components

$$\psi_{bt} = \delta_1\psi_1 + \delta_2\psi_2, \quad \psi_{bc} = \psi_2 - \psi_1.$$

The nondimensionnal two layers quasi-geostrophic evolution equation reads

$$\partial_t\nabla^2\psi_{bt} + \beta\partial_x\psi_{bt} + \mathsf{Bu}J\left(\psi_{bt}, \nabla^2\psi_{bt}\right) + 2\mathsf{Bu}\delta_2\delta_1 J\left(\psi_{bc}, \nabla^2\psi_{bc}\right) - W_E - D_{bt} = 0,$$

for the barotropic mode while the baroclinic one verifies

$$\partial_t\left[\mathsf{Bu}\nabla^2\psi_{bc} - \psi_{bc}\right] + \mathsf{Bu}\beta\partial_x\psi_{bc} + \mathsf{Bu}J\left(\psi_{bc}, \psi_{bt}\right) + \mathsf{Bu}^2\left(\delta_1^2 - \delta_2^2\right)J\left(\psi_{bc}, \nabla^2\psi_{bc}\right) + \mathsf{Bu}^2 J\left(\psi_{bc}, \nabla^2\psi_{bt}\right) + \mathsf{Bu}^2 J\left(\psi_{bt}, \nabla^2\psi_{bc}\right) + \mathsf{Bu}\left[\delta_1^{-1}W_E + D_{bc}\right] = 0,$$

where time is rescaled by the characteristic time scale of barotropic waves $T_0 = (\beta_0 L)^{-1}$; W_E is the Ekman pumping, D_i's represent diss processes and

$$\mathsf{Bu} = \frac{R_d^2}{L^2}, \quad R_d = \frac{\gamma_1 H_1 H_2}{f_0^2 H}, \quad \delta_i = \frac{H_i}{H}, \quad \gamma_1 = \frac{\rho_2 - \rho_1}{\rho_0}g.$$

The boundary conditions are

$$\forall\, \mathbf{x} \in \delta\mathcal{D},\ i \in (bt, bc),\ \psi_i(\mathbf{x}) = \psi_{b,i}(t),$$

yielding the mass conservation constraint

$$\iint_{\mathcal{D}} dx\, dy\, (\psi_1 - \psi_2) = \iint_{\mathcal{D}} dx\, dy\, \psi_{bc} = \text{constant}.$$

The barotropic stream function boundary condition is taken to be ψ_{bt} the domain frontier $\delta\mathcal{D}$.

Large scale approximation: Bu $= O(\epsilon) \ll 1$

We solve this system by a weakly non-linear expansion for the stream fu of the form

$$\psi_{bt} = \psi_{bt,0} + \epsilon\psi_{bt,1} + \epsilon^2\psi_{bt,2} + \ldots, \quad \psi_{bc} = \psi_{bc,0} + \epsilon\psi_{bc,1} + \epsilon^2\psi_{bc,2}$$

assuming a multiple time scale expansion as in (3).
At first order, we obtain that baroclinic Rossby modes have slower ev time scale than barotropic ones which obeys (5).
At next order in $\mathsf{Bu} = O(\epsilon)$, we obtain that the barotropic steady flow the Sverdrup balance (4) since the baroclinic component is $\bar{\psi}_{bc,0} = -\delta$ which corresponds to a resting lower layer (in absence of dissipation).
The baroclinic flow also admits a time varying component on time sca

$$\partial_{t_1}\left[\mathsf{Bu}\nabla^2\tilde{\psi}_{bc,0} - \tilde{\psi}_{bc,0}\right] + J\left(\tilde{\psi}_{bc,0}, \bar{\psi}_{bt,0} + \beta y\right) = 0.$$

Note that we kept the higher order linear dispersive term for regulari poses. Solutions of (7) are all oscillatory and form an orthogonal the basin. This result is a direct consequence of the inhibition of ba instability at large scale where potential and kinetic energy are decou

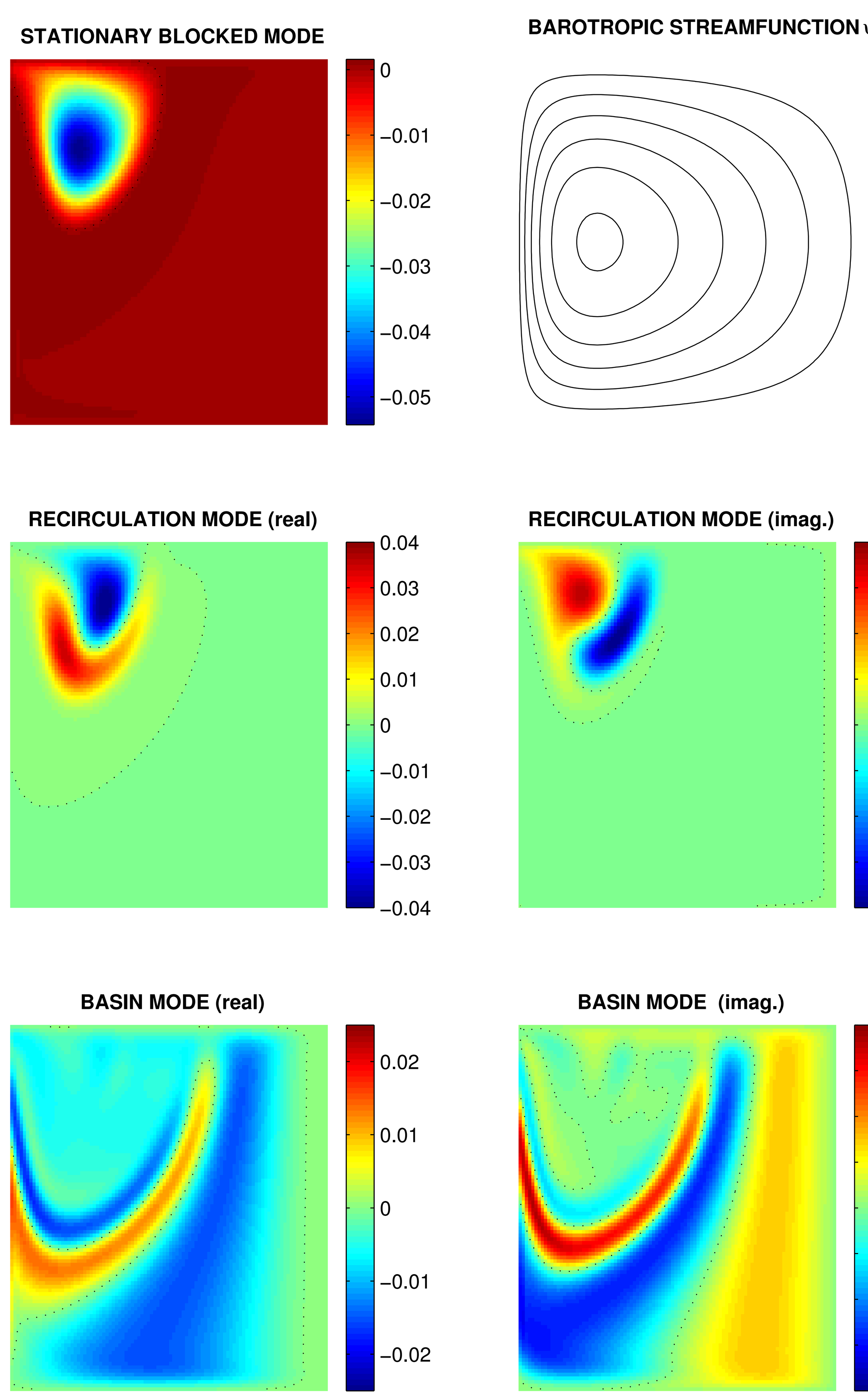

Figure 2: *Stationary modes (upper left), recirculating modes (middle), anc modes (lower) in a subtropical gyre (upper right). Real and imaginary par oscillatory modes are displayed.*

These baroclinic modes are of three type

- Rossby basin modes deformed by the barotropic Sverdrup flow,
- Recirculating modes, localized in the recirculating gyre, whose freq lies outside the Rossby wave range (lower frequencies),
- Stationary modes, localized in the recirculating gyre, are constant contours of $\bar{\psi}_{bt,0} + \beta y$.

References